

# MSSM-Like Models with R-Parity from Strings

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*Based on*

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# Outline of my Talk

- Why we are not happy with the Standard Model . . .
- Hints at Physics beyond the Standard Model
- String theory as an ultraviolet completion of GUTS
- MSSM-like theories from strings
- Conclusions

# The Standard Model

Gauge group:

$$\text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y$$

Particle content:

$Q$	$(\mathbf{3}, \mathbf{2})_{1/3}$	$L$	$(\mathbf{1}, \mathbf{2})_{-1}$	$H$	$(\mathbf{1}, \mathbf{2})_1$
$\bar{u}$	$(\overline{\mathbf{3}}, \mathbf{1})_{-4/3}$	$\bar{e}$	$(\mathbf{1}, \mathbf{1})_2$	$\bar{H}$	$(\mathbf{1}, \mathbf{2})_{-1}$
$\bar{d}$	$(\overline{\mathbf{3}}, \mathbf{1})_{2/3}$	$\bar{\nu}$	$(\mathbf{1}, \mathbf{1})_0$		

# Why are we not happy with SM?

## (i) Too many free parameters

Gauge sector: 3 couplings $g'$ , $g$ , $g_3$	3
Quark sector: 6 masses, 3 mixing angles, 1 CP phase	10
Lepton sector: 6 masses, 3 mixing angles and 1-3 phases	10
Higgs sector: Quartic coupling $\lambda$ and vev $v$	2
$\theta$ parameter of QCD	1

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26

# Why are we not happy with SM?

## (ii) Structure of gauge symmetry

Why the product structure  $SU(3)_c \times SU(2)_L \times U(1)_Y$ ?

Why 3 different coupling constants  $g', g, g_3$ ?

## (iii) Structure of family multiplets

One family is

$$(\mathbf{3},\mathbf{2})_{1/3} + (\overline{\mathbf{3}},\mathbf{1})_{-4/3} + (\mathbf{1},\mathbf{1})_{-2} + (\overline{\mathbf{3}},\mathbf{1})_{2/3} + (\mathbf{1},\mathbf{2})_{-1} + (\mathbf{1},\mathbf{1})_0$$

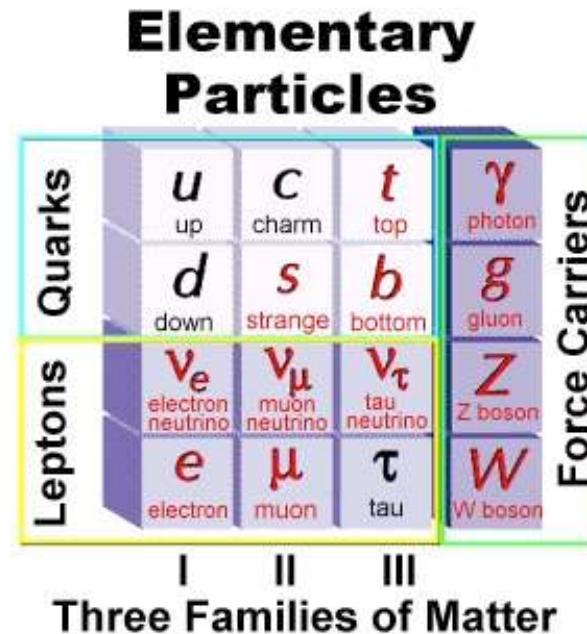
$Q$                      $\bar{u}$                      $\bar{e}$                      $\bar{d}$                      $L$                      $\bar{\nu}$

Can the particles be reorganized in a single representation?

# Why are we not happy with SM?

## (iv) Repetition of families

Earth, sun, stars, etc. are built from quarks and leptons of one generation. Why is this pattern for 1 generation replicated 3 times? Horizontal symmetries?



# Why are we not happy with SM?

## (v) Texture of Yukawa couplings

Minimal mixing in quark sector

$$V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \simeq \begin{pmatrix} 0.97 & 0.22 & 0.00 \\ 0.22 & 0.97 & 0.04 \\ 0.00 & 0.04 & 0.99 \end{pmatrix}$$

Yukawa coupling of top-quark  $\simeq 1$ , and thus  $m_t \simeq 200$  GeV.  
But why are the other quarks so light?

