# AdS spacetimes and Kaluza-Klein consistency

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based on work with

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# Consistent KK reductions

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2. $AdS_5 \times_w M_6$ solutions with dual $d = 4$, $N = 1$ SCFTs
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3. $AdS_5 \times_w N_6$ solutions with dual $d = 4$, $N = 2$ SCFTs
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4. Conclusions and outlook

# Motivation

- A powerful method to construct solutions to sugra theories in a higher dimension $D$ is to uplift solutions of simpler sugras in lower dimension $d$.
- For this uplift to be well defined, there must exist a consistent Kaluza-Klein (KK) reduction from the sugra in dimension $D$ to the sugra in dimension $d$.
- To determine if such KK reduction is consistent is an interesting problem by its own.

## KK consistency

- Upon compactification on an internal manifold $M_{D-d}$, the $D$-dimensional fields give rise to $d$-dimensional fields: a finite set of *light* $L$ and a KK tower of *heavy* $H$ fields.
- The $D$-dimensional e.o.m.'s can be rewritten in terms of these:

$$\Box L \sim a_{mn} L^m H^n$$

$$\Box H \sim b_{mn} L^m H^n$$

- A truncation keeping $L$ and discarding $H$ ($H = 0$) will be consistent only if $b_{m0} = 0$.
- Then, the fields $L$ satisfy $d$-dimensional e.o.m.'s: $\Box L = 0$. Any solution to the $d$-dimensional theory can then be uplifted to $D$ dimensions (via the 'KK ansatz').

## Some cases of consistent reductions

- Only in a few cases there is a group-theoretical argument behind the consistency of the truncation (when all the singlets under a convenient symmetry group are retained):

- (Toroidal) dimensional reductions,

- Compactifications on group manifolds.

- But in general, the compactification on arbitrary manifolds will be inconsistent.

# $AdS \times Sphere$ compactifications

- Remarkable consistent compactifications of $D = 10, 11$ sugra are associated with the (maximally supersymmetric) solutions $AdS_7 \times S^4$, $AdS_4 \times S^7$ and $AdS_5 \times S^5$:

- The compactification of $D = 11$ sugra on $S^4$ can be consistently truncated to $SO(5)$ gauged (maximal) supergravity in $d = 7$ [Nastase, Vaman, van Nieuwenhuizen, hep-th/9905075, 9911238].

- The compactification of $D = 11$ sugra on $S^7$ can be consistently truncated to $SO(8)$ gauged (maximal) supergravity in $d = 4$ [De Wit, Nicolai, NPB 281 (1987) 211] .

- Similarly, the compactification of IIB sugra on $S^5$ is expected to consistently yield $SO(6)$ (maximal) gauged $d = 5$ sugra [Cvetič, Duff, Hoxha, Liu, Lu, Lu, Martinez-Acosta, Pope, Sati, Tran, hep-th/9903214; Lu, Pope, Tran, hep-th/9909203; Cvetič, Lu, Pope, Sadrzadeh, Tran, hep-th/0003103] .

# A conjecture about consistency

- String/M-theory on all these backgrounds is dual, via the AdS/CFT correspondence, to a superconformal field theory (SCFT) in the boundary of $AdS$.
- Indeed, we would like to view the compactifications on those backgrounds as special cases of the following conjecture:
- For any supersymmetric $AdS_d \times_w M_{D-d}$ solution of $D = 10$ or $D = 11$ supergravity there is a consistent Kaluza-Klein truncation on $M_{D-d}$ to a gauged supergravity theory in $d$-dimensions for which the fields are dual to those in the superconformal current multiplet of the $(d-1)$-dimensional dual SCFT [Gauntlett, OV, arXiv:0707.2315].

## A conjecture about consistency

- Equivalently, the fields of the gauged supergravity are those that contain the $d$-dimensional graviton and fill out an irreducible representation of the superisometry algebra of the $D = 10$ or $D = 11$ supergravity solution $AdS_d \times_w M_{D-d}$.

