Exploring Holographic Approaches to QCD

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U.G., E. Kiritsis, F.Nitti arXiv:0707.1349
U.G., E. Kiritsis arXiv:0707.1324

Physics of Strong Interactions

- QCD perturbation theory, in the UV
- Non-perturbative phenomena in the IR
  Lattice QCD

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  Holographic approaches
  String theory may have real impact!

QCD in this talk:

- Pure Yang-Mills at $N_c \gg 1$
- QCD in the quenced limit: $N_f/N_c \ll 1$

Holographic Approaches to QCD

"TOP - BOTTOM APPROACH"

- 10D critical string theory
- D-brane configurations
- Decoupling limit of open and closed string sectors
- Treatable in the supergravity limit, $\ell_s \to 0$

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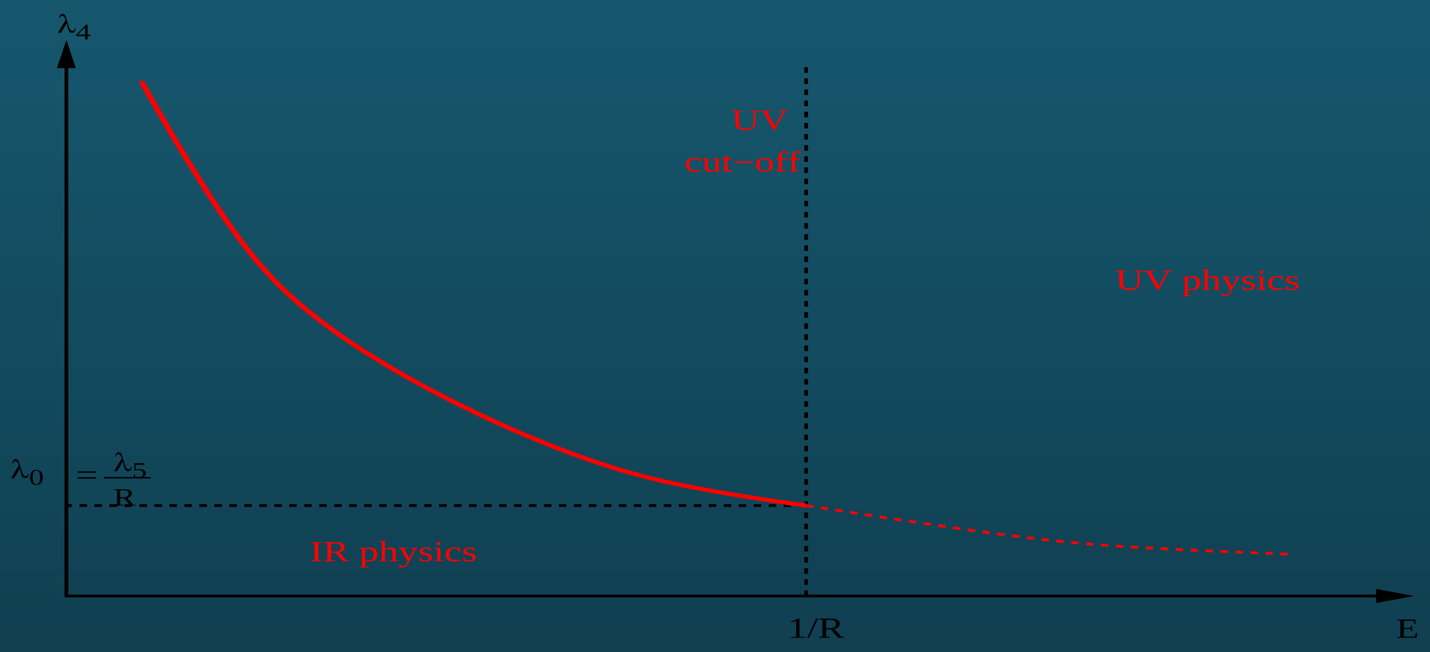
EXAMPLES

- Klebanov-Strassler, Polchinski-Strassler, Maldacena-Nunez, orbifold constructions, etc. for $\mathcal{N} = 1, 2$ gauge theories
- Witten's model for pure Yang-Mills

Witten's Model '98

- YM_5 on D4 Branes
- Antiperiodic boundary conditions on for the fermions on S^1 $m_\psi \sim \frac{1}{R}, \qquad m_\phi \sim \frac{\lambda_4}{R}$
- UV cut-off in the 4D theory at $E = 1/R$
- Pure YM_4 in the IR

