Calabi-Yau Metrics and the Spectrum of the Laplacian

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Let's consider our favourite CY threefold:

$$Q = \left\{ z_0^5 + z_1^5 + z_2^5 + z_3^5 + z_4^5 = 0 \right\} \subset \mathbb{P}^4$$

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The metric is completely determined by the Kähler potential $K(z,\bar{z})$:

$$g_{i\bar{j}}(z,\bar{z}) = \partial_i \bar{\partial}_{\bar{j}} K(z,\bar{z})$$
$$\omega = g_{i\bar{j}}(z,\bar{z})\,\mathrm{d}z^i\,\mathrm{d}\bar{z}^{\bar{j}} = \partial\bar{\partial} K(z,\bar{z}).$$

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Locally, K is a real function.
ω is a $(1, 1)$-form.

Fubini-Study Metric

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$SU(5)$ acts on the 5 homogeneous coordinates.

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$SU(5)$ acts on the 5 homogeneous coordinates. Unique $SU(5)$ invariant Kähler metric comes from

$$K_{\mathrm{FS}} = \ln \sum_{i=0}^{4} z_i \bar{z}_{\bar{i}}$$

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$$K_{\mathrm{FS}} = \ln \sum_{i=0}^{4} z_i \bar{z}_{\bar{i}}$$

Generalize to

$$K_{\mathrm{FS}} = \ln \sum_{\alpha,\bar{\beta}=0}^{4} h^{\alpha\bar{\beta}} z_\alpha \bar{z}_{\bar{\beta}}$$

with h a hermitian 5×5 matrix.

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K_{FS} lives on $\mathbb{P}^4$, but we can restrict to $Q \subset \mathbb{P}^4$.

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Let's try [Donaldson]

$$K(z, \bar{z}) = \ln \sum_{\substack{\sum i_\ell = k \\ \sum \bar{j}_\ell = k}} h^{(i_1,\dots,i_k),(\bar{j}_1,\dots,\bar{j}_k)} \underbrace{z_1^{i_1} \cdots z_k^{i_k}}_{\text{degree } k} \underbrace{\bar{z}_1^{\bar{j}_1} \cdots \bar{z}_k^{\bar{j}_k}}_{\text{degree } k}$$

for some hermitian $N \times N$ matrix h
$N = \binom{5+k-1}{k} = \{\#\deg k \text{ monomials}\}$

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Let s_α be a basis for

$$\mathbb{C}[z_0, \ldots, z_4] \Big/ \left\langle z_0^5 + z_1^5 + z_2^5 + z_3^5 + z_4^5 = 0 \right\rangle \Big|_{\text{degree } k}$$

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and try this Ansatz for the metric on the quintic:

$$K(z, \bar{z}) = \ln \sum_{\alpha, \bar{\beta}} h^{\alpha\bar{\beta}} s_\alpha \bar{s}_{\bar{\beta}}$$

More Technical

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s_α: Sections of $\mathcal{O}_Q(k)$

$$0 \to H^0\Big(\mathbb{P}^4, \mathcal{O}(k-5)\Big) \to H^0\Big(\mathbb{P}^4, \mathcal{O}(k)\Big) \to H^0\Big(Q, \mathcal{O}_Q(k)\Big) \to 0$$

$h^{\alpha\bar\beta}$: Metric on the line bundle $\mathcal{O}_Q(k)$

$$(\sigma, \tau) \mapsto \frac{\sigma(z)\bar\tau(\bar z)}{\sum h^{\alpha\bar\beta} s_\alpha(z)\bar s_{\bar\beta}(\bar z)}$$

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Metric on the line bundle

$$(\sigma, \tau) \in C^\infty(Q, \mathbb{C})$$

gives a value at each point.

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Metric on the line bundle

$$(\sigma, \tau) \in C^\infty(Q, \mathbb{C})$$

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This defines a metric on the space of sections $H^0(Q, \mathcal{O}_Q(k))$:

$$\langle \sigma, \tau \rangle = \int_Q (\sigma, \tau)(z, \bar{z}) \, \mathrm{dVol}$$

(does not depend on points of Q)

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h is "balanced" if the matrices representing the metrics coincide, that is:

$$\Big(\big\langle s_\alpha, s_\beta \big\rangle\Big)_{1 \leq \alpha, \bar{\beta} \leq N} = h^{-1}$$

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$$\Big(\big\langle s_\alpha, s_\beta \big\rangle\Big)_{1\leq\alpha,\bar\beta\leq N} = h^{-1}$$

Theorem 1. *Let h be the balanced metric for each k. Then the sequence of metrics*

$$\omega_k = \partial\bar\partial \ln \sum h^{\alpha\bar\beta} s_\alpha \bar s_{\bar\beta}$$

converges to the Calabi-Yau metric as $k \to \infty$.

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h is "balanced" if the matrices representing the metrics coincide, that is:

$$\underbrace{\Big(\big\langle s_\alpha, s_\beta \big\rangle\Big)_{1\leq\alpha,\bar\beta\leq N}}_{\text{Depends nonlinearly on } h} = h^{-1}$$

Theorem 1. *Let h be the balanced metric for each k. Then the sequence of metrics*

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How to solve

$$\Big(\langle s_\alpha, s_\beta \rangle\Big)^{-1} = h?$$

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Donaldson's T-operator:

$$\begin{aligned} T(h)_{\alpha\bar\beta} &= \langle s_\alpha, s_\beta \rangle \\ &= \int_Q \frac{s_\alpha \bar s_{\bar\beta}}{\sum h^{\alpha\bar\beta} s_\alpha(z) \bar s_{\bar\beta}(\bar z)} \,\mathrm{dVol} \end{aligned}$$

T-Operator

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One can show that iterating $T(h_n)^{-1} = h_{n+1}$ converges! Fixed point is balanced metric.

