Exploring Holographic Approaches to QCD

Umut Gürsoy

CPhT, École Polytechnique
LPT, École Normale Supérieure

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Physics of Strong Interactions

- QCD perturbation theory, in the UV
- Non-perturbative phenomena in the IR
- Lattice QCD
Physics of Strong Interactions

- QCD perturbation theory, in the UV
- Non-perturbative phenomena in the IR
  Lattice QCD
- Dynamical phenomena, finite Temperature, real-time correlation functions,
  Applications to RHIC physics:
  Holographic approaches
  String theory may have real impact!
Physics of Strong Interactions

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- Non-perturbative phenomena in the IR
  Lattice QCD
- Dynamical phenomena, finite Temperature,
  real-time correlation functions,
  Applications to RHIC physics:
  Holographic approaches
  String theory may have real impact!

QCD in this talk:
- Pure Yang-Mills at $N_c \gg 1$
- QCD in the quenced limit: $N_f/N_c \ll 1$
Holographic Approaches to QCD

“TOP - BOTTOM APPROACH”

- 10D critical string theory
- D-brane configurations
- Decoupling limit of open and closed string sectors
- Treatable in the supergravity limit, $\ell_s \to 0$
Holographic Approaches to QCD

“TOP - BOTTOM APPROACH”

- 10D critical string theory
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EXAMPLES

- Klebanov-Strassler, Polchinski-Strassler, Maldacena-Nunez, orbifold constructions, etc. for $\mathcal{N} = 1, 2$
gauge theories
- Witten’s model for pure Yang-Mills
- $YM_5$ on D4 Branes
- Antiperiodic boundary conditions on for the fermions on $S^1$: $m_\psi \sim \frac{1}{R}$, $m_\phi \sim \frac{\lambda}{R}$
- UV cut-off in the 4D theory at $E = 1/R$
- Pure $YM_4$ in the IR
Witten’s model

\[ \lambda_4 = \frac{\lambda_5}{R} \]

UV cut-off

IR physics

UV physics

Pure YM

Weak curvature

Confinement, mass gap

Mixing with KK

High curvature corrections

No asymptotic freedom
Witten’s model

- Pure $YM_4$
- $\lambda_0 \gg 1$
- Weak curvature
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Witten’s model

- Pure $YM_4$
  - $\lambda_0 \gg 1$
  - Weak curvature
  - Confinement, mass gap
- Mixing with KK
  - $\lambda_0 \ll 1$
  - High curvature corrections
  - No asymptotic freedom
Confining Gauge Theories in the SG Regime

- UV and the IR physics disconnected
  - Effects of the logarithmic running not captured

- Mixing of pure gauge sector with the KK sector
  - To disentangle need $\lambda_0 \ll 1$
  - Then $\ell_s$ corrections!

- Problems with the glueball spectra (Witten ’98, Ooguri et al. ’98; )
The glueball spectra

Normalizable modes of $\phi(r, \vec{x}) = f(r)e^{i\vec{k} \cdot \vec{x}}$

$\Leftrightarrow$ spectrum of $\mathcal{O}(\vec{x}|\text{vac})$

KK like spectra $m_n^2 \propto n^2$ for $n \gg 1$!
Solution to problems

- Full $\sigma$-model $S_{WS}[\lambda_0]$ on Wittens’s background $M_{10}$

- Compute corrections as $M_{10}[\lambda_0]$
  $\Rightarrow$ Compute e.g. the glueball spectra perturbatively in $\lambda_0^{-1}$

- Generally need to sum over all series

- 2D lattice for $S_{WS}[\lambda_0]$?

VERY HARD OPEN PROBLEM
General Lessons: confining gauge theories

- Effects of either KK modes or $\ell_s$ corrections
- UV completion non-unique
- Only low lying excitations in QCD, up to spin 2
General Lessons: confining gauge theories

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- Only low lying excitations in QCD, up to spin 2
- Still certain quantities receive very little corrections. *e.g.* glueball mass ratios, $m_{0^{++}}/m_{0^{--}}$, etc. Ooguri et al. ’98
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IS IT POSSIBLE TO CONSTRUCT AN EFFECTIVE THEORY FOR THE IR PHYSICS OF LOW LYING EXCITATIONS BY PARAMETERIZING THE UV REGION?