- This is essentially a restricted version of the conjecture in [Duff, Pope, Nucl. Phys. **B255** (1985) 355].

- General arguments supporting it were subsequenty put forward in [Pope, Stelle, Phys. Lett. **B198** (1987) 151].

# A conjecture about consistency

- For example, the $AdS_5 \times S^5$ solution of type IIB, which has superisometry algebra $SU(2,2|4)$, is dual to $N=4$ superYang-Mills theory in $d=4$.
- The superconformal current multiplet of the latter theory includes the energy momentum tensor, $SO(6)$ R-symmetry currents, along with scalars and fermions.
- These are dual to the metric, $SO(6)$ gauge fields along with scalar and fermion fields, and are precisely the fields of the maximally supersymmetric $SO(6)$ gauged supergravity in $d=5$.
- Here we will give evidence of this conjecture for the cases of $AdS_5 \times_w M_6$ solutions in $D=11$, which are dual to $N=1$ and $N=2$ SCFTs in 4 dimensions.

# $AdS_5 \times_w M_6$ solutions with dual $d = 4$, $N = 1$ SCFTs

# $AdS_5 \times_w M_6$ solutions with dual $d = 4$, $N = 1$ SCFTs

- Consider the $AdS_5 \times_w M_6$ solutions in $D = 11$, which are dual to $N = 1$ SCFTs in 4 dimensions.
- These SCFTs all have a $U(1)$ R-symmetry and so we expect that $D = 11$ sugra on $M_6$ gives a $d = 5$ sugra with
    - $N = 1$ supersymmetry
    - a metric $ds_5^2$ (dual to the energy-momentum tensor of the SCFT)
    - and a $U(1)$ gauge field $A$ (dual to the R-symmetry current).
- This is precisely the content of minimal $d = 5$ gauged sugra.
- Hence we expect that the reduction of $D = 11$ supergravity on $M_6$ truncates consistently to $D = 5$ minimal gauged supergravity.

## $AdS_5 \times_w M_6$ solutions of $D=11$ sugra with $N=1$ susy

The most general solution of the $D=11$ sugra equations containing an $AdS_5$ factor in the metric was analysed in [Gauntlett, Martelli, Sparks, Waldram, hep-th/0402153] using $G$-structure techniques [Gauntlett, Martelli, Pakis, Waldram, hep-th/0205050].

The most general form of the $D=11$ bosonic fields $ds^2_{11}$, $G_4 = dA_3$ containing $AdS_5$, compatible with $SO(4,2)$ symmetry is

$$\begin{aligned} ds^2_{11} &= e^{2\lambda}[ds^2(AdS_5) + ds^2(M_6)] \;, \\ G_4 &\in \Omega_4(M_6, \mathbb{R}) \\ \lambda &\in \Omega_0(M_6, \mathbb{R}) \quad \text{(warp factor)} \end{aligned}$$

and subject to the field equations.

# $N = 1$ supersymmetry

- In order to have $N = 1$ supersymmetry, the solution must admit a Killing spinor $\epsilon$, solution to the Killing spinor equation

  $$\mathcal{D}\epsilon = 0$$

  where $\mathcal{D}$ is the supercovariant derivative, involving the ordinary Riemannian covariant derivative and the $D = 11$ sugra fields.

# $N = 1$ supersymmetry

- The $D = 11$ Killing spinor splits as

  $$\epsilon = \varepsilon \otimes e^{\lambda/2} \xi ,$$

  where $\varepsilon$ is a Killing spinor on $AdS_5$ and $\xi$ is a (non-chiral) spinor on $M_6$.

- The Killing spinor equation also splits into
  - an equation for $\varepsilon$ on $AdS_5$, immediately satisfied, and
  - equations for $\xi$ on $M_6$, which specify a particular $G$-structure on $M_6$ (with $G = SU(2)$).