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- Pick a basis of sections s_α

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- Pick a basis of sections s_α
- Iterate $h = T(h)^{-1}$ where

$$T(h)_{\alpha\bar{\beta}} = \int_Q \frac{s_\alpha \bar{s}_{\bar{\beta}}}{s_\alpha h^{\alpha\bar{\beta}} \bar{s}_{\bar{\beta}}} \mathrm{dVol}$$

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- The approximate Calabi-Yau metric is

$$g_{i\bar j} = \partial_i \bar\partial_{\bar j} \ln \sum s_\alpha h^{\alpha\bar\beta} \bar{s}_{\bar\beta}$$

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$$g_{i\bar{j}} = \partial_i \bar{\partial}_{\bar{j}} \ln \sum s_\alpha h^{\alpha\bar{\beta}} \bar{s}_{\bar{\beta}}$$

Runs easily on “our” 10 dual-core AMD Opteron cluster (Evelyn Thomson, ATLAS).

What is the Volume Form?

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The T-operator contains dVol:

$$T(h)_{\alpha\bar{\beta}} = \int_Q \frac{s_\alpha \bar{s}_{\bar{\beta}}}{s_\alpha h^{\alpha\bar{\beta}} \bar{s}_{\bar{\beta}}} \, \mathrm{dVol}$$

We could use the volume form computed from $h^{\alpha\bar{\beta}}$.

What is the Volume Form?

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$$T(h)_{\alpha\bar{\beta}} = \int_Q \frac{s_\alpha \bar{s}_{\bar{\beta}}}{s_\alpha h^{\alpha\bar{\beta}} \bar{s}_{\bar{\beta}}} \, \mathrm{dVol}$$

We could use the volume form computed from $h^{\alpha\bar{\beta}}$. But we actually know the *exact* Calabi-Yau volume form

$$\mathrm{dVol} = \Omega \wedge \bar{\Omega}, \qquad \Omega = \oint \frac{\mathrm{d}^4 x}{Q(x)}$$

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Defining coordinate patches would painful!

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Defining coordinate patches would painful! [Douglas,Karp,Lukic,Reinbacher]: Use random points $\{p_1, \ldots, p_N\}$ such that

$$\sum f(p_i)\frac{1}{N} \xrightarrow{N\to\infty} \int_Q f(x)\,\mathrm{dVol}$$

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$$\sum f(p_i) \frac{1}{N} \xrightarrow{N\to\infty} \int_Q f(x)\,\mathrm{dVol}$$

Pick "random" lines

$$\ell \simeq \mathbb{P}^1 \subset \mathbb{P}^4 \quad \Rightarrow \quad \ell \cap Q = \{5 \text{ pt}\}.$$

The "random" distribution of ℓ's determines the distribution of points!

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Its easy to make everything $SU(5)$-uniformly distributed. Then

$$\sum f(p_i)\frac{1}{N} \xrightarrow{N\to\infty} \int_Q f(x)\,\omega_{\mathrm{FS}}^3$$

by symmetry! But we want the Calabi-Yau volume form...

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$$\sum f(p_i)\frac{1}{N} \xrightarrow{N\to\infty} \int_Q f(x)\,\omega_{\mathrm{FS}}^3$$

by symmetry! But we want the Calabi-Yau volume form... So we have to weight the points by

$$\sum f(p_i)\underbrace{\left(\frac{\Omega\wedge\bar{\Omega}(p_i)}{\omega_{\mathrm{FS}}^3(p_i)}\right)}_{\in\mathbb{R}}\frac{1}{N} \xrightarrow{N\to\infty} \int_Q f(x)\,\mathrm{dVol}_{\mathrm{CY}}$$

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How do we test whether the metric is the Calabi-Yau metric? We could compute the Ricci tensor, but its easier to test that