Witten's model



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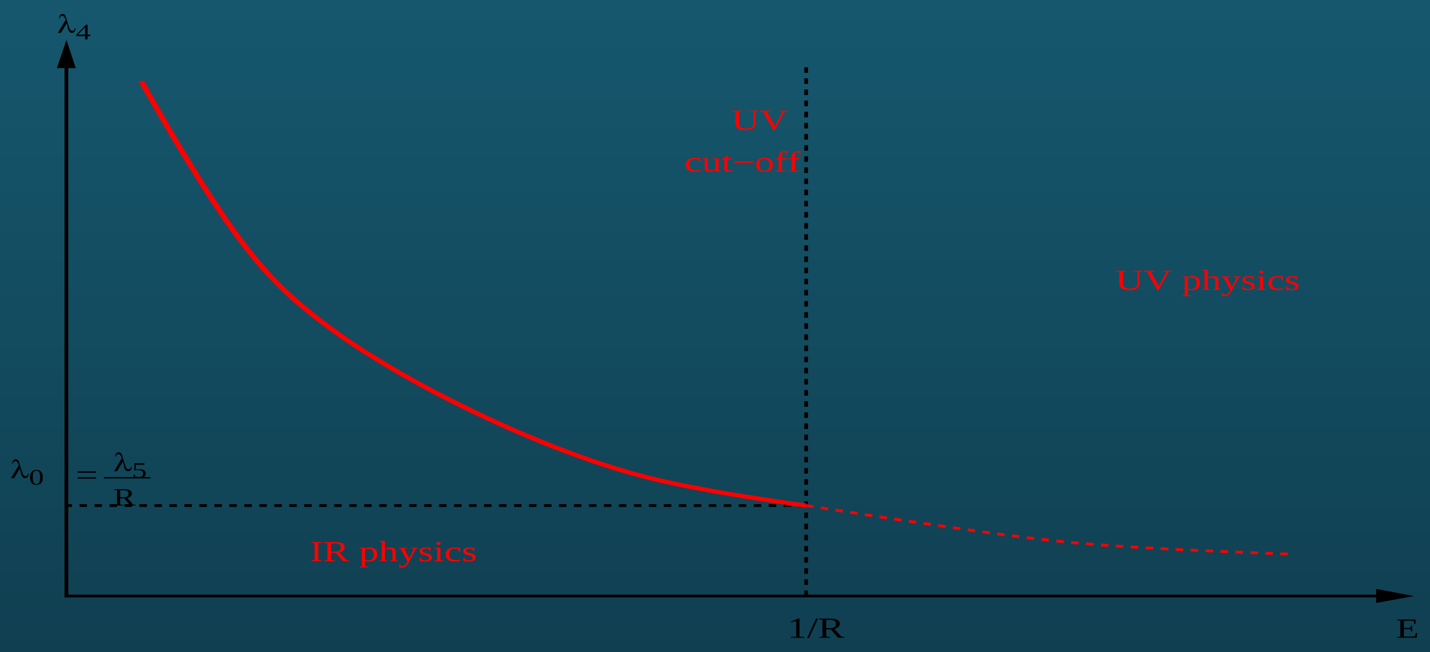

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- $\lambda_0 \gg 1$
- Weak curvature
- Confinement, mass gap

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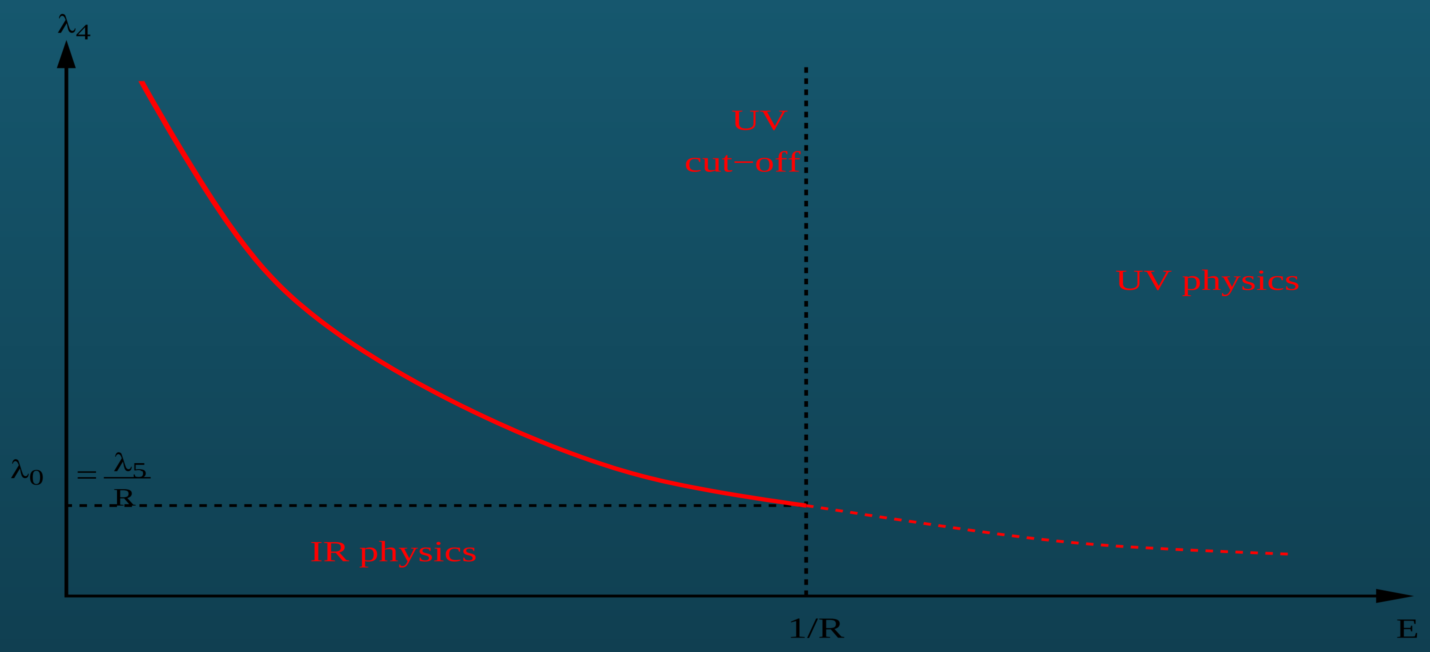

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- Mixing with KK
- $\lambda_0 \ll 1$
- High curvature corrections
- No asymptotic freedom