## The $G$-structure on $M_6$

The existence of $\xi$ defines a $G$-structure on $M_6$, alternatively specified by a set of bilinears on $\xi$ (e.g., $\tilde{K}^2_m = \frac{1}{2}\bar{\xi}\gamma_m\gamma_7\xi$):

$$\begin{aligned} K^1,\ \tilde{K}^2 &\in \Omega_1(M_6,\mathbb{R}) \\ J &\in \Omega_2(M_6,\mathbb{R}) \\ \Omega &\in \Omega_2(M_6,\mathbb{C}) \\ \cos\zeta &\in \Omega_0(M_6,\mathbb{R}) \end{aligned}$$

# The $G$-structure on $M_6$

- The Killing spinor equations for $\xi$ translate into a set of differential and algebraic equations among these bilinear forms and the warp factor $\lambda$ and four-form $G_4$.
- *e.g.*, $\nabla_{(\mu}\tilde{K}^2_{\nu)} = 0$, *i.e.*, $\tilde{K}^2 \equiv \cos\zeta K^2$ defines a Killing vector (related to the R-symmetry of the dual CFT)
- The equations among the bilinear forms constrain the internal geometry, *i.e.*, the metric on $M_6$, the flux $G_4$ and the warp factor $\lambda$.

## The metric on $M_6$

- The existence of two vectors $K^1$, $K^2$ on $M_6$ allows one to choose the convenient frame

$$ds^2(M_6) = e^{-6\lambda} e^i \otimes e^i + (K^1)^2 + (K^2)^2, \quad i = 1,2,3,4$$

and coordinates $y$, $\psi$ can be introduced such that $K^1 \sim dy$, $K^2 \sim d\psi$.

- $e^{-6\lambda} e^i \otimes e^i$ defines a family of four-dimensional manifolds $M_4$ with Kähler metrics parametrised by $y$ .

- $e^{-6\lambda} J$ is the Kähler form on $M_4$ and is independent of $y$.

- The explicit expression of the four-form flux $G_4$ will not be needed in this discussion.

## Kaluza-Klein ansatz: the metric

- The 'KK ansatz' must express the $D=11$ fields $ds^2_{11}$, $G_4 = dA_3$ in terms of the $d=5$ fields $ds^2_5$, $F_2 = dA$.
- It is natural to think of the $d=5$ $U(1)$ gauge field $A$ as arising from the $U(1)$ isometry of $M_6$ generated by $\tilde{K}^2$.
- Thus, we take the usual KK ansatz for the metric:

In $ds^2_{11} = e^{2\lambda}[ds^2(AdS_5) + ds^2(M_6)]$

replace $ds^2(AdS_5) \longrightarrow ds^2_5$ , $\quad ds^2(M_6) \longrightarrow ds^2(\hat{M}_6)$

to get

$$ds^2_{11} = e^{2\lambda}[ds^2_5 + ds^2(\hat{M}_6)]$$

where $\hat{M}_6$ denotes the deformation of $M_6$ parametrised by $A$ as

$$K^2 \quad \longrightarrow \quad \hat{K}^2 = K^2 + \tfrac{1}{3}\cos\zeta A$$

## Kaluza-Klein ansatz: the four-form

KK ansatz for the four-form:

$$G'_4 = \hat{G}_4 + F_2 \wedge \hat{\beta}_2 + *_5 F_2 \wedge \hat{\beta}_1 .$$

Here,

- $F_2 = dA$
- hatted quantities are forms on $\hat{M}_6$ (*i.e.* with $K^2 \quad \longrightarrow \quad \hat{K}^2 = K^2 + \frac{1}{3}\cos\zeta A$)
- $G_4$ is the four-form on $M_6$ corresponding to the undeformed background $AdS_5 \times_w M_6$
- $\beta_1$, $\beta_2$ are forms on $M_6$ to be determined.

# Consistent truncation

- Direct substitution shows that the KK ansatz satisfies the $D = 11$ field equations provided that
  - the $d = 5$ fields satisfy the equations of minimal $d = 5$ gauged supergravity, and
  - a set of differential and algebraic equations among $\beta_1$, $\beta_2$ and the warp factor $\lambda$ and four-form $G_4$ is satisfied.