$$\Omega \wedge \bar{\Omega} \sim \omega^3$$

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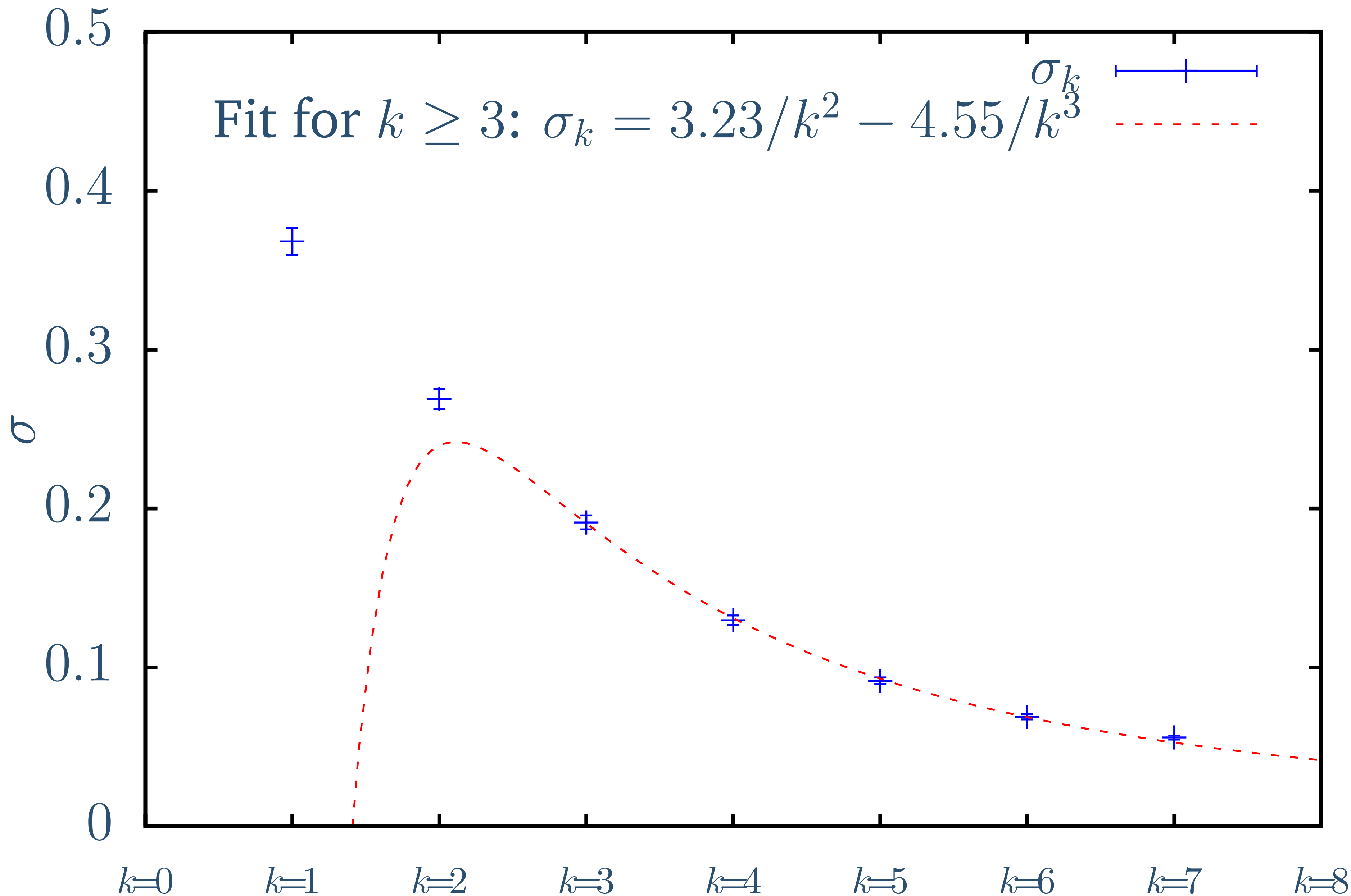
$$\Omega \wedge \bar{\Omega} \sim \omega^3$$

So normalize both volume forms and define

$$\sigma_k = \int_Q \left| 1 - \frac{\Omega(z) \wedge \bar{\Omega}(\bar{z})}{\omega^3(z, \bar{z})} \right| \mathrm{dVol}$$

On a Calabi-Yau manifold $\sigma_k = O(k^2)$

Resulting Plot



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The $\mathbb{Z}_5 \times \mathbb{Z}_5$ Quotient

Symmetric Quintics

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The Fermat quintic is part of a 5-dimensional family of quintics with a free $\mathbb{Z}_5 \times \mathbb{Z}_5$ group action.

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The Fermat quintic is part of a 5-dimensional family of quintics with a free $\mathbb{Z}_5 \times \mathbb{Z}_5$ group action.

It is numerically much easier to work on the four-generation quotient $Q \Big/ \left(\mathbb{Z}_5 \times \mathbb{Z}_5\right)$.

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It is numerically much easier to work on the four-generation quotient $Q \Big/ \big(\mathbb{Z}_5 \times \mathbb{Z}_5\big)$.

To do this, we only have to replace the sections s_α of $\mathcal{O}_Q(k)$ by invariant sections!

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$$g_1 \begin{pmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$$

$$g_2 \begin{pmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & e^{\frac{2\pi i}{5}} & 0 & 0 & 0 \\ 0 & 0 & e^{2\frac{2\pi i}{5}} & 0 & 0 \\ 0 & 0 & 0 & e^{3\frac{2\pi i}{5}} & 0 \\ 0 & 0 & 0 & 0 & e^{4\frac{2\pi i}{5}} \end{pmatrix} \begin{pmatrix} z_0 \\ z_1 \\ z_2 \\ z_3 \\ z_4 \end{pmatrix}$$

Note that $g_1 g_2 g_1^{-1} g_2^{-1} = e^{\frac{2\pi i}{5}}$, so they generate the Heisenberg group

$$0 \longrightarrow \mathbb{Z}_5 \longrightarrow G \longrightarrow \mathbb{Z}_5 \times \mathbb{Z}_5 \longrightarrow 0$$

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The invariant sections are

$$\mathbb{C}[z_0, z_1, z_2, z_3, z_4]^G$$

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The invariant sections are

$$\mathbb{C}[z_0, z_1, z_2, z_3, z_4]^G = \bigoplus_{i=0}^{100} \eta_i \mathbb{C}[\theta_1, \theta_2, \theta_3, \theta_4, \theta_5]$$

("Hironaka decomposition") where

$$\begin{aligned}
\theta_1 &\stackrel{\text{def}}{=} z_0^5 + z_1^5 + z_2^5 + z_3^5 + z_4^5 \\
\theta_2 &\stackrel{\text{def}}{=} z_0 z_1 z_2 z_3 z_4 \\
\theta_3 &\stackrel{\text{def}}{=} z_0^3 z_1 z_4 + z_0 z_1^3 z_2 + z_0 z_3 z_4^3 + z_1 z_2^3 z_3 + z_2 z_3^3 z_4 \\
\theta_4 &\stackrel{\text{def}}{=} z_0^{10} + z_1^{10} + z_2^{10} + z_3^{10} + z_4^{10} \\
\theta_5 &\stackrel{\text{def}}{=} z_0^8 z_2 z_3 + z_0 z_1 z_3^8 + z_0 z_2^8 z_4 + z_1^8 z_3 z_4 + z_1 z_2 z_4^8
\end{aligned}$$