# Why are we not happy with SM?

## (vi) Texture of Yukawa couplings

Maximal mixing in lepton sector

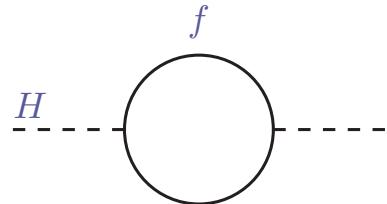
$$U_{\text{PMNS}} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \simeq \begin{pmatrix} 0.8 & 0.5 & 0.0 \\ -0.4 & 0.6 & 0.7 \\ 0.4 & -0.6 & 0.7 \end{pmatrix}$$

Why are neutrinos so light?

$$\Delta m_\nu^2 \sim 10^{-2} - 10^{-5} \text{ eV}, \quad \sum m_\nu \lesssim 2 \text{ eV}$$

# Why are we not happy with SM?

## (vii) Hierarchy problem



$$\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2} \Lambda_{\text{UV}}^2 + \dots$$

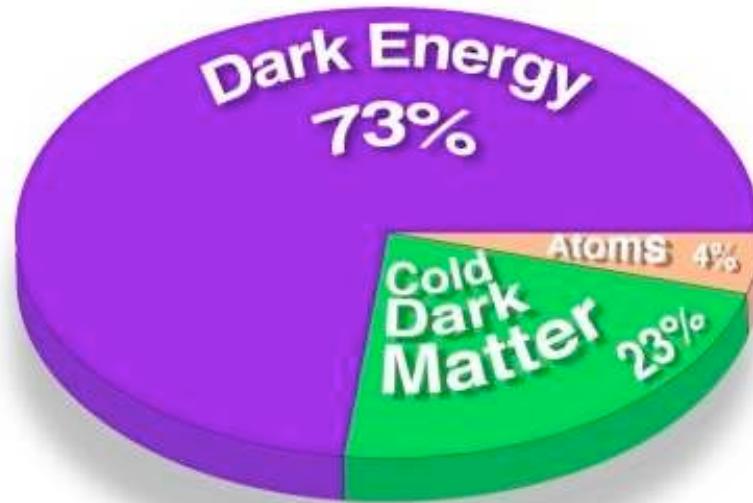
- Stability of the vacuum
- Requires incredible fine-tuning to set things right

Another way to put it: Why are there 2 fundamental scales at all?

# Why are we not happy with SM?

## (viii) Dark Matter

23% of our universe is made up of dark matter and the Standard Model offers no candidate particle ...



# Why are we not happy with SM?

## (ix) Dark Energy

73% of our universe is made up of dark energy



Cosmological constant as calculated from QFT is the worst-predicted quantity in particle physics

# Why are we not happy with SM?

## (x) Gravity

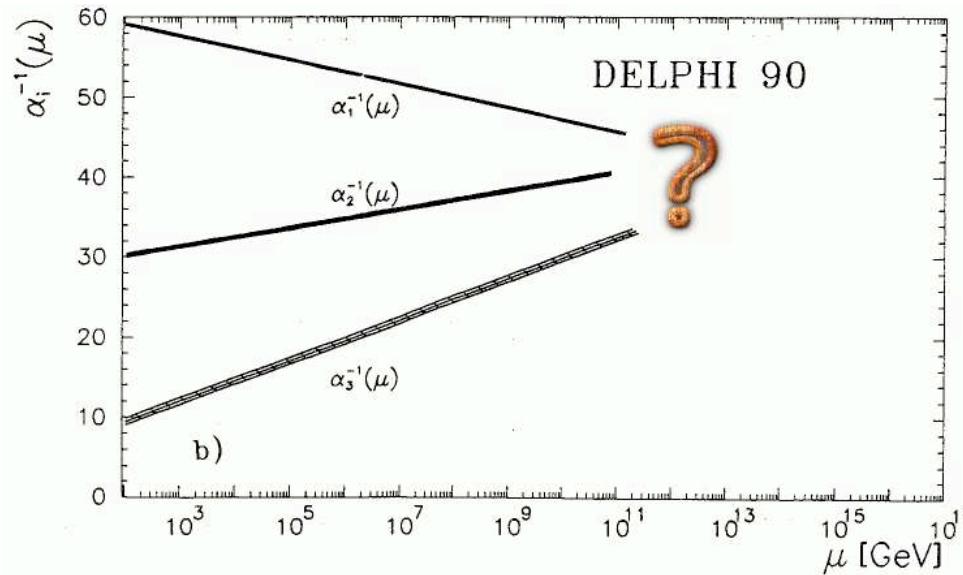
- Scales relevant in everyday life  $\leadsto$  Newton's theory
- Satellites, solar system, etc.  $\leadsto$  Still Newton's theory
- Cosmological scales  $\leadsto$  Einstein's theory of GR
- Very small scales  $\leadsto$  Need quantum theory of gravitation
- Don't know how to quantize gravity and how to unify with SM