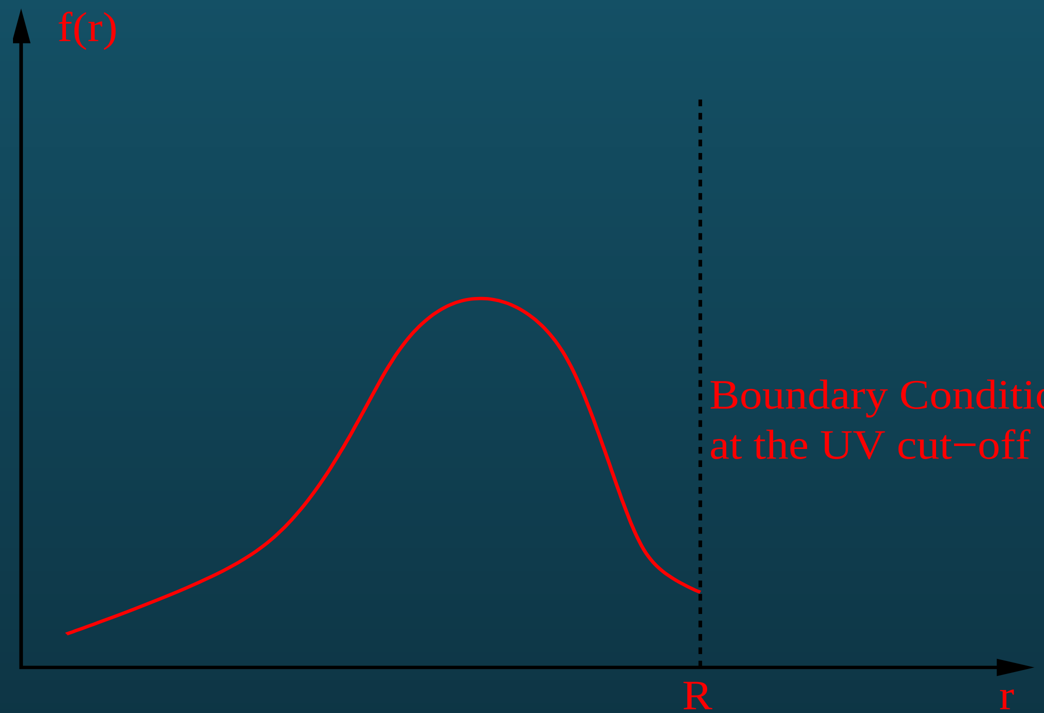
Confining Gauge Theories in the SG Regime

- UV and the IR physics disconnected
  - Effects of the logarithmic running not captured
- Mixing of pure gauge sector with the KK sector
  - To disentangle need $\lambda_0 \ll 1$
  - Then ℓ_s corrections !
- Problems with the glueball spectra (Witten ’98, Ooguri et al. ’98;)

The glueball spectra

Normalizable modes of $\phi(r,\vec{x}) = f(r)e^{i\vec{k}\cdot\vec{x}}$

$\Leftrightarrow$ spectrum of $\mathcal{O}(\vec{x})|vac\rangle$



KK like spectra $m_n^2 \propto n^2$ for $n \gg 1$!

Solution to problems

- Full σ-model $S_{WS}[\lambda_0]$ on Wittens's background $\mathcal{M}_{10}$
- Compute corrections as $\mathcal{M}_{10}[\lambda_0]$
  $\Rightarrow$ Compute *e.g.* the glueball spectra perturbatively in λ_0^{-1}
- Generally need to sum over all series
- 2D lattice for $S_{WS}[\lambda_0]$?

VERY HARD OPEN PROBLEM

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- Effects of either KK modes or ℓ_s corrections
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IS IT POSSIBLE TO CONSTRUCT AN EFFECTIVE THEORY FOR THE IR PHYSICS OF LOW LYING EXCITATIONS BY PARAMETERIZING THE UV REGION ?

- tunable parameters
- Fixed by input from gauge theory + experiment (or lattice)

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Build an effective theory for the lowest lying excitations by introducing MINIMUM ingredients

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Guideline

- insights from AdS/CFT:
  - Space-time symmetries $T_{\mu\nu} \Leftrightarrow g_{\mu\nu}$
  - Energy $\Leftrightarrow$ radial direction r
  - $\Lambda_{QCD} \Leftrightarrow$ broken translation invariance in r

5D space-time $ds^2 = e^{2A(r)}\left(dx^2 + dr^2\right)$

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  - Non-perturbative effects through glueball condensates

  e.g. $\langle \mathrm{Tr} F^2 \rangle$, $\langle \mathrm{Tr} F \wedge F \rangle$, etc.

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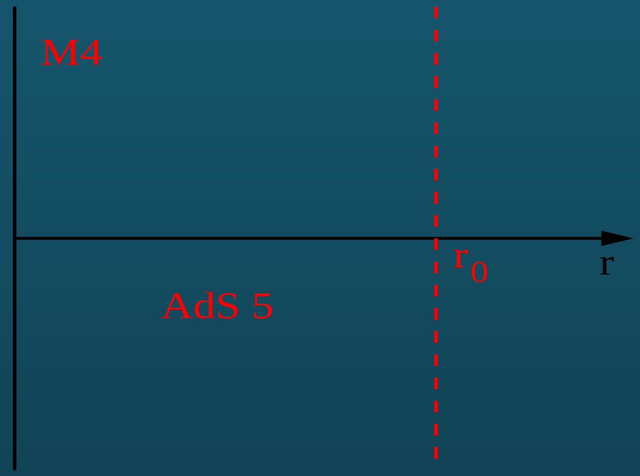
- insights from 5D non-critical string theory

Simplest model: AdS/QCD

Polchinski-Strassler '02; Erlich et al. '05; Da Rold, Pomarol '05

AdS_5 with an IR cut-off



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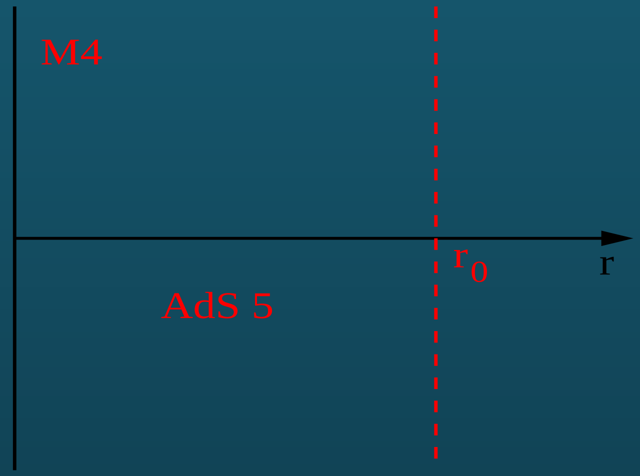

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- Mesons by adding $D4 - \overline{D}4$ branes in probe approximation
- Fluctuations of the fields on $D4$: meson spectrum
  e.g. $A^L_\mu + A^R_\mu \Leftrightarrow$ vector meson spectrum
- surprisingly successful: certain qualitative features, meson spectra %13 of the lattice