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… and the "secondary invariants" η_i are polynomials in degrees 0, 5, 10, 15, 20, 25, 30:

$$\eta_1 \stackrel{\text{def}}{=} 1$$

$$\eta_2 \stackrel{\text{def}}{=} z_0^2 z_1 z_2^2 + z_1^2 z_2 z_3^2 + z_2^2 z_3 z_4^2 + z_3^2 z_4 z_0^2 + z_4^2 z_0 z_1^2$$

$$\vdots$$

$$\eta_{100} \stackrel{\text{def}}{=} z_0^{30} + z_1^{30} + z_2^{30} + z_3^{30} + z_4^{30}$$

All invariants are in degrees divisible by 5!

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&\vdots \\
\eta_{100} &\overset{\text{def}}{=} z_0^{30} + z_1^{30} + z_2^{30} + z_3^{30} + z_4^{30}
\end{aligned}
$$

All invariants are in degrees divisible by 5!

No invariant sections in $\mathcal{O}_Q(k)$ unless $5|k$?

Invariant vs. Equivariant

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$\mathcal{O}_Q(k)$ is not $\mathbb{Z}_5 \times \mathbb{Z}_5$-equivariant unless $5|k$.

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$\mathcal{O}_Q(k)$ is not $\mathbb{Z}_5 \times \mathbb{Z}_5$-equivariant unless $5|k$.

Under the quotient map $q : Q \to Q/(\mathbb{Z}_5 \times \mathbb{Z}_5)$,

$$q^*\Big(\mathcal{O}_{Q/(\mathbb{Z}_5\times\mathbb{Z}_5)}(1)\Big) = \mathcal{O}_Q(5)$$

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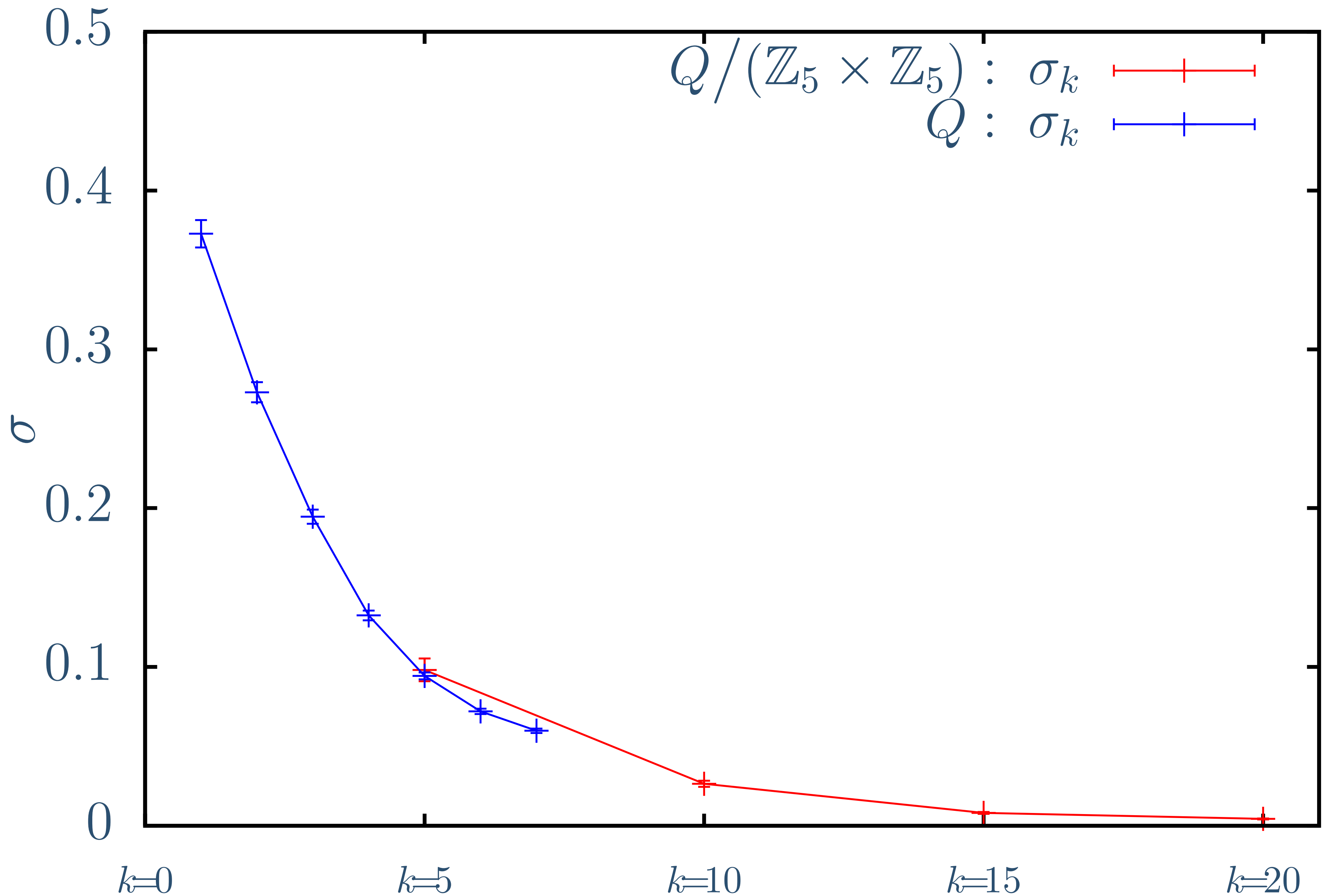
Under the quotient map $q : Q \to Q/(\mathbb{Z}_5 \times \mathbb{Z}_5)$,

$$q^*\Big(\mathcal{O}_{Q/(\mathbb{Z}_5\times\mathbb{Z}_5)}(1)\Big) = \mathcal{O}_Q(5)$$