- tunable parameters
- Fixed by input from gauge theory + experiment (or lattice)
The Bottom Up approach

Build an effective theory for the lowest lying excitations by introducing MINIMUM ingredients
The Bottom Up approach

Build an effective theory for the lowest lying excitations by introducing MINIMUM ingredients

Guideline

- insights from AdS/CFT:
  - Space-time symmetries $T_{\mu\nu} \Leftrightarrow g_{\mu\nu}$
  - Energy $\Leftrightarrow$ radial direction $r$
  - $\Lambda_{QCD} \Leftrightarrow$ broken translation invariance in $r$

5D space-time $ds^2 = e^{2A(r)} \left(dx^2 + dr^2\right)$
The Bottom Up approach

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- insights from SVZ sum rules
  - Non-perturbative effects through glueball condensates
  - e.g. $\langle Tr F^2 \rangle$, $\langle Tr F \wedge F \rangle$, etc.
The Bottom Up approach

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    e.g. $\langle \text{Tr} F^2 \rangle$, $\langle \text{Tr} F \wedge F \rangle$, etc.

- insights from 5D non-critical string theory
Simplest model: $AdS/QCD$

Polchinski-Strassler ’02; Erlich et al. ’05; Da Rold, Pomarol ’05

$AdS_5$ with an IR cut-off

- color confinement
- mass gap
- $\Lambda_{QCD} \sim \frac{1}{r_0}$
Simplest model: $AdS/QCD$

AdS$_5$ with an IR cut-off

- color confinement
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- Mesons by adding $D4 - \overline{D4}$ branes in probe approximation
- Fluctuations of the fields on $D4$: meson spectrum
  e.g. $A^L_\mu + A^R_\mu \leftrightarrow$ vector meson spectrum
- surprisingly successful: certain qualitative features, meson spectra $\%13$ of the lattice
Problems

- No running gauge coupling, no asymptotic freedom
- Ambiguity with the IR boundary conditions at $r_0$
- No linear confinement, $m_n^2 \sim n^2$, $n \gg 1$
- No magnetic screening
  - Soft wall models Karch et al. ’06
  - $AdS_5$ with dilaton $\phi(r) \Leftrightarrow$ linear confinement
- No gravitational origin, not solution to diffeo-invariant theory
- No obvious connection with string theory
Our Purpose

- Use insight from non-critical string theory
- Construct a 5D gravitational set-up
- To parametrize the $\ell_s$ corrections in the UV, use to gauge theory input $\beta$-function
- Classify and investigate the solutions
- Improvement on $AdS/QCD$
- Gain insights for possible mechanisms in QCD
- General, model independent results:
  - Color confinement ⇔ mass gap
  - A proposal for the strong CP problem ??
  - Finite Temperature physics
Outline

- Two derivative effective action in 5D, $S[g, \phi, a]$
- Constrain small $\phi$ asymptotics, asymptotic freedom
- UV physics parametrized by the perturbative $\beta$-function
  - $\beta$-function $\leftrightarrow$ superpotential
  - $\ell_s$-corrections $\leftrightarrow$ scheme-dependent $\beta$-coefficients
- Constrain large $\phi$ asymptotics, color confinement
- Fluctuations in $g, \phi, a$, the glueball spectrum
- Mesons
- Axion sector
- Discussion, Finite Temperature results, Outlook
Construction of the effective action

- **Ingredients in $S_{eff}$**

  Pure $YM_4$

  $\left\{\begin{array}{l}
  SU(N_c) \iff F_5 \propto N_c \\
  g_{YM} \iff e^\phi \\
  \theta_{YM} \iff a
  \end{array}\right.$
Construction of the effective action