Problems

- No running gauge coupling, no asymptotic freedom
- Ambiguity with the IR boundary conditions at r_0
- No linear confiniment, $m_n^2 \sim n^2$, $n \gg 1$
- No magnetic screening
  Soft wall models Karch et al. '06
  AdS_5 with dilaton $\phi(r) \Leftrightarrow$ linear confinement
- No gravitational origin, not solution to diffeo-invariant theory
- No obvious connection with string theory

Our Purpose

- Use insight from non-critical string theory
- Construct a 5D gravitational set-up
- To parametrize the ℓ_s corrections in the UV,
  use to gauge theory input β-function
- Classify and investigate the solutions
- Improvement on AdS/QCD
- Gain insights for possible mechanisms in QCD
- General, model independent results:
  - Color confiniment $\Leftrightarrow$ mass gap
  - A proposal for the strong CP problem ??
  - Finite Temperature physics

Outline

- Two derivative effective action in 5D, $S[g, \phi, a]$
- Constrain small ϕ asymptotics, asymptotic freedom
- UV physics parametrized by the perturbative β-function
  - β-function $\Leftrightarrow$ superpotential
  - ℓ_s-corrections $\Leftrightarrow$ scheme-dependent β-coefficients
- Constrain large ϕ asymptotics, color confinement
- Fluctuations in g, ϕ, a, the glueball spectrum
- Mesons
- Axion sector
- Discussion, Finite Temperature results, Outlook

Construction of the effective action

- Ingredients in S_{eff}

$$\text{Pure } YM_4 \quad \left\{ \begin{array}{lcl} SU(N_c) & \Leftrightarrow & F_5 \propto N_c \\ g_{YM} & \Leftrightarrow & e^{\phi} \\ \theta_{YM} & \Leftrightarrow & a \end{array} \right.$$

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- Two-derivative action Klebanov-Maldacena '04

$$S_S = M^3 \int d^5x \sqrt{g_S} \left\{ e^{-2\phi}[R + (\partial\phi)^2 - \frac{\delta c}{\ell_s^2}] - F_5^2 - F_1^2 \right\}$$

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Define $\lambda \equiv N_c e^{\phi} \equiv e^{\Phi}$,

go to the Einstein frame $g_s = \lambda^{\frac{4}{3}} g_E$

dualize F_5

Einstein frame Action

The action in the Einstein frame,

$$S_E = M^3 N_c^2 \int d^5x \sqrt{g_E} \left\{ R + (\partial\Phi)^2 - V(\Phi) - N_c^{-2} F_1^2 \right\}$$

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- Axion suppressed by N_c^{-2}. Do not back-react on the geometry.

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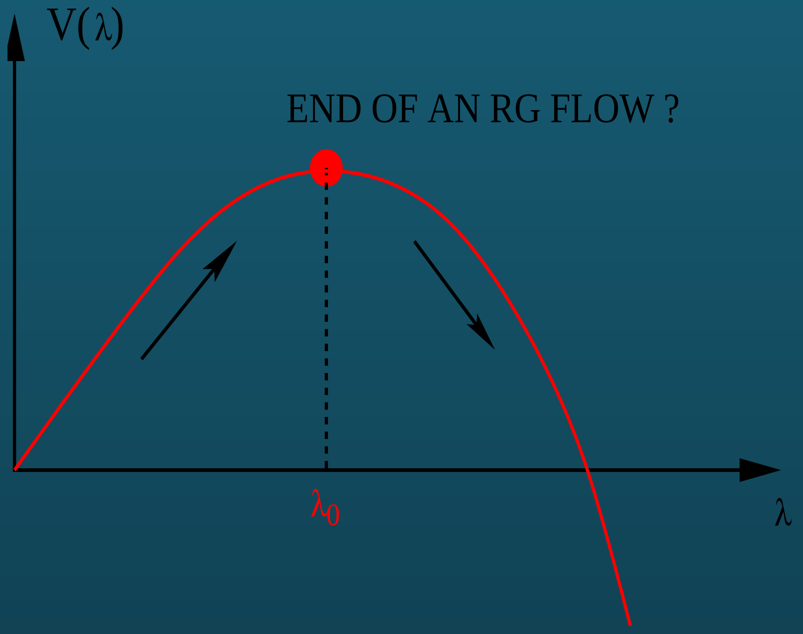
The naive dilaton potential

$$V(\lambda) = \frac{\lambda^{\frac{4}{3}}}{\ell_s^2} \left(1 - \lambda^2\right)$$

- The potential is of order ℓ_s $\Leftrightarrow$ string σ-model corrections are substantial.

The naive potential



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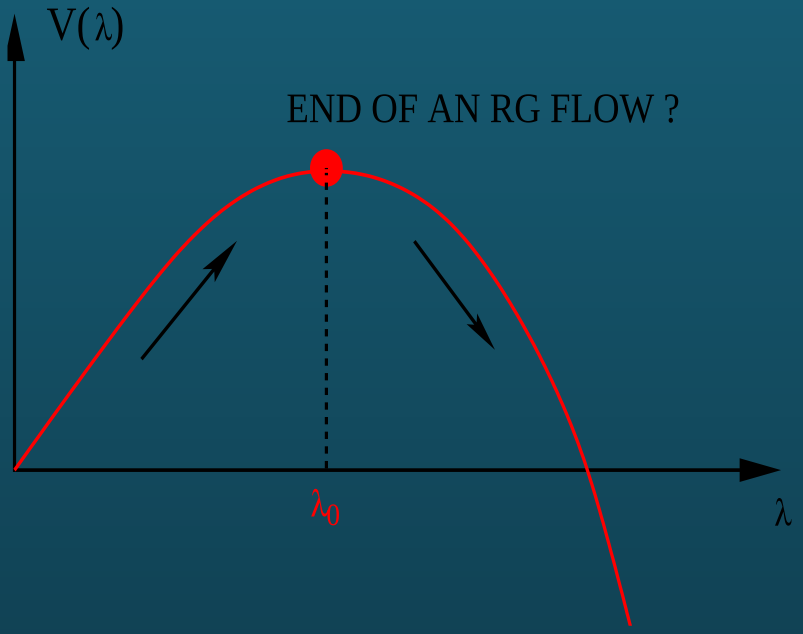