The first Chern classes of bundles coming from the quotient are divisible by 5, that is,

$$q^* : \underbrace{H^2(Q/(\mathbb{Z}_5 \times \mathbb{Z}_5), \mathbb{Z})}_{\mathbb{Z}\oplus\mathbb{Z}_5^2} \xrightarrow{\times 5} \underbrace{H^2(Q, \mathbb{Z})}_{\mathbb{Z}}$$

Result



0.5
0.4
0.3
0.2
0.1
0
σ
$Q/(\mathbb{Z}_5 \times \mathbb{Z}_5)$: σ_k
Q : σ_k
k=0
k=5
k=10
k=15
k=20

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The Laplace-Beltrami Operator

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Just knowing the Calabi-Yau metric is useless!

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Would like to know the Eigenvalues and Eigenmodes of the Laplace operator.

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Just knowing the Calabi-Yau metric is useless!

Would like to know the Eigenvalues and Eigenmodes of the Laplace operator.

$\Rightarrow$ Complete KK reduction 10d $\longrightarrow$ 4d, including normalization of fields, numeric values of the Yukawa couplings, threshold corrections, and proton decay operators.

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For now, only the scalar Laplace operator

$$\Delta \big| \phi_i \big\rangle = \lambda_i \big| \phi_i \big\rangle$$

Matrix Elements

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In terms of some (non-orthogonal) basis of functions $\{f_s\}$, we can write

$$\big|\phi_i\big\rangle = \sum_t \big|f_t\big\rangle\big\langle f_t\big|\tilde{\phi}_i\big\rangle$$

and

$$\begin{aligned} \Delta\big|\phi_i\big\rangle &= \lambda_i\big|\phi_i\big\rangle \\ \Rightarrow \quad \big\langle f_s\big|\Delta\big|f_t\big\rangle \underbrace{\big\langle f_t\big|\tilde{\phi}_i\big\rangle}_{\vec{v}} &= \lambda_i\big\langle f_s\big|f_t\big\rangle \underbrace{\big\langle f_t\big|\tilde{\phi}_i\big\rangle}_{\vec{v}} \end{aligned}$$

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Using an approximate finite basis $\{f_s\}$, we only have to solve the generalized Eigenvalue problem

$$\langle f_s | \Delta | f_t \rangle \, \vec{v} = \lambda_i \langle f_s | f_t \rangle \, \vec{v}$$

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$$\langle f_s \big| \Delta \big| f_t \rangle \, \vec{v} = \lambda_i \langle f_s \big| f_t \rangle \, \vec{v}$$

Nice basis: Recall that $\mathbb{P}^4 = S^7/U(1)$
So take the $U(1)$-invariant spherical harmonics on S^7.

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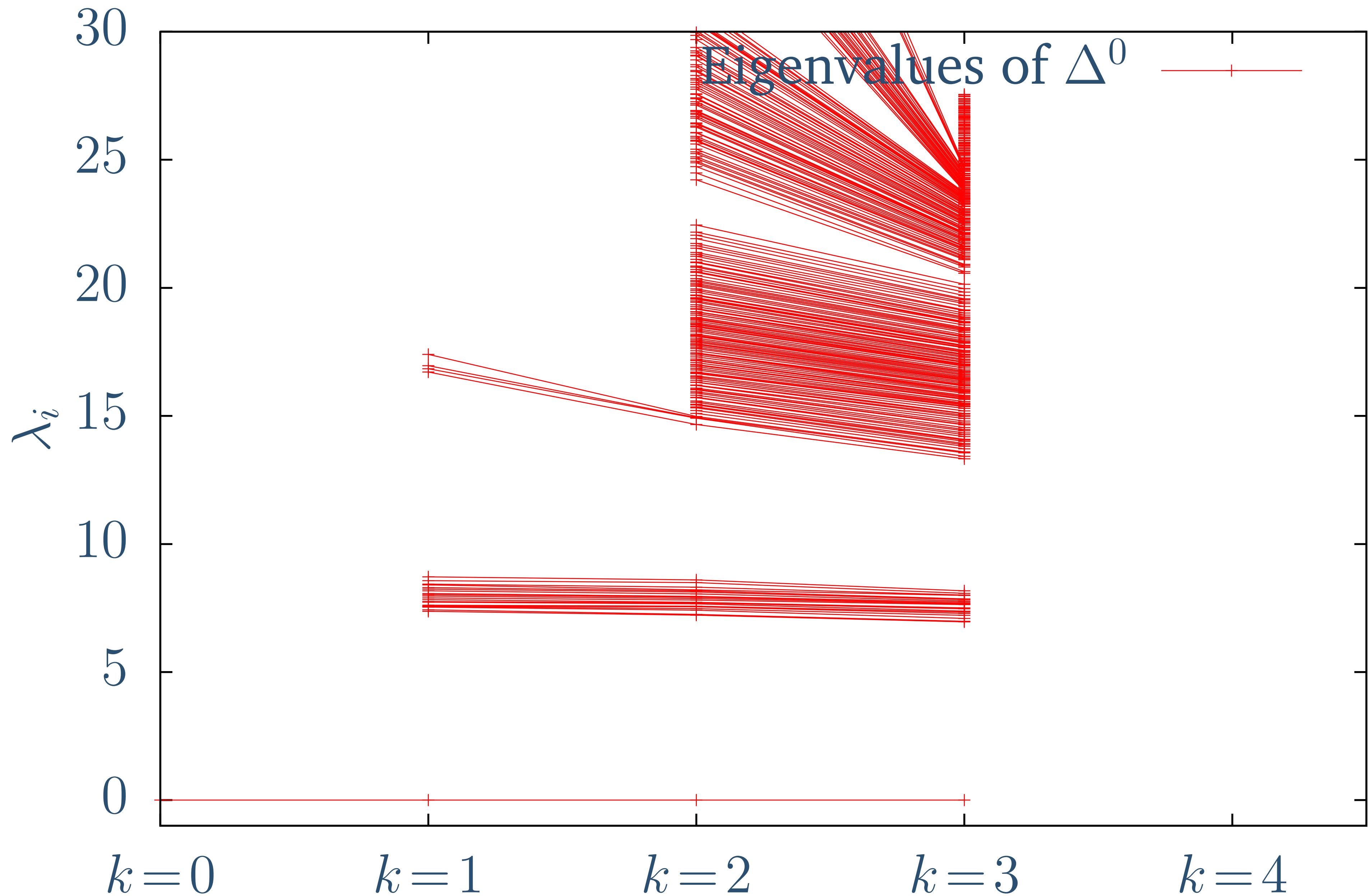
In homogeneous coordinates, the spherical harmonics are