- **Ingredients in** $S_{\text{eff}}$

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  &SU(N_c) \quad \Leftrightarrow \quad F_5 \propto N_c \\
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  \]

- **Two-derivative action** Klebanov-Maldacena ’04

  \[
  S_S = M^3 \int d^5x \sqrt{g_S} \left\{ e^{-2\phi} [R + (\partial\phi)^2 - \frac{\delta c}{\ell^2_S}] - F_5^2 - F_1^2 \right\}
  \]
Construction of the effective action

- **Ingredients in \( S_{\text{eff}} \)**

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\]

Define \( \lambda \equiv N_c e^\phi \equiv e^\Phi \),
go to the Einstein frame \( g_s = \lambda^{\frac{4}{3}} g_E \)
dualize \( F_5 \)
Einstein frame Action

The action in the Einstein frame,

\[ S_E = M^3 N_c^2 \int d^5 x \sqrt{g_E} \left\{ R + (\partial \Phi)^2 - V(\Phi) - N_c^{-2} F_1^2 \right\} \]
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- \( N_c \) appears as an overall factor. **String-loop corrections are small in the large** \( N_c \) **limit.**
- Axion suppressed by \( N_c^{-2} \). Do not back-react on the geometry.
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The naive dilaton potential

\[ V(\lambda) = \frac{\chi^4}{\ell_s^2} (1 - \lambda^2) \]

- The potential is of order \( \ell_s \leftrightarrow \) string \( \sigma \)-model corrections are substantial.
The naive potential

\[ V(\lambda) \]

END OF AN RG FLOW?

\[ \lambda_{\text{do}} \]

It cannot be UV end of an RG-flow

Asymptotic freedom, Asymptotic AdS in the UV

The naive theory is not rich enough to describe QCD RG-flow
The naive potential

- Fluctuation analysis near $\lambda_0 \Rightarrow$ no dimension 4 operator, $\text{Tr} F^2$
  It can not be UV end of an RG-flow
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  $$V(\lambda) \to V_0 + V_1 \lambda + \cdots \quad \text{as} \quad \lambda \to 0$$
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String corrections to the naive potential

- Higher derivative corrections to the $F_5$ kinetic term (after going to the Einstein frame and dualizing)

\[ F_5^2 \Rightarrow \sum_n F_5^{2n} \lambda^{2(n-1)} a_n \quad \text{as} \quad \lambda \to 0 \]

with unknown coefficients $a_n$. 
String corrections to the naive potential

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- The naive potential is corrected as,

$$V(\lambda) = \frac{\lambda^4}{3\ell^2} \sum_{n=0}^{\infty} a_n \lambda^{2n}$$

Still no constant term $V_0$
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- The naive potential is corrected as,

\[ V(\lambda) = \frac{\lambda^{\frac{4}{3}}}{\ell_s^2} \sum_{n=0}^{\infty} a_n \lambda^{2n} \]

Still no constant term $V_0$

- $V_0$ can be generated through higher derivative corrections to $R$
eq\text{e.g. in an } f(R) \text{ type gravity}  

U.G. E. Kiritsis ’07
Hard to obtain general form of curvature corrections

We adopt a phenomenological approach and take a bold step:
General action

Hard to obtain general form of curvature corrections

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The sole effect of the curvature corrections near $\lambda \ll 1$ is to generate $V_0$ and modify the coefficients $a_n$. 
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Give up a “derivation” from NCST and simply conjecture:

$$S_E = M^3 N_c^2 \int d^5 x \sqrt{g} \left\{ R + (\partial \Phi)^2 - V(\lambda) - \frac{Z_a(\lambda)}{N_c^2}(\partial a)^2 \right\}$$
General action

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with a conjectured dilaton potential:

$$V(\lambda) = \sum_{n=0}^{\infty} V_n \lambda^n$$
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with a conjectured dilaton potential:

$$V(\lambda) = \sum_{n=0}^{\infty} V_n \lambda^n$$

- $V_n$ include corrections to $F_5$. We will relate $V_n$ to $\beta$-coefficients
Properties of general solutions will be:

- AdS in the UV, $\lambda \to 0$

- Curvature singularity in the IR, $\lambda \to \infty$
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Construct the theory and justify a posteriori:
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Construct the theory and justify a posteriori:

- \[ \frac{\ell_{AdS}}{\ell_s} \gg 1 \]
Properties of general solutions will be:

- AdS in the UV, $\lambda \to 0$
- Curvature singularity in the IR, $\lambda \to \infty$

Construct the theory and justify a posteriori:

- $\frac{\ell_{AdS}}{\ell_s} \gg 1$ ?
- Is the singularity of repulsive type? Does the strings or particles probe in the singular region?
Solutions to the Einstein-dilaton system

Look for solutions domain-wall type of solutions

\[ ds^2 = e^{2A(u)} dx^2 + du^2, \quad \Phi = \Phi(u) \]

- \( A(u) \to -\frac{u}{\ell} \) as \( u \to -\infty \), \( \Phi(u) \to -\infty \) as \( u \to -\infty \)
Solutions to the Einstein-dilaton system

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The superpotential

- \( V = W^2 - \left( \frac{\partial W}{\partial \Phi} \right)^2, \quad A' = -W, \quad \Phi' = \frac{\partial W}{\partial \Phi} \)
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Three integration constants:

- Asymptotic freedom, \( V \to V_0 \), get rid of one
- Reparametrization invariance \( u \to u + \delta u \) get rid of another
Solutions to the Einstein-dilaton system

Look for solutions domain-wall type of solutions

\[ ds^2 = e^{2A(u)} dx^2 + du^2, \quad \Phi = \Phi(u) \]

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Three integration constants:

- Asymptotic freedom, \( V \to V_0 \), get rid of one
- Reparametrization invariance \( u \to u + \delta u \) get rid of another

A single integration constant in the system:

\[ A_0 \leftrightarrow \Lambda_{QCD} \text{ in the gauge theory} \]
Holographic dictionary I

ENERGY ⇔ SCALE FACTOR

\[ ds^2 = e^{2A(r)} \left( dx^2 + dr^2 \right) \]
Holographic dictionary I

ENERGY $\Leftrightarrow$ SCALE FACTOR

$$ds^2 = e^{2A(r)} \left( dx^2 + dr^2 \right)$$

- Measures the energy of gravitational excitations observed at the boundary $r = 0$
- Monotonically decreasing function
- Agrees with the AdS/CFT relation $E = 1/r$ near boundary

We propose $E = \exp A$
Holographic dictionary I

**ENERGY ⇔ SCALE FACTOR**

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- Monotonically decreasing function
- Agrees with the AdS/CFT relation \( E = 1/r \) near boundary

We propose \( E = \exp A \)

- **String corrections:**
  If replace \( R \rightarrow f(\ell_s^2 R) = \sum_n f_n R^n \), with unknown \( f_n \),

\[
\frac{d \log E}{dA} = 1 + f_1 \lambda^2 + f_2 \lambda^4 + \cdots
\]
’t HOOFT COUPLING ⇔ DILATON
Insert a probe D3 brane in the geometry. The DBI action:

\[ S_{D3} \propto \int e^{-\Phi} F^2 \quad \Rightarrow \quad \lambda = e^{\Phi} = g_{YM}^2 N_c = \lambda_t \]
Holographic dictionary II

’t HOOFT COUPLING $\leftrightarrow$ DILATON

Insert a probe D3 brane in the geometry. The DBI action:

$$S_{D3} \propto \int e^{-\Phi} F^2 \quad \Rightarrow \quad \lambda = e^{\Phi} = g_{YM}^2 N_c = \lambda_t$$

String corrections:

Higher order couplings of $F_5$ to $D3$ probe brane

$$S_{D3} = \frac{T_3}{\ell_s^4} \int \sqrt{g} e^{-\phi} Z(e^{2\phi} F_5^2) F^2$$

with $Z(x) = 1 + \sum_{n=1}^{\infty} c_n x^n$

The identification receives corrections as,

$$\lambda_t = \lambda (1 + c_1 \lambda^2 + c_2 \lambda^4 + \cdots)$$
If one ignores the string corrections:

\[ \beta(\lambda) = \frac{d\lambda}{d \ln E} = -b_0 \lambda^2 + b_1 \lambda^3 + \cdots = -\frac{9}{4} \lambda^2 \frac{d \ln W(\lambda)}{d\lambda} \]
\( \beta \)-FUNCTION ⇔ SUPERPOTENTIAL

- If one ignores the string corrections:

\[
\beta(\lambda) = \frac{d\lambda}{d \ln E} = -b_0 \lambda^2 + b_1 \lambda^3 + \cdots = -\frac{9}{4} \lambda^2 \frac{d \ln W(\lambda)}{d\lambda}
\]

- One can solve for the dilaton potential

\[
V(\lambda) = V_0 \left( 1 - \left( \frac{\beta}{3\lambda} \right)^2 \right) e^{\int_0^\lambda \frac{d\lambda \beta}{3\lambda}}
\]
\[ \beta(\lambda) = \frac{d\lambda}{d \ln E} = -b_0 \lambda^2 + b_1 \lambda^3 + \cdots = -\frac{9}{4} \lambda^2 \frac{d \ln W(\lambda)}{d \lambda} \]

\[ V(\lambda) = V_0 \left( 1 - \left( \frac{\beta}{3\lambda} \right)^2 \right) e^{\int_0^\lambda \frac{d\lambda}{3\lambda} \beta} \]

- One-to-one correspondence between the coefficients \( V_n \) and \( b_n \). We parameterize our ignorance about the UV part of the dual geometry by the \( b_n \).
The string corrections:

\[ \beta_t \rightarrow \beta_{st} = -b_0 \lambda^2 + b_1 \lambda^3 \]
\[ + (b_2 - 4c_1 b_0 + f_1 b_0) \lambda^4 \]
\[ + (b_3 + 4c_1 b_0 - f_1 b_1) \lambda^5 + \cdots \]

\[ c_n \] from corrections to the probe brane, and the \( F_5 \) kinetic term

\[ f_n \] from curvature corrections

\[ \ell_s \] corrections appear with the scheme dependent \( \beta \)-coefficients!
Geometry near the boundary

For $\beta = -b_0 \lambda^2 + b_1 \lambda^3 + \cdots$,

- The dilaton

$$b_0 \lambda = -\frac{1}{\log r \Lambda} + \frac{b_1}{b_0^2} \frac{\log(-\log r \Lambda)}{\log^2(r \Lambda)} + \cdots$$

- The scale factor $ds^2 = e^{2A}(dx^2 + dr^2)$

$$e^{2A} = \frac{\ell^2}{r^2} \left( 1 + \frac{8}{9} \frac{1}{\log(r \Lambda)} - \frac{8}{9} \frac{b_1}{b_0^2} \frac{\log(-\log r \Lambda)}{\log^2(r \Lambda)} + \cdots \right)$$

- AdS with logarithmic corrections. Subleading term is model independent.
Geometry near the boundary

For $\beta = -b_0 \lambda^2 + b_1 \lambda^3 + \cdots$,

- The dilaton

$$b_0 \lambda = -\frac{1}{\log r \Lambda} + \frac{b_1}{b_0^2} \frac{\log(-\log r \Lambda)}{\log^2(r \Lambda)} + \cdots$$

- The scale factor $ds^2 = e^{2A}(dx^2 + dr^2)$

$$e^{2A} = \frac{\ell^2}{r^2} \left( 1 + \frac{8}{9} \frac{1}{\log(r \Lambda)} - \frac{8}{9} \frac{b_1}{b_0^2} \frac{\log(-\log r \Lambda)}{\log^2(r \Lambda)} + \cdots \right)$$

- AdS with logarithmic corrections. Subleading term is model independent.