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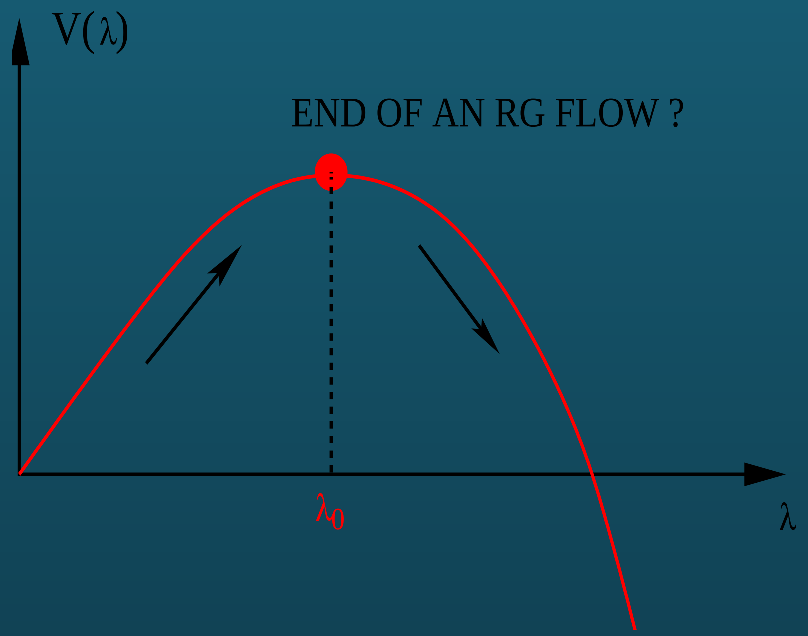

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$$V(\lambda) \to V_0 + V_1\lambda + \cdots \qquad as \;\; \lambda \to 0$$

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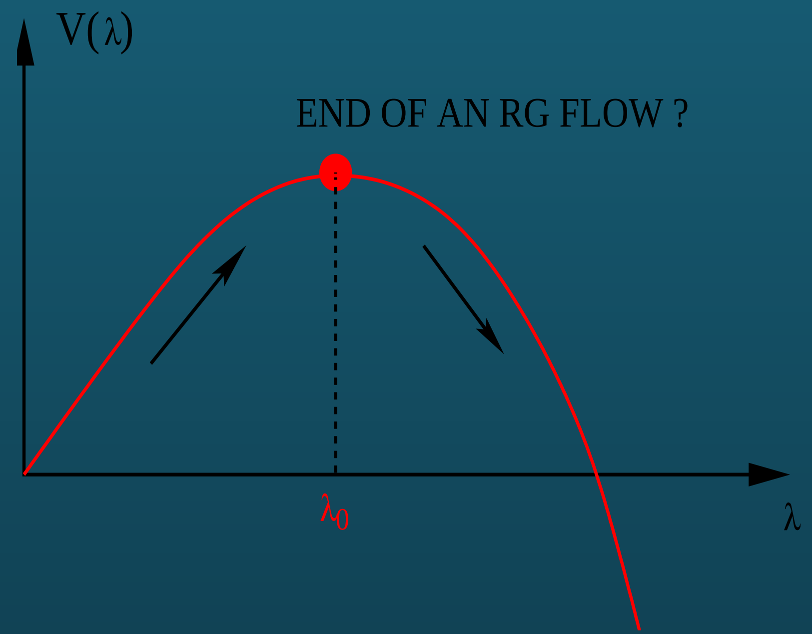

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- The naive theory is not rich enough to describe QCD RG-flow

String corrections to the naive potential

- Higher derivative corrections to the F_5 kinetic term (after going to the Einstein frame and dualizing)

$$F_5^2 \Rightarrow \sum_n F_5^{2n} \lambda^{2(n-1)} a_n \qquad as \;\; \lambda \rightarrow 0$$

with unknown coefficients a_n.

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- V_0 can be generated through higher derivative corrections to R *e.g.* in an $f(R)$ type gravity U.G, E. Kiritsis '07

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- V_n include corrections to F_5. We will relate V_n to β-coefficients

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Construct the theory and justify a posteriori:

- $$\frac{\ell_{AdS}}{\ell_s} \gg 1 \ \ ?$$
- Is the singularity of repulsive type? Does the strings or particles probe in the singular region?

Solutions to the Einstein-dilaton system

Look for solutions domain-wall type of solutions

$$ds^2 = e^{2A(u)} dx^2 + du^2, \qquad \Phi = \Phi(u)$$

- $A(u) \to -\frac{u}{\ell} \;\; as \;\; u \to -\infty, \qquad \Phi(u) \to -\infty \;\; as \;\; u \to -\infty$

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- $V = W^2 - \left(\frac{\partial W}{\partial \Phi}\right)^2, \qquad A' = -W, \quad \Phi' = \frac{\partial W}{\partial \Phi}$

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A single integration constant in the system:

$A_0 \Leftrightarrow \Lambda_{QCD}$ in the gauge theory

Holographic dictionary I

ENERGY $\Leftrightarrow$ SCALE FACTOR

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- String corrections:
  If replace $R \to f\left(\ell_s^2 R\right) = \sum_n f_n R^n$, with unknown f_n,

$$\frac{d \log E}{dA} = 1 + f_1 \lambda^2 + f_2 \lambda^4 + \cdots$$

Holographic dictionary II

't HOOFT COUPLING $\Leftrightarrow$ DILATON

Insert a probe D3 brane in the geometry. The DBI action:

$$S_{D3} \propto \int e^{-\Phi} F^2 \quad \Rightarrow \quad \lambda = e^{\Phi} = g_{YM}^2 N_c = \lambda_t$$