- These equations are actually of the same form than those among the bilinear forms defining the $G$-structure on $M_6$.

- Indeed $\beta_1$, $\beta_2$ have a solution in terms of some of the spinor bilinears on $M_6$:

$$\beta_1 = -\tfrac{1}{3} e^{3\lambda} \cos\zeta K^1$$

$$\beta_2 = \tfrac{1}{3} e^{3\lambda} \left(-\sin\zeta J + K^1 \wedge K^2\right) \ .$$

# Consistency and the Einstein equation

The $D = 11$ Einstein equations reduce to

$$R_{\alpha\beta} = -4g_{\alpha\beta} + k_1 F_{\alpha\gamma} F_\beta{}^\gamma - k_2 g_{\alpha\beta} F_{\gamma\delta} F^{\gamma\delta},$$

$$d(*_5 F) + k_3 F \wedge F = 0,$$

where, in general, $k_1$, $k_2$, $k_3$ are functions of $M_6$ (given by combinations of the components of $\beta_1$, $\beta_2$).

- This is a potential source of inconsistency of the KK reduction [Duff, Nilsson, Pope, Warner, Phys. Lett. B **149** (1984) 90; Hoxha, Martínez-Acosta, Pope, hep-th/0005172] . .
- However, for $\beta_1$, $\beta_2$ conveniently chosen, $k_1$, $k_2$, $k_3$ are constants, and the $D = 11$ Einstein equation reduces to the right equations in $D = 5$.
- Moreover, this provides yet another check on the constants appearing in the e.o.m. of minimal gauged supergravity in $D = 5$.

## Consistent truncation

- To summarise, the $d=5$ fields $ds_5^2$, $F_2$ can be embedded into the $D=11$ fields $ds_{11}^2$, $G_4$ through the KK ansatz

$$ds_{11}^2 = e^{2\lambda}[ds_5^2 + ds^2(\hat{M}_6)] \ , \qquad \tilde{K}^2 \quad \longrightarrow \quad \hat{\tilde{K}}^2 = \tilde{K}^2 + A$$

$$G_4' = \hat{G}_4 + F_2 \wedge \tfrac{1}{3} e^{3\lambda}(-\sin\zeta J + K^1 \wedge \hat{K}^2) - *_5 F_2 \wedge \tfrac{1}{3} e^{3\lambda} \cos\zeta K^1$$

- This shows the consistency of the truncation, at the level of the bosonic equations [Gauntlett, O Colgain, OV, hep-th/0611219] .

- The $D=11$ gravitino variations also reduce consistently to the $d=5$ gravitino variation [Gauntlett, O Colgain, OV, hep-th/0611219] .

# $AdS_5 \times_w M_6$ solutions with dual $d=4$, $N=2$ SCFTs

## $AdS_5 \times_w N_6$ solutions with dual $d = 4$, $N = 2$ SCFTs

- Consider the $AdS_5 \times_w N_6$ solutions in $D = 11$, which are dual to $N = 2$ SCFTs in 4 dimensions.
- These SCFTs have now a $U(1) \times SU(2)$ R-symmetry.
- Along with $U(1) \times SU(2)$ gauge fields $B$, $A^i$, $i = 1, 2, 3$, the corresponding gauged supergravity multiplet contains a scalar $X$ and a complex two-form $C_2$.
- This are the fields of Romans' $D = 5$, $N = 4$ gauged supergravity [Romans, Nucl. Phys. B **267** (1986) 433.].

# Consistent truncation to $D = 5$, $N = 4$ gauged supergravity

- Hence we expect that the reduction of $D = 11$ supergravity on $N_6$ truncates consistently to Romans' $D = 5$, $N = 4$ gauged supergravity.
- A consistent truncation of IIB supergravity on $S^5$ down to $D = 5$, $N = 4$ gauged supergravity was found in [ Lu, Pope, Tran, hep-th/9909203].
- A consistent truncation of $D = 11$ down to $D = 5$, $N = 4$ gauged supergravity was found in [ Cvetič, Lu, Pope, hep-th/0007109].