Holographic renormalization Bianchi, Freedman, Skenderis ’01

AdS with log-corrections, ongoing with E. Kiritsis, Y. Papadimitriou
Geometry in the interior

For $A(r) \to \frac{\ell'}{r}$ as $r \to 0$, Einstein’s equations lead to the following IR behaviours:

- **AdS**, $A(r) \to \frac{\ell'}{r}$ with $l' \leq l$
- Singularity (in the Einstein frame) at a finite point $r = r_0$
- Singularity at infinity $r = \infty$
Geometry in the interior

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- AdS, \( A(r) \to \frac{l'}{r} \) with \( l' \leq l \)
- Singularity (in the Einstein frame) at a finite point \( r = r_0 \)
- Singularity at infinity \( r = \infty \)

Phenomenologically preferred asymptotics

- color confinement
- magnetic screening
- linear spectra \( m_n^2 \sim n \) for large \( n \)
Constraints on the IR geometry

Color confinement

J Maldacena ’98; S. Rey, J. Yee ’98

Solve for the string embedding and compute its action:

\[ E_{q\bar{q}} T = S_{WS} \]
String action: \[ S_{WS} = \ell_s^{-2} \int \sqrt{\text{det} g_{ab}} + \int \sqrt{\text{det} g_{ab} R^{(2)}} \Phi(X) \]
in the string frame, \[ g_{ab} = g^S_{\mu\nu} \partial_a X^\mu \partial_b X^\nu . \]
String action: \( S_{WS} = \ell_s^{-2} \int \sqrt{\text{det} g_{ab}} + \int \sqrt{\text{det} g_{ab}} R^{(2)} \Phi(X) \)

in the string frame, \( g_{ab} = g^S_{\mu\nu} \partial_a X^\mu \partial_b X^\nu \).

- Coupling to dilaton bounded as \( L \to \infty \), linear potential,
  if at least one minimum of \( A_S = A_E + \frac{2}{3} \Phi \)
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- Similar consideration for the magnetic charges, using probe D1
Confining backgrounds

- Space ending at finite \( r_0 \)
- Space ending at \( r = \infty \) with metric vanishing as \( e^{-Cr} \) or faster
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In terms of the superpotential, a diffeo-invariant characterization:

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**Confinement $\Leftrightarrow Q > 2/3 \text{ or } Q = 2/3, P > 0$**

The phenomenologically preferred backgrounds for infinite $r$:

$$A \sim -Cr^\alpha \quad \Leftrightarrow \quad Q = 2/3, P = \frac{\alpha - 1}{\alpha}$$

Linear confinement in the glueball spectrum for $\alpha = 2$

- Borderline case $\alpha = 1$ is linear dilaton background!
Glueballs

Spectrum of 4D glueballs $\Leftrightarrow$ Spectrum of normalizable fluctuations of the bulk fields.
Spin 2: $h_{\mu\nu}^{TT}$; Spin 0: mixture of $h_\mu^\mu$ and $\delta\Phi$; Pseudo-scalar: $\delta a$. 
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Quadratic action for fluctuations:

$$S \sim \frac{1}{2} \int d^4x dr e^{2B(r)} \left[ \ddot{\zeta}^2 + (\partial_\mu \zeta)^2 \right]$$

$$\ddot{\zeta} + 3\dot{B}\dot{\zeta} + m^2\zeta = 0, \quad \partial^\mu \partial_\mu \zeta = -m^2\zeta$$
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- Scalar: $B(r) = 3/2A(r) + \log(\dot{\Phi}/\dot{A})$
- Tensor: $B(r) = 3/2A(r)$
- Pseudo-scalar: $B(r) = 3/2A(r) + 1/2 \log Z_A$
Reduction to a Schrödinger problem

Define: \( \zeta(r) = e^{-B(r)} \Psi(r) \) Schrödinger equation:

\[
\mathcal{H}\Psi = -\ddot{\Psi} + V(r)\Psi = m^2 \Psi \quad V_s(r) = \dot{B}^2 + \ddot{B}
\]
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Schrödinger equation:

\[
\mathcal{H} \Psi = -\ddot{\Psi} + V(r) \Psi = m^2 \Psi \quad V_s(r) = \dot{B}^2 + \ddot{B}
\]