$$\frac{\big(\text{degree } k \text{ monomial}\big)\overline{\big(\text{degree } k \text{ monomial}\big)}}{\big(|z_0|^2+|z_1|^2+|z_2|^2+|z_3|^2+|z_4|^2\big)^k}$$

So, for example $k=1$ on $\mathbb{P}^1$:

Homog.	$\frac{z_0\bar{z}_0}{\lvert z_0\rvert^2+\lvert z_1\rvert^2}$	$\frac{z_1\bar{z}_0}{\lvert z_0\rvert^2+\lvert z_1\rvert^2}$	$\frac{z_0\bar{z}_1}{\lvert z_0\rvert^2+\lvert z_1\rvert^2}$	$\frac{z_1\bar{z}_1}{\lvert z_0\rvert^2+\lvert z_1\rvert^2}$
Inhomog.	$\frac{1}{1+\lvert x\rvert^2}$	$\frac{x}{1+\lvert x\rvert^2}$	$\frac{\bar{x}}{1+\lvert x\rvert^2}$	$\frac{x\bar{x}}{1+\lvert x\rvert^2}$

Result from Matrix Elements



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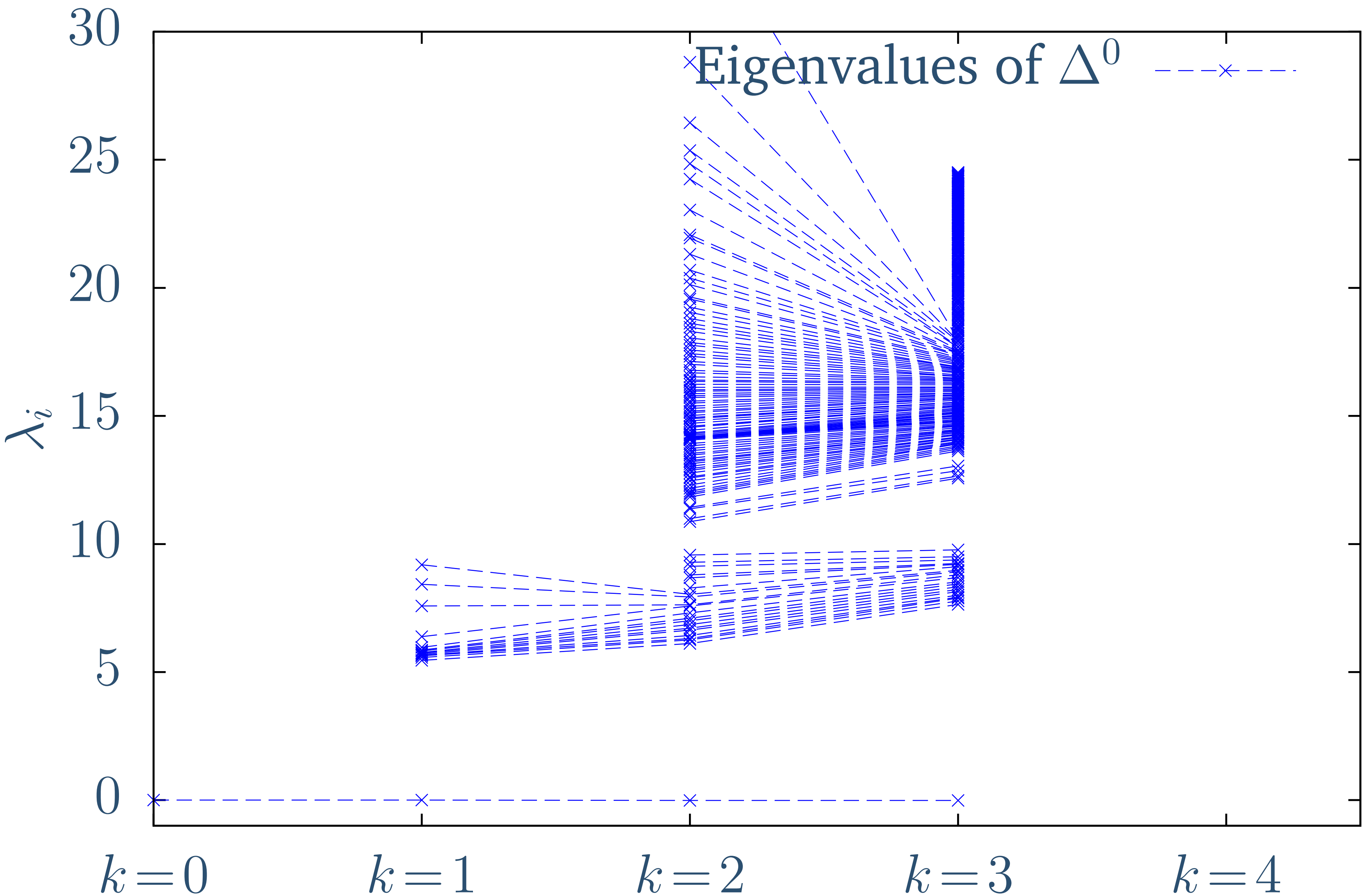
Donaldson originally already proposed a different way to compute the Eigenmodes of the scalar Laplacian.