# $AdS_5 \times_w N_6$ solutions of $D = 11$ sugra with $N = 2$ susy

The most general $AdS_5$ solutions of $D = 11$ supergravity that are dual to $N = 2$ SCFTs in $d = 4$ where first derived by LLM [Lin, Lunin, Maldacena, hep-th/0409174]. and rederived by [Gauntlett, Mac Conamhna, Mateos, Waldram, arXiv:hep-th/0605146].

- The metric is

$$ds_{11}^2 = \lambda^{-1} ds^2(AdS_5) + ds^2(N_6), \qquad \lambda \in \Omega_0(M_6, \mathbb{R})$$

- A frame $(e^1, \ldots, e^6)$ can be introduced for $N_6$ with

$$e^4 = \frac{\lambda}{2m\sqrt{1-z}} d\rho$$

$$(e^5)^2 + (e^6)^2 = \frac{\lambda^2 \rho^2}{4m^2} \mathrm{d}\mu^i \mathrm{d}\mu^i$$

  where $z \equiv \lambda^3 \rho^2$, $\mu^i \mu^i = 1$ parametrise an $S^2$ and $e^3$ is a $U(1)$ Killing vector; in all, $ds^2(N_6)$ has $U(1) \times SU(2)$ isometry.

# $N = 2$ supersymmetry

- $N = 2$ supersymmetry places the following constraints on the frame:

$$\mathrm{d}\left(\lambda^{-1}\sqrt{1-z}e^1\right) = m\lambda^{-1/2}\left(\lambda^{3/2}\rho e^{14} + e^{23}\right),$$

$$\mathrm{d}\left(\lambda^{-1}\sqrt{1-z}e^2\right) = m\lambda^{-1/2}\left(\lambda^{3/2}\rho e^{24} - e^{13}\right),$$

$$\mathrm{d}\left(\frac{\lambda^{1/2}}{\sqrt{1-z}}e^3\right) = -\frac{2m\lambda}{1-z}e^{12} - \frac{3\lambda\rho}{(1-z)^{3/2}}\left[(\mathrm{d}\lambda)_4 e^{12} - (\mathrm{d}\lambda)_2 e^{14} + (\mathrm{d}\lambda)_1 e^{24}\right],$$

- The four-form flux is given by

$$G_4 = -\frac{1}{8m^2}\epsilon_{ijk}\mu^i d\mu^j \wedge \mathrm{d}\mu^k \wedge \left[\mathrm{d}\left(\lambda^{1/2}\rho\sqrt{1-z}e^3\right) + 2m\left(\lambda\rho e^{12} + \lambda^{-1/2}e^{34}\right)\right].$$

# Field content of $D = 5$, $N = 4$ gauged supergravity

Romans' $D = 5$, $N = 4$ gauged supergravity [Romans, Nucl. Phys. B **267** (1986) 433.]. consists of a metric $ds_5^2$, a scalar field $X$, $U(1) \times SU(2)$ gauge fields $B$, $A^i$ with $i = 1, 2, 3$ and a complex two form $C_2$ which is charged with respect to the $U(1)$ gauge field. The corresponding field strengths for these potentials are

$$
\begin{aligned}
G_2 &= dB \\
F_2^i &= dA^i + \frac{g}{\sqrt{2}} \epsilon_{ijk} A^j \wedge A^k \\
F_3 &= dC_2 - igB \wedge C_2
\end{aligned}
$$