- The normalizability condition: \( \int dr \ |\Psi|^2 < \infty \)
- Normalizability in the UV, picks normalizable UV asymptotics for \( \zeta \)
- Normalizability in the IR, restricts discrete \( m^2 \), for confining \( V_s \).
Mass gap

\[ \mathcal{H} = (\partial_r + \partial_r B)(-\partial_r + \partial_r B) = \mathcal{P}^\dagger \mathcal{P} \geq 0 : \]

- Spectrum is non-negative
- Can prove that no normalizable zero-modes
- If \( V(r) \to \infty \) as \( r \to +\infty \): Mass Gap
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- Can prove that no normalizable zero-modes
- If \( V(r) \to \infty \) as \( r \to +\infty \): Mass Gap
- This precisely coincides with the condition from color confinement
- \( \text{e.g. for the infinite geometries } A(r) \to -Cr^\alpha : \) color confinement AND mass gap for \( \alpha \geq 1 \).
Choose a specific model. Take a superpotential such that

\[
W \sim \begin{cases} 
W_0 \left(1 + \frac{4}{3} b_0 \lambda + \ldots\right) & \lambda \to 0 \\
W_0 \lambda^{2/3} (\log \lambda)^{1/4} & \lambda \to \infty \quad (\alpha = 2)
\end{cases}
\]

For example:

\[
W = \left(1 + \frac{2}{3} b_0 \lambda\right)^{2/3} \left[1 + \frac{4(2b_0^2 + 3b_1)}{9} \log(1 + \lambda^2)\right]^{1/4}
\]

Then, compute numerically metric, dilaton, mass spectrum.

- Parameters of the model: \(b_0\) and \(A_0\). We fix \(b_1/b_0^2 = 51/121\), pure YM value.
Comparison with one lattice study  Meyer, ’02

<table>
<thead>
<tr>
<th>$J^{PC}$</th>
<th>Lattice (MeV)</th>
<th>Our model (MeV)</th>
<th>Mismatch</th>
</tr>
</thead>
<tbody>
<tr>
<td>0^{++}</td>
<td>1475 (4%)</td>
<td>1475</td>
<td>0</td>
</tr>
<tr>
<td>2^{++}</td>
<td>2150 (5%)</td>
<td>2055</td>
<td>4%</td>
</tr>
<tr>
<td>0^{+++}</td>
<td>2755 (4%)</td>
<td>2753</td>
<td>0</td>
</tr>
<tr>
<td>2^{++*}</td>
<td>2880 (5%)</td>
<td>2991</td>
<td>4%</td>
</tr>
<tr>
<td>0^{++**}</td>
<td>3370 (4%)</td>
<td>3561</td>
<td>5%</td>
</tr>
<tr>
<td>0^{+++*}</td>
<td>3990 (5%)</td>
<td>4253</td>
<td>6%</td>
</tr>
</tbody>
</table>

$0^{++} : Tr F^2; \quad 2^{++} : Tr F_{\mu\nu} F^{\mu\nu}.$
Summary of general results

- Mass gap ⇔ Color confinement

- Universal asymptotic mass ratios: $m_{0^{++}}/m_{2^{++}} \to 1$ as $n \gg 1$
  
  In accord with old string models of QCD

- Fit the lattice data with single parameter $b_0 \approx 4.2$

- Strong dependence on $\alpha$, linear spectrum for $\alpha = 2$ only

- Spectrum changes drastically if replace logarithmic running in the UV with e.g. a fixed point.
Meson sector

- Real challenge for phenomenology
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- Add fundamental matter in the quenched limit $N_f/N_c \ll 1$ by $N_f \, D4 - \overline{D4}$ branes.
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\[
\begin{array}{c}
\text{D 4} \\
| \\
| \\
| \\
| \\
| \\
T=0 \\
| \\
| \\
| \\
| \\
| \\
T=0' \\
| \\
| \\
\end{array}
\]
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Choose a Tachyon potential $V_T \sim e^{-T^2}$
- DBI action $\Rightarrow$ solve the eq. for $T$
- No backreaction on the geometry
- Compute from $\delta A_\mu$ on D4 $\Rightarrow$ vector meson spectrum
Meson sector cont.