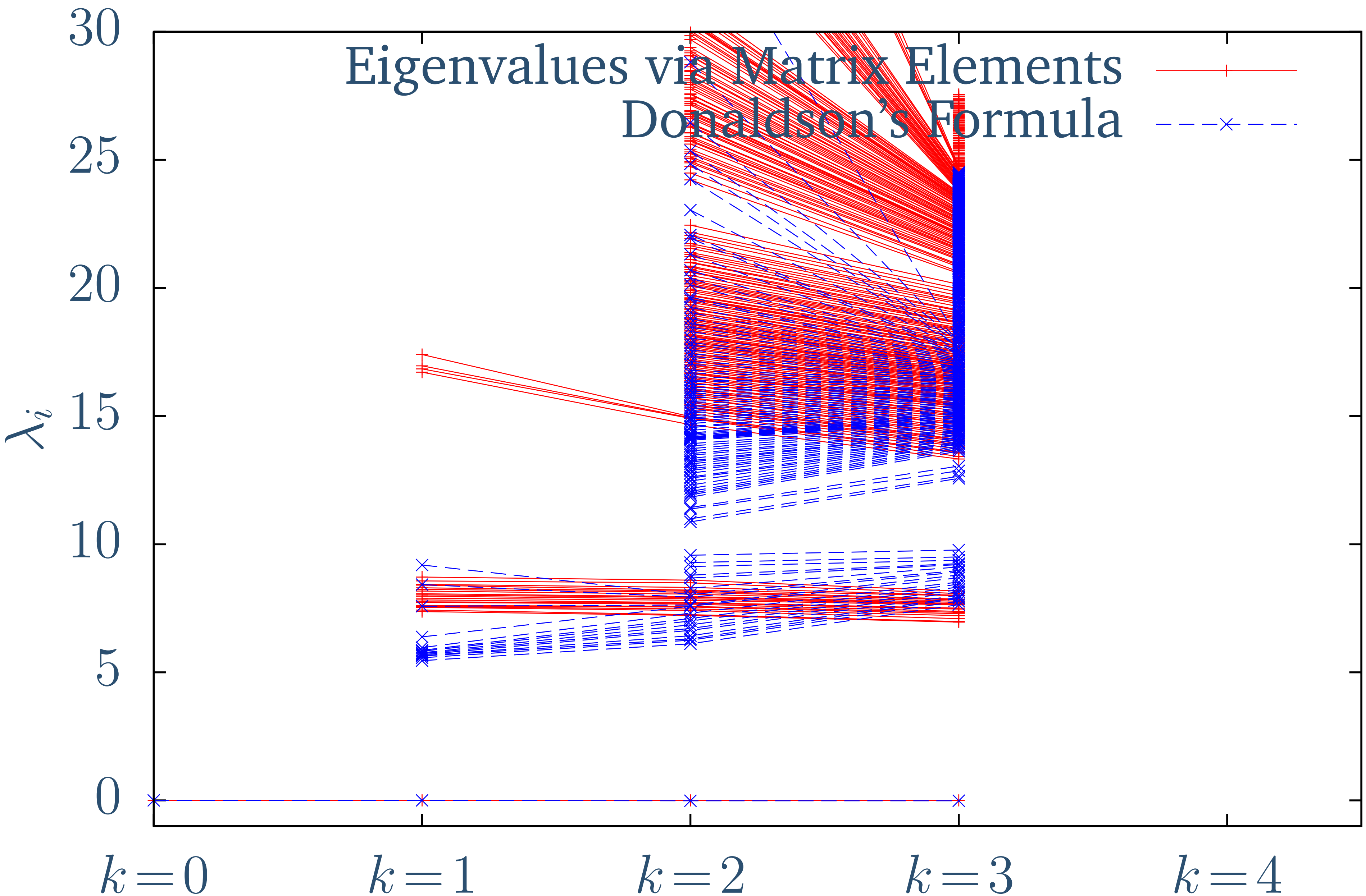
It does not generalize to the Laplacian on differential forms.

Nevertheless interesting to compare to!

Donaldson's Formula



Results Combined



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The first massive Eigenmode seems to have degeneracy 20.

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This can be explained partially by symmetry, the Fermat quintic has the discrete symmetry group $G_F = S_5 \ltimes \mathbb{Z}_4$ of order 75000.