Holographic dictionary II

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String corrections:

Higher order couplings of F_5 to $D3$ probe brane

$$S_{D3} = \frac{T_3}{\ell_s^4} \int \sqrt{g} e^{-\phi} Z(e^{2\phi} F_5^2) F^2$$

with $Z(x) = 1 + \sum_{n=1}^{\infty} c_n x^n$

The identification receives corrections as,

$$\lambda_t = \lambda(1 + c_1 \lambda^2 + c_2 \lambda^4 + \cdots)$$

Holographic dictionary III

β-FUNCTION $\Leftrightarrow$ SUPERPOTENTIAL

- If one ignores the string corrections:

$$\beta(\lambda) = \frac{d\lambda}{d\ln E} = -b_0\lambda^2 + b_1\lambda^3 + \cdots = -\frac{9}{4}\lambda^2 \frac{d\ln W(\lambda)}{d\lambda}$$

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- One can solve for the dilaton potential

$$V(\lambda) = V_0 \left(1 - \left(\frac{\beta}{3\lambda}\right)^2\right) e^{\int_0^\lambda \frac{d\lambda\beta}{3\lambda}}$$

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- One-to-one correspondence between the coefficients V_n and b_n. We parameterize our ignorance about the UV part of the dual geometry by the b_n.

Holographic dictionary IV

The string corrections:

$$\begin{aligned}\beta_t \to \beta_{st} &= -b_0\lambda^2 + b_1\lambda^3 \\ &+ \left(b_2 - 4c_1 b_0 + f_1 b_0\right)\lambda^4 \\ &+ \left(b3 + 4c_1 b_0 - f_1 b_1\right)\lambda^5 + \cdots\end{aligned}$$

c_n from corrections to the probe brane, and the F_5 kinetic term

f_n from curvature corrections

ℓ_s corrections appear with the scheme dependent β-coefficients!

Geometry near the boundary

For $\beta = -b_0\lambda^2 + b_1\lambda^3 + \cdots$,

- The dilaton

$$b_0\lambda = -\frac{1}{\log r\Lambda} + \frac{b_1}{b_0^2}\frac{\log(-\log r\Lambda)}{\log^2(r\Lambda)} + \cdots$$

- The scale factor $ds^2 = e^{2A}(dx^2 + dr^2)$

$$e^{2A} = \frac{\ell^2}{r^2}\left(1 + \frac{8}{9}\frac{1}{\log(r\Lambda)} - \frac{8}{9}\frac{b_1}{b_0^2}\frac{\log(-\log r\Lambda)}{\log^2(r\Lambda)} + \cdots\right)$$

- AdS with logarithmic corrections. Subleading term is model independent.

Geometry near the boundary

For $\beta = -b_0\lambda^2 + b_1\lambda^3 + \cdots$,

- The dilaton

$$b_0\lambda = -\frac{1}{\log r\Lambda} + \frac{b_1}{b_0^2}\frac{\log(-\log r\Lambda)}{\log^2(r\Lambda)} + \cdots$$

- The scale factor $ds^2 = e^{2A}(dx^2 + dr^2)$

$$e^{2A} = \frac{\ell^2}{r^2}\left(1 + \frac{8}{9}\frac{1}{\log(r\Lambda)} - \frac{8}{9}\frac{b_1}{b_0^2}\frac{\log(-\log r\Lambda)}{\log^2(r\Lambda)} + \cdots\right)$$

- AdS with logarithmic corrections. Subleading term is model independent.

Holographic renormalization Bianchi, Freedman, Skenderis '01

AdS with log-corrections, ongoing with E. Kiritsis, Y. Papadimitriou

Geometry in the interior

For $A(r) \rightarrow \frac{\ell}{r} \qquad as \qquad r \rightarrow 0$,
Einstein's equations lead to the following IR behaviours:

- AdS, $A(r) \rightarrow \frac{\ell'}{r}$ with $l' \leq l$
- Singularity (in the Einstein frame) at a finite point $r = r_0$
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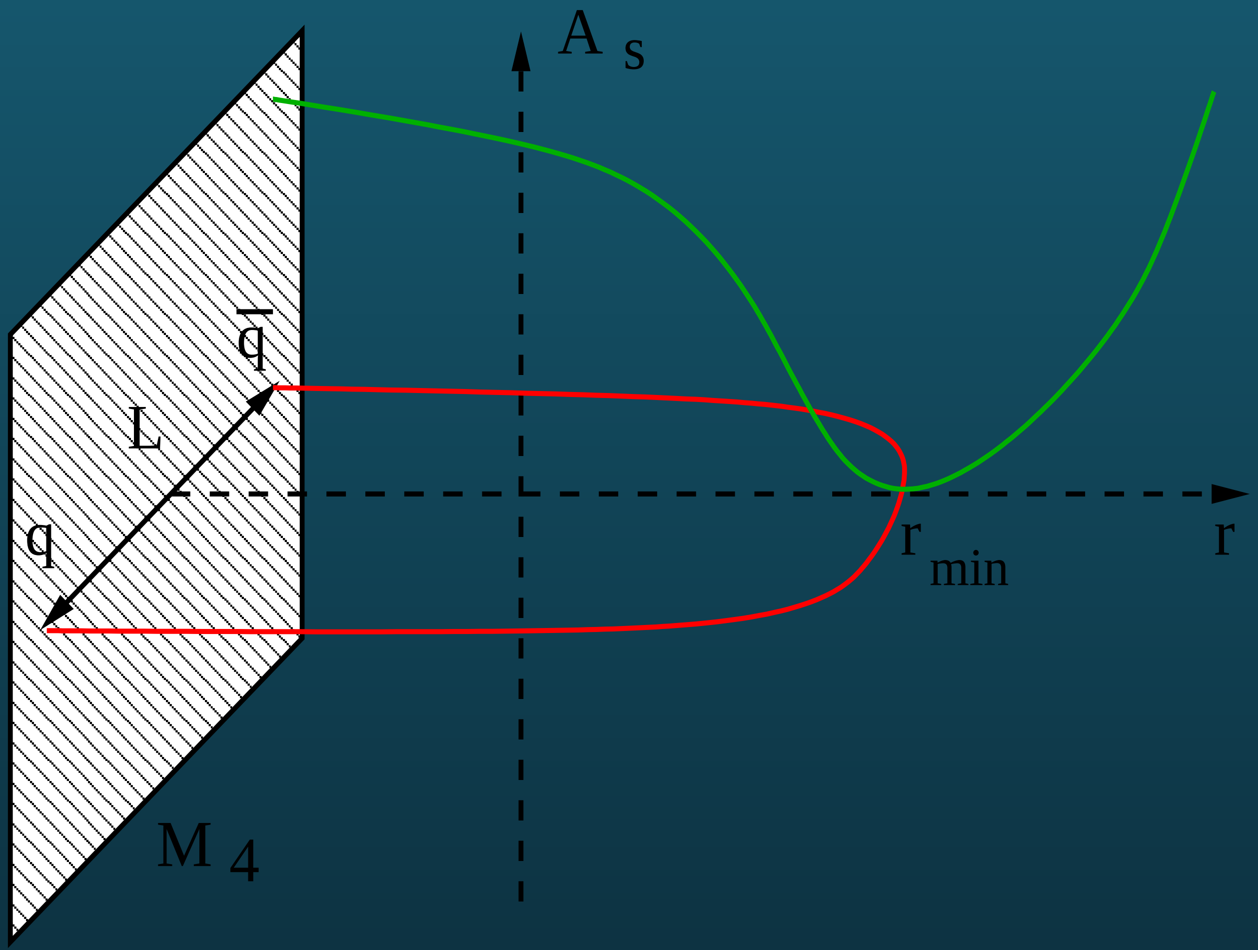
Phenomenologically preferred asymptotics