Fluctuations on D4 ⇔ Vector mesons from Schrödinger eq. with

\[ V = (B')^2 + B'' , \quad B = \frac{A - \Phi}{2} + \frac{1}{2} \log V_T(T(r)) \]
Meson sector cont.

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$$V = (B')^2 + B''$$

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- Always linear confinement regardless the background, due to $V_T$

- Typical mass scales for the mesons and the glueballs different in general:

$$\Lambda_{\text{glue}} = \Lambda, \quad \Lambda_{\text{meson}} = \Lambda (\ell \Lambda)^{\alpha-2}$$

A single scale in the spectrum for $\alpha = 2$
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A single scale in the spectrum for \( \alpha = 2 \)

- Highly non-linear T equation, proved very hard to solve numerically (issue of the initial conditions.) Ongoing work with F. Nitti, A. Paredes, E. Kiritsis
The axion sector

- Axion action $S_A = \frac{M^3}{2} \int \sqrt{g} Z_A(\lambda)(\partial a)^2$

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- General solution:

  $$a(r) = \theta_0 + C_a \int_0^r \frac{dr}{\ell} \frac{e^{-3A}}{Z_A(\lambda)}$$

  $$\rightarrow \theta_0 + \frac{C_a}{4Z_a \ell^4} r^4 + \cdots \quad \text{as} \quad r \rightarrow 0$$

the axionic glueball condensate
The axion sector cont.

- Vacuum energy from $E = S_A[a] \propto a(r) \bigg|_{r_0}$

Effects of CP violation e.g. electric dipole moment of neutron, $0^+ \rightarrow 0^+$, etc., the axion $a$.

Renormalized effects of the $\mu$-parameter vanishes in the IR! Pseudo-scalar glueball screens the $0$-glueball, a hint at resolution of the strong CP problem?
The axion sector cont.

- Vacuum energy from \( E = S_A[a] \propto a(r) \left|_{r=0}^{r_0} \right. \)
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- Pseudo-scalar glueball screens the \( \theta_0 \) in the IR, a hint at resolution of the strong CP problem?
Summary and discussion

- A holographic model for QCD
  - Effectively describe the uncontrolled physics in the UV by a general dilaton potential, with parameters $\beta$-function coefficients
  - Focused on a model with two parameters $b_0$ and $A_0$. Improvement on AdS/QCD: linear confinement, magnetic screening, agreement with lattice, mesons can be treated

- Asymptotic AdS in the UV with log-corrections, $\frac{\ell_{\text{AdS}}}{\ell_s} \approx 7$

- Singularity in the IR. But $R_S \to 0$: A log-corrected linear dilaton background in the IR.

- Dilaton diverges in the IR, that region is not probed neither by probe strings nor by bulk excitations

- Some qualitative results: confinement $\Leftrightarrow$ mass gap, universal mass ratios for $n \gg 1$, a suggestion for the resolution of CP problem
Outlook

- Precise computations in the axionic sector, predictions for experiments
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  At finite $T$, thermal gas (zero $T$ geometry with Euclidean time compactified) and two Black-hole geometries (big and small)

$$ds^2 = e^{2A(r)} \left( -f(r) dt^2 + d\vec{x}^2 + \frac{dr^2}{f(r)^2} \right)$$

- General results: Hawking-Page transition at $T_c$. For $T > T_c$ big black-hole dominates
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- Color confinement $\Leftrightarrow$ confinement-deconfinement transition at $T_c \neq 0$. 

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Outlook cont.

- Finite baryon chemical potential, phase diagram of large N Yang-Mills in $T, \mu$
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THANK YOU!