## (xi) Many other problems

Baryon asymmetry in the universe, charge quantization, ...

# Hints at Physics Beyond the SM?

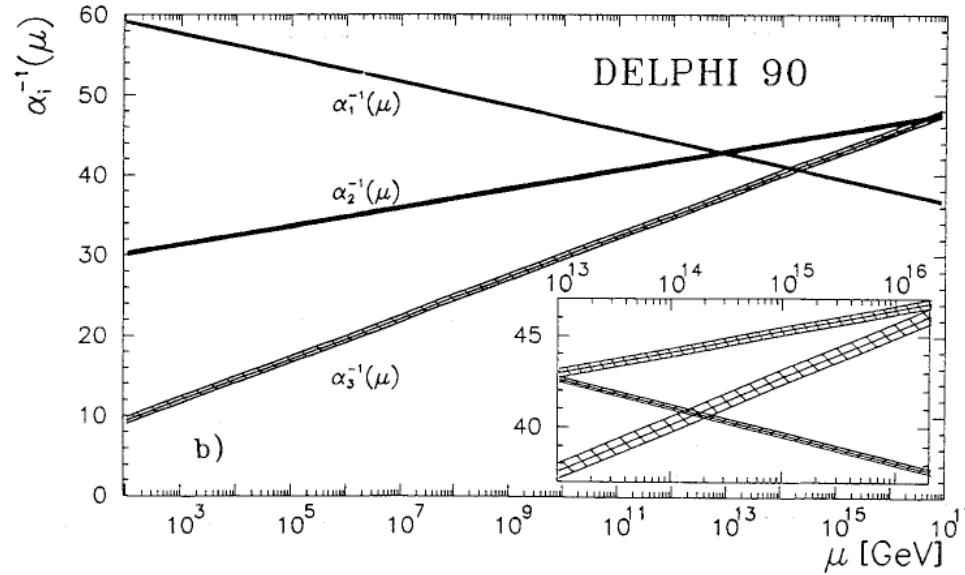
H. Georgi and S. L. Glashow, “Unity of all elementary particle forces,” *Phys. Rev. Lett.* **32** (1974)



- ★ Running gauge couplings seem to meet at  $10^{15}$  GeV ✓
- ~ Grand Unification: 1 gauge group, one 1 constant !