# Kaluza-Klein ansatz: the metric

The KK ansatz for the metric is

$$ds^2_{11} = \lambda^{-1} X^{-1/3} \Delta^{1/3} ds^2_5 + ds^2(\hat{N}_6)$$

where

$$\Delta = Xz + X^{-2}(1-z)$$

$$\mathrm{d}s^2(\hat{N}_6) = X^{2/3}\Delta^{1/3}\left[(e^1)^2 + (e^2)^2 + (e^4)^2\right] + X^{5/3}\Delta^{-2/3}(\hat{e}^3)^2 + X^{-4/3}\Delta^{-2/3}\frac{\lambda^2\rho^2}{4g^2} D\mu^i D\mu^i$$

and

$$\hat{e}^3 = e^3 + \frac{\sqrt{1-z}}{\lambda^{1/2}} B$$

$$D\mu^i = d\mu^i - \sqrt{2} g \epsilon_{ijk} A^k \mu^j$$

$(m = -g)$

# Kaluza-Klein ansatz: the four-form

For the four-form, the KK ansatz is

$$G_4 = \tilde{G}_4 + G_2 \wedge \beta_2 + F_2^i \wedge \beta_2^i + *_5 F_2^i \wedge \beta_{1i} + (C_2 \wedge \alpha_2 + F_3 \wedge \alpha_1 + c.c.)$$

where

$$\begin{aligned}\tilde{G}_4 \quad &= \quad -\frac{1}{8g^2}\epsilon_{ijk}\mu^i D\mu^j \wedge D\mu^k \wedge \Bigg[\mathrm{d}\left(X^{-2}\Delta^{-1}\rho(1-z)\right)\frac{\lambda^{1/2}}{\sqrt{1-z}}\hat{e}^3 \\ &+ \quad X^{-2}\Delta^{-1}\rho(1-z)\mathrm{d}\left(\frac{\lambda^{1/2}}{\sqrt{1-z}}e^3\right) - 2g\left(\lambda\rho e^{12} + \lambda^{-1/2}\hat{e}^{34}\right)\Bigg].\end{aligned}$$

and...

# Kaluza-Klein ansatz: the four-form

$$G_4 = \tilde{G}_4 + G_2 \wedge \beta_2 + F_2^i \wedge \beta_2^i + *_5 F_2^i \wedge \beta_{1i} + (C_2 \wedge \alpha_2 + F_3 \wedge \alpha_1 + c.c.)$$

$$\begin{aligned}
\beta_2 &= \frac{1}{8g^2}\rho z X \Delta^{-1} \epsilon_{ijk}\mu^i D\mu^j \wedge D\mu^k \\
\beta_{2i} &= -\frac{1}{2\sqrt{2}g}\left[X^{-2}\Delta^{-1}\rho\lambda^{1/2}\sqrt{1-z}D\mu^i \wedge \hat{e}^3 - 2m\mu_i(\lambda\rho e^{12} + \lambda^{-1/2}\hat{e}^{34})\right] \\
\beta_1^i &= -\frac{X^{-2}}{2\sqrt{2}g}\left(\mu^i d\rho + \rho D\mu^i\right) \\
\alpha_1 &= \frac{1}{8g^2}\lambda^{-1}\sqrt{1-z}(e^1 - ie^2) \\
\alpha_2 &= -\frac{1}{8g}(e^1 - ie^2)\left(\lambda\rho e^4 + i\lambda^{-1/2}\hat{e}^3\right)
\end{aligned}$$

# Consistent truncation

The KK ansatz satisfies the $D=11$ field equations provided the $d=5$ fields satisfy the equations of $D=5$, $N=4$ gauged supergravity:

$$
\begin{aligned}
d(X^{-1} *dX) &= \tfrac{1}{3}X^4 *G_2 \wedge G_2 - \tfrac{1}{6}X^{-2}\,(*F_2^i \wedge F_2^i + *\bar{C}_2 \wedge C_2) \\
&\quad -\tfrac{4}{3}g^2\,(X^2 - X^{-1})\,\mathrm{vol}_5\,, \\
d(X^4 *G_2) &= -\tfrac{1}{2}F_2^i \wedge F_2^i - \tfrac{1}{2}\bar{C}_2 \wedge C_2\,, \\
D(X^{-2} *F_2^i) &= -F_2^i \wedge G_2\,, \\
X^2 *F_3 &= -i\,g\,C_2\,, \\
R_{\mu\nu} &= 3X^{-2}\,\partial_\mu X\,\partial_\nu X - \tfrac{4}{3}g^2\,(X^2 + 2X^{-1})\,g_{\mu\nu} \\
&\quad +\tfrac{1}{2}X^4\,(G_\mu{}^\rho G_{\nu\rho} - \tfrac{1}{6}g_{\mu\nu}\,G_2^2) + \tfrac{1}{2}X^{-2}\,(F_\mu^i{}^\rho F_{\nu\rho}^i - \tfrac{1}{6}g_{\mu\nu}\,(F_2^i)^2) \\
&\quad +\tfrac{1}{2}X^{-2}\,(\bar{C}_{(\mu}{}^\rho\, C_{\nu)\rho} - \tfrac{1}{6}g_{\mu\nu}\,|C_2|^2)\,,
\end{aligned}
$$

which proves the consistency of the truncation [Gauntlett, OV, arXiv:0711.xxxx].

# $AdS_5 \times_w M_6$ solutions with dual $d = 4$, $N = 2$ SCFTs

# Further examples in $D = 11$

Other examples suport our conjecture about consistent KK reductions.

- The $D = 11$ solutions of the form $AdS_4 \times SE_7$, where $SE_7$ is Sasaki-Einstein are dual to 3d $N = 2$ SCFTs, and the reduction of $D = 11$ on $M_7$ consistently truncates to $d = 4$, $N = 2$ gauged sugra [Gauntlett, OV, arXiv:0707.2315] .

- The $D = 11$ solutions of the form $AdS_4 \times_w M_7$, corresponding to M5-branes wrapping SLAG 3 cycles [Gauntlett, Mac Conamhna, Mateos, Waldram hep-th/0605146] , also allow for a consistent reduction of $D = 11$ sugra on $M_7$ to $d = 4$, $N = 2$ gauged sugra [Gauntlett, OV, arXiv:0707.2315] .

## Further examples in IIB

- IIB sugra on $d = 5$ Sasaki-Einstein spaces is dual to a 4d $N = 1$ SCFT, and consistently truncates to minimal $d = 5$ gauged sugra [Buchel, Liu, hep-th/0608002] .
- The IIB solutions of the form $AdS_5 \times_w M_5$ with $N = 1$ susy and all fluxes active [Gauntlett, Martelli, Sparks, Waldram, hep-th/0510125] also allow for a consistent reduction of IIB on $M_5$ to $d = 5$ minimal gauged sugra [Gauntlett, OV, arXiv:0707.2315] .

## Conclusions

- Supersymmetric solutions $AdS_d \times_w M_{D-d}$ in $D = 11, 10$ have been conjectured to give rise to a consistent truncation of $D = 11, 10$ sugra on $M_{D-d}$ down to a pure, gauged sugra in $d$ dimensions whose fields are dual to those defining the $(d-1)$-dimensional dual SCFT.

- Consistent truncations have been explicitly shown to exist for the most general solutions in $D = 11$ sugra with $d = 4$, $N = 1$ and $N = 2$ dual SCFTs.

- Other examples including $AdS_4$, $AdS_5$ in IIB and $D = 11$ give further evidence.

# Outlook

- The conditions that allow $D = 11, 10$ solutions with $AdS_d$ factors allow for consistent truncations: how about the other way around?
- Better understanding of the opposite statement could lead to the characterisation of new $AdS$ solutions (e.g.: the most general $AdS_5$ solutions in IIB dual to $d = 4$, $N = 2$ SCFTs).
- Our explicit KK ansatze allow for the uplift of lower $d$ solutions to higher $D$, that would need to be interpreted in $D$ dimensions.
- It would be interesting to recast the known KK truncations on spheres in this language.
- It would be interesting to prove the conjecture, both from the sugra and CFT sides.