- color confininement
- magnetic screening
- linear spectra $m_n^2 \sim n$ for large n

Constraints on the IR geometry

Color confinement

J Maldacena ’98; S. Rey, J. Yee ’98



Solve for the string embedding and compute its action:

$$E_{q\bar{q}}T = S_{WS}$$

Color Confinement - Magnetic Screening

String action: $S_{WS} = \ell_s^{-2} \int \sqrt{\det g_{ab}} + \int \sqrt{\det g_{ab}} R^{(2)} \Phi(X)$
in the string frame, $g_{ab} = g^S{}_{\mu\nu} \partial_a X^\mu \partial_b X^\nu$.

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$$E_{q\bar{q}} = T_s L = \frac{e^{A_S(r_{min})}}{\ell_s^2} L$$

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- Similar consideration for the magnetic charges, using probe D1

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- Space ending at finite r_0
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The phenomenologically preferred backgrounds for infinite r:

$$A \sim -Cr^{\alpha} \quad \Leftrightarrow \quad Q = 2/3, P = \frac{\alpha - 1}{\alpha}$$

Linear confiniment in the glueball spectrum for $\alpha = 2$

- Borderline case $\alpha = 1$ is linear dilaton background!

Glueballs

Spectrum of 4D glueballs $\Leftrightarrow$ Spectrum of normalizable flucutations of the bulk fields.

Spin 2: $h_{\mu\nu}^{TT}$; Spin 0: mixture of h_μ^μ and $\delta\Phi$; Pseudo-scalar: δa.

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Quadratic action for fluctuations:

$$S \sim \frac{1}{2}\int d^4x dr e^{2B(r)}\left[\dot{\zeta}^2 + (\partial_\mu\zeta)^2\right]$$

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- Scalar : $B(r) = 3/2A(r) + \log(\dot{\Phi}/\dot{A})$
- Tensor : $B(r) = 3/2A(r)$
- Pseudo-scalar: $B(r) = 3/2A(r) + 1/2\log Z_A$

Reduction to a Schrödinger problem

Define: $\zeta(r) = e^{-B(r)}\Psi(r)$ Schrödinger equation:

$$\mathcal{H}\Psi \equiv -\ddot{\Psi} + V(r)\Psi = m^2\Psi \qquad V_s(r) = \dot{B}^2 + \ddot{B}$$

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- The normalizability condition: $\int dr\,|\Psi|^2 < \infty$
- Normalizability in the UV, picks normalizable UV asymptotics for ζ
- Normalizability in the IR, restricts discrete m^2, for confining V_s.

Mass gap

$$\mathcal{H} = (\partial_r + \partial_r B)(-\partial_r + \partial_r B) = \mathcal{P}^\dagger \mathcal{P} \geq 0 :$$

- Spectrum is non-negative
- Can prove that no normalizable zero-modes
- If $V(r) \to \infty$ as $r \to +\infty$: Mass Gap

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- Spectrum is non-negative
- Can prove that no normalizable zero-modes
- If $V(r) \to \infty$ as $r \to +\infty$: Mass Gap
- This precisely coincides with the condition from color confinement
- *e.g.* for the infinite geometries $A(r) \to -Cr^\alpha$:
  color confiniment AND mass gap for $\alpha \geq 1$.

Numerics

Choose a specific model. Take a superpotential such that

$$W \sim \begin{cases} W_0 \left(1 + \frac{4}{9} b_0 \lambda + \ldots\right) & \lambda \to 0 \\ \\ W_0 \lambda^{2/3} (\log \lambda)^{1/4} & \lambda \to \infty \quad (\alpha = 2) \end{cases}$$

For example:

$$W = \left(1 + \frac{2}{3} b_0 \lambda\right)^{2/3} \left[1 + \frac{4(2b_0^2 + 3b_1)}{9} \log(1 + \lambda^2)\right]^{1/4}$$

Then, compute numerically metric, dilaton, mass spectrum.

- Parameters of the model: b_0 and A_0. We fix $b_1/b_0^2 = 51/121$, pure YM value.