# Hints at Physics Beyond the SM?

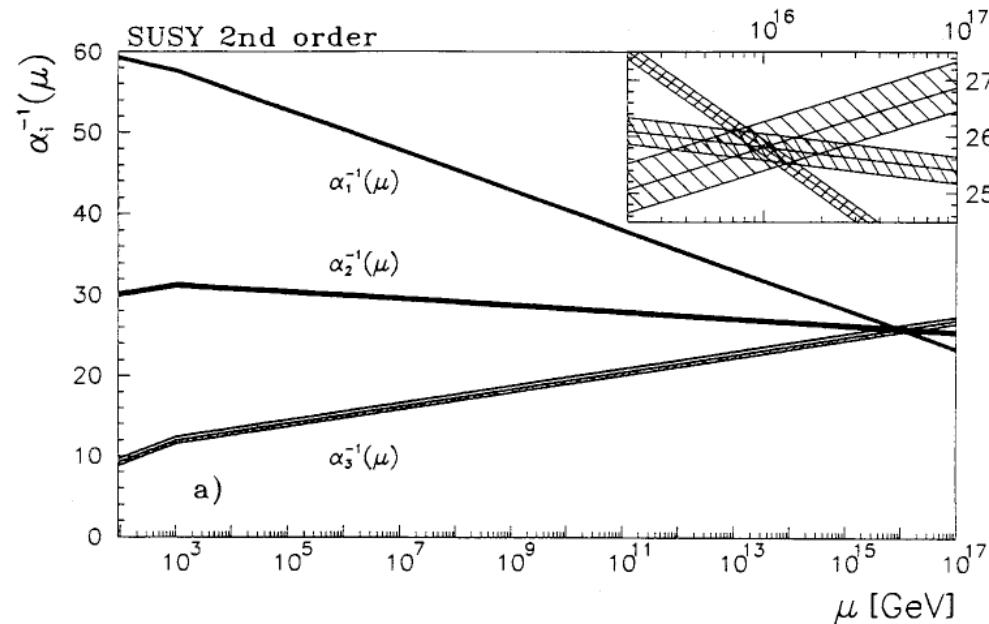
H. Georgi and S. L. Glashow, “Unity of all elementary particle forces,” *Phys. Rev. Lett.* **32** (1974)



★ Looking more closely, couplings do not unify X  
Are we missing something?

# Hints at Physics Beyond the SM?

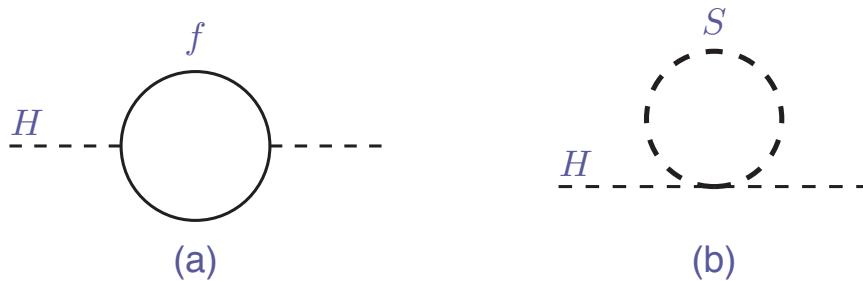
S. Dimopoulos, S. Raby, F. Wilczek, “Unification of couplings,” *Phys. Today*. **44N10** (1991) 25-33.



- ★ Supersymmetry helps: Unification at  $\sim 3 \times 10^{16}$  GeV ✓
- ~ Strong motivation for GUTS and SUSY!

# Another Motivation for SUSY

- Remember the hierarchy problem



For fermions :  $\Delta m_H^2 = -\frac{|\lambda_f|^2}{8\pi^2} \Lambda_{\text{UV}}^2 + \dots$

For scalars :  $\Delta m_H^2 = \frac{\lambda_S}{16\pi^2} [\Lambda_{\text{UV}}^2 - 2m_S^2 \ln(\Lambda_{\text{UV}}/m_S) + \dots]$

- SUSY correlates 1 fermion to 2 real scalars  
~ Quadratic divergencies cancel !
- “Technical” solution to hierarchy problem:  
Still 2 mass scales in theory, but energy dependence ‘mild’

# Grand Unification

Assume some grand unified gauge group at  $3 \times 10^{16}$ :

- Big enough to include  $SU(3)_c \times SU(2)_L \times U(1)_Y$
- “Minimal” otherwise: Rank 4

All Lie algebras: classical and exceptional

$SU(n-1)$	$E_6$	$G_2$
$Sp(2n)$	$E_7$	$F_4$
$SO(2n+1)$	$E_8$	
$SO(2n)$		

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Now look at the minimal ones:

$SU(2)^4$ ,  $SO(5)^2$ ,  $SU(3)^2$ ,  $(G_2)^2$ ,  $SO(8)$ ,  $SO(9)$ ,  $Sp(8)$ ,  $F_4$ ,  $SU(5)$

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- Do not contain  $SU(3)$  as a subset

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- Do not contain  $SU(3)$  as a subset
- Cannot define sensible charge operator
- No complex representations
- $SU(5)$  is unique candidate !!!