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The first massive Eigenmode should transform in one of the 106 irreps:

Dimension d	1	4	5	6	20	30	40	120
Irreps in dim d	10	10	10	5	20	40	10	1

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Dimension d	1	4	5	6	20	30	40	120
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There are irreps only in eight different dimensions, and 20 is one of these possibilities.

Varying Moduli

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Consider the one-parameter family of $\mathbb{Z}_5 \times \mathbb{Z}_5$-symmetric quintics:

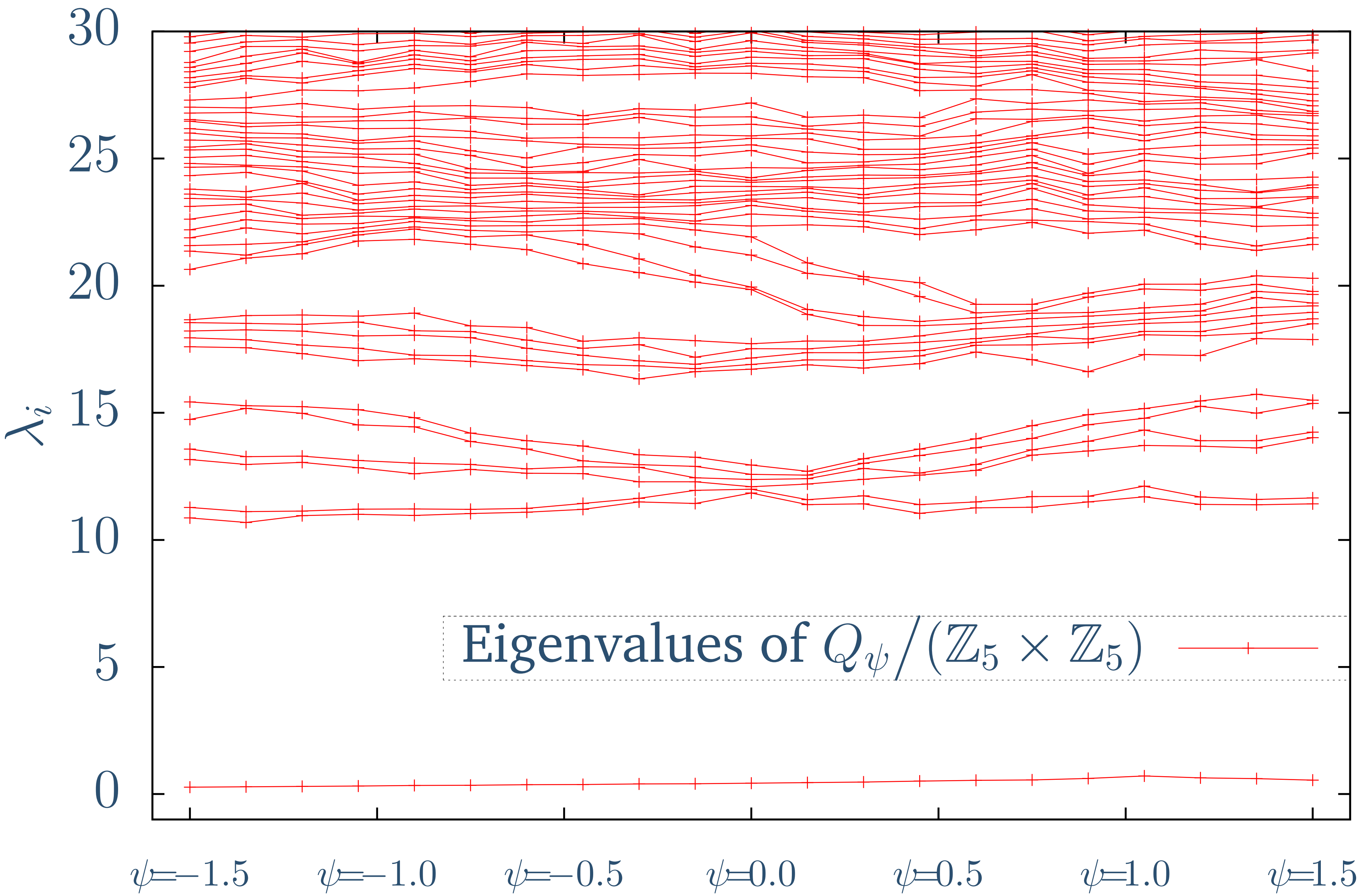
$$Q_\psi = \sum_{i=0}^{4} z_i^5 - 5\psi \prod_{i=0}^{4} z_i$$

For any given ψ, we can compute the spectrum of the Laplace operator.

Work on the quotient $Q_\psi / (\mathbb{Z}_5 \times \mathbb{Z}_5)$.

Moduli Space

$$Q_\psi = \sum z_i^5 - 5\psi \prod z_i$$



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The first massive Eigenvalue of the *scalar* Laplacian on a Calabi-Yau manifold is

$$\frac{\pi^2}{D^2} \leq \lambda_1 \leq \frac{2d(d+4)}{D^2}$$

where d is the dimension and D the diameter.

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where d is the dimension and D the diameter.

More precisely:

$$\frac{1}{4}h^2 \leq \lambda_1 \leq (\text{const.})\left(\rho h + h^2\right)$$

where h is Cheeger's isoperimetric constant and ρ is the minimal Ricci curvature.

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- We can now compute the Calabi-Yau metric numerically (including CICY).

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- We can now compute the Calabi-Yau metric numerically (including CICY).
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- Vector bundles, fluxes, sLag's, ...