Comparison with one lattice study Meyer, '02

J^{PC}	Lattice (MeV)	Our model (MeV)	Mismatch
0^{++}	1475 (4%)	1475	0
2^{++}	2150 (5%)	2055	4%
0^{++*}	2755 (4%)	2753	0
2^{++*}	2880 (5%)	2991	4%
0^{++**}	3370 (4%)	3561	5%
0^{++***}	3990 (5%)	4253	6%

$$0^{++} : TrF^2; \qquad 2^{++} : TrF_{\mu\rho}F^{\rho}_{\nu}.$$

Summary of general results

- Mass gap $\Leftrightarrow$ Color confinement
- Universal asymptotic mass ratios: $m_{0++}/m_{2++} \rightarrow 1$ as $n >> 1$
  In accord with old string models of QCD
- Fit the lattice data with single parameter $b_0 \approx 4.2$
- Strong dependence on α, linear spectrum for $\alpha = 2$ only
- Spectrum changes drastically if replace logarithmic running in the UV with *e.g.* a fixed point.

Meson sector

- Real challenge for phenomenology

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- Add fundamental matter in the quenched limit $N_f/N_c \ll 1$ by N_f $D4 - \overline{D4}$ branes.

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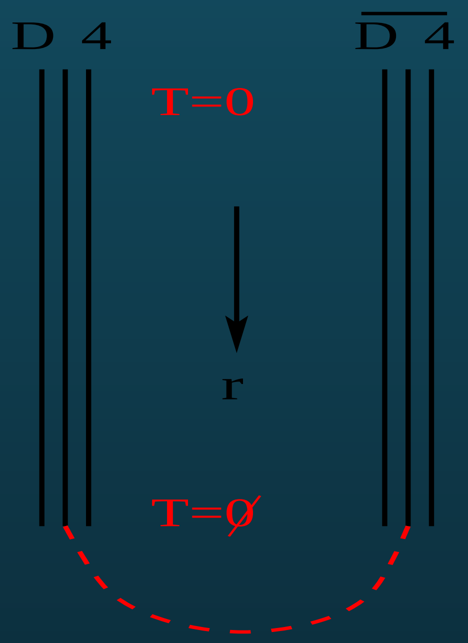

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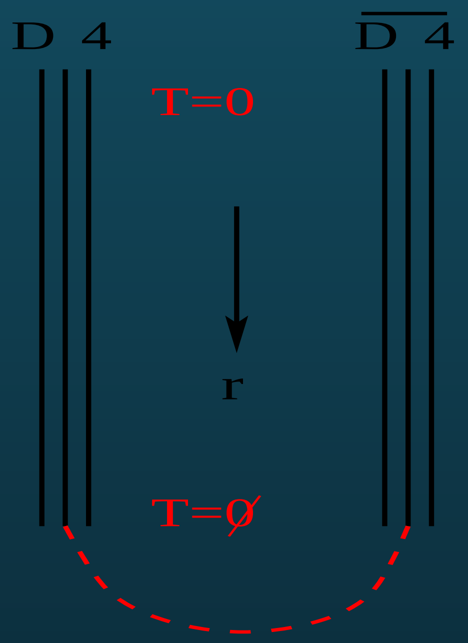

- Choose a Tachyon potential $V_T \sim e^{-T^2}$
- DBI action $\Rightarrow$ solve the eq. for T
- No backreaction on the geometry
- Compute from δA_μ on D4 $\Rightarrow$ vector meson spectrum

Meson sector cont.

Fluctuations on D4 $\Leftrightarrow$ Vector mesons from Schrödinger eq. with

$$V = (B')^2 + B'', \qquad B = \frac{A - \Phi}{2} + \frac{1}{2}\log V_T(T(r))$$

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$$V = (B')^2 + B'', \qquad B = \frac{A - \Phi}{2} + \frac{1}{2} \log V_T(T(r))$$

- Always linear confinement regardless the background, due to V_T
- Typical mass scales for the mesons and the glueballs different in general:

$$\Lambda_{glue} = \Lambda, \qquad \Lambda_{meson} = \Lambda(\ell\Lambda)^{\alpha-2}$$

A single scale in the spectrum for $\alpha = 2$

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A single scale in the spectrum for $\alpha = 2$

- Highly non-linear T equation, proved very hard to solve numerically (issue of the initial conditions.) Ongoing work with F. Nitti, A. Paredes, E. Kiritsis

The axion sector

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- General solution:

$$a(r) = \theta_0 + C_a \int_0^r \frac{dr}{\ell} \frac{e^{-3A}}{Z_A(\lambda)}$$

$$\to \theta_0 + \frac{C_a}{4 Z_a \ell^4} r^4 + \cdots \qquad as \qquad r \to 0$$

the axionic glueball condensate

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- Effects of CP violation *e.g.* electric dipole moment of neutron, 0^{+-} decay into 0^{++} etc. $\Leftrightarrow$ the axion a
- Renormalized effects of the θ-parameter vanishes in the IR!
- Pseudo-scalar glueball screens the θ_0 in the IR, a hint at resolution of the strong CP problem?

Summary and discussion

- A holographic model for QCD
  - Effectively describe the uncontrolled physics in the UV by a general dilaton potential, with parameters β-function coefficients
  - Focused on a model with two parameters b_0 and A_0. Improvement on AdS/QCD: linear confinement, magnetic screening, agreement with lattice, mesons can be treated
- Asymptotic AdS in the UV with log-corrections, $\frac{\ell_{Ads}}{\ell_s} \approx 7$
- Singularity in the IR. But $R_S \to 0$: A log-corrected linear dilaton background in the IR.
- Dilaton diverges in the IR, that region is not probed neither by probe strings nor by bulk excitations
- Some qualitative results: confinement $\Leftrightarrow$ mass gap, universal mass ratios for $n \gg 1$, a suggestion for the resolution of CP problem

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- Color confiniment $\Leftrightarrow$ confiniment-deconfiniment transition at $T_c \neq 0$.

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THANK YOU !