# Grand Unification

Minimal SU(5) very predictive, but also very constrained . . .

~ Suffers from some problems (see later in talk)

We need to introduce some “extra degrees of freedom” into the theory

- Consider next-to-minimal algebras
- Prefer simple Lie algebras, i.e. no direct products
- Must have complex representations for particles and anti-particles

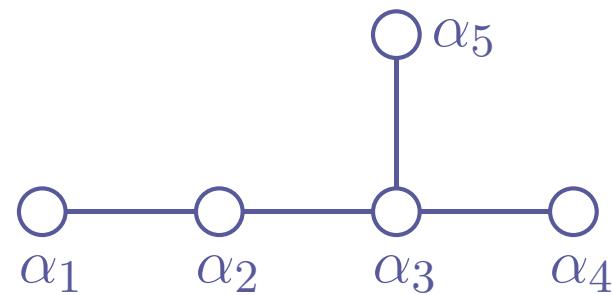
Candidates:

$$\text{SU}(n) \text{ for } n \geq 5, \quad \text{SO}(4n + 2) \text{ for } n \geq 2, \quad \text{E}_6$$

# Supersymmetric Grand Unification

Assume  $\text{SO}(10)$  at fundamental scale  $\sim 10^{16}$  GeV

H. Fritzsch and P. Minkowski, “Unified interactions of leptons and hadrons,” *Ann. Phys.* **93** (1975)

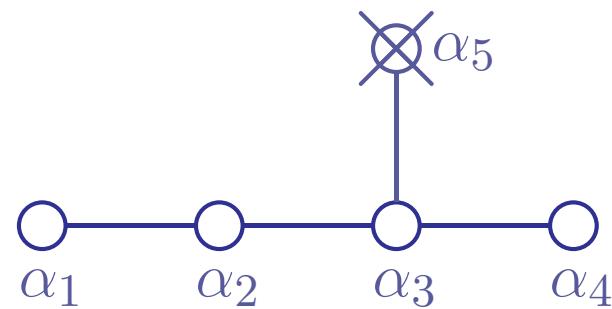


$\text{SO}(10)$

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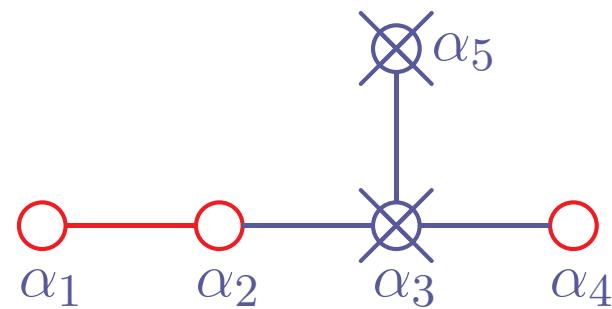


$$\text{SO}(10) \rightarrow \text{SU}(5) \times \text{U}(1)_X$$

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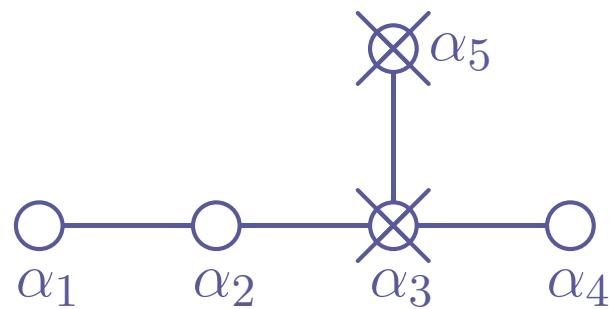


$$\text{SO}(10) \rightarrow \text{SU}(5) \times \text{U}(1)_X \rightarrow \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y \times \text{U}(1)_{\text{B-L}}$$

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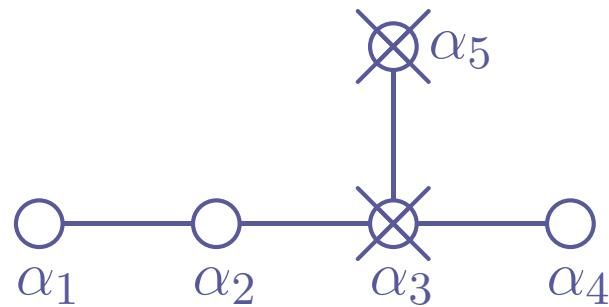
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16

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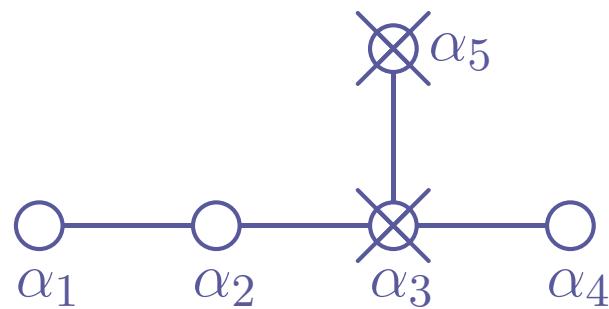
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$$16 \rightarrow 10 + \bar{5} + 1$$

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$$\text{SO}(10) \rightarrow \text{SU}(5) \times \text{U}(1)_X \rightarrow \text{SU}(3) \times \text{SU}(2) \times \text{U}(1)_Y \times \text{U}(1)_{\text{B-L}}$$

$$16 \rightarrow \mathbf{10} + \overline{\mathbf{5}} + \mathbf{1}$$

$$\rightarrow (\mathbf{3}, \mathbf{2})_{1/3} + (\overline{\mathbf{3}}, \mathbf{1})_{-4/3} + (\mathbf{1}, \mathbf{1})_{-2} + (\overline{\mathbf{3}}, \mathbf{1})_{2/3} + (\mathbf{1}, \mathbf{2})_{-1} + (\mathbf{1}, \mathbf{1})_0$$

$Q$

$\bar{u}$

$\bar{e}$

$\bar{d}$

$L$

$\bar{\nu}$

# Predictions from Grand Unification

	SU(5)	SO(10)
• Gauge coupling unification		
• 1 family of quarks and leptons in 1 irrep		
• Quantization of electric charge		
• $\sin^2 \theta_w$ in agreement w/experiment		
• $\frac{m_b}{m_\tau}$ ratio		
• Right-handed neutrino		
• Smallness of neutrino masses		

# Problems of Grand Unification

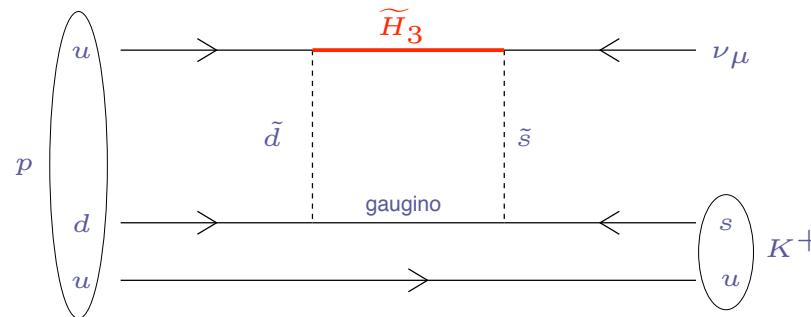
- Large representations required:

24 to break  $SU(5)$   $\leadsto$  Many more particles

45 for realistic mass matrices  $\leadsto$  Even more particles

- Doublet-triplet splitting problem

$$\mathbf{10} \rightarrow \mathbf{5} + \overline{\mathbf{5}} \rightarrow (\mathbf{1},\mathbf{2}) + (\mathbf{3},\mathbf{1}) + (\mathbf{1},\mathbf{2}) + (\overline{\mathbf{3}},\mathbf{1})$$

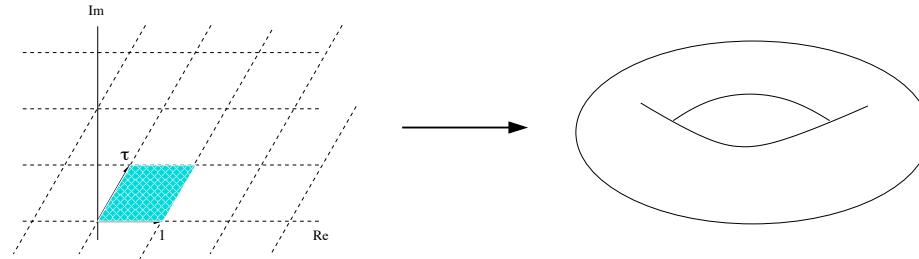


- Both problems can be avoided by extra dimensions !

# The Idea of an Orbifold

Consider QFT in 6 dimensions: 4 large and 2 small

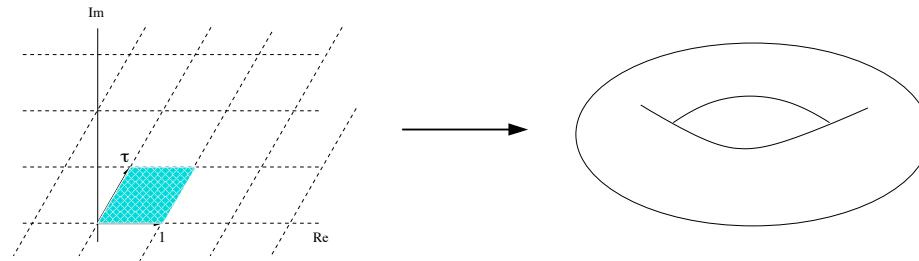
- Compactify 2 dimensions on torus



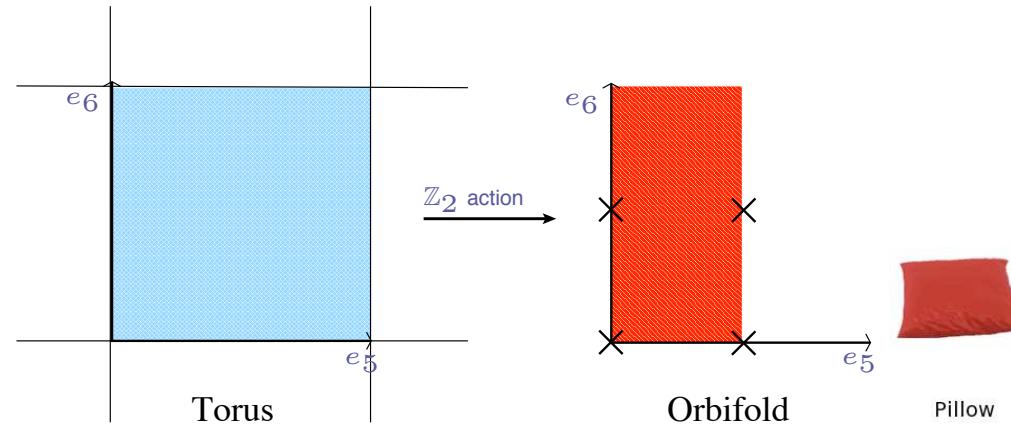
# The Idea of an Orbifold

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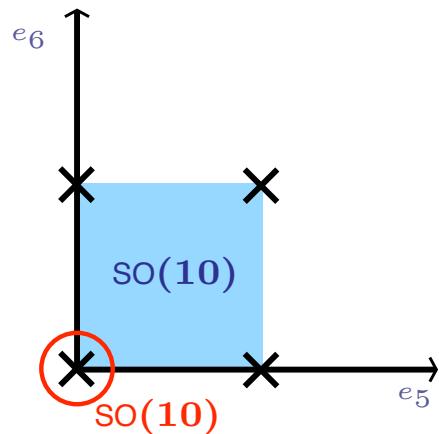
- Compactify 2 dimensions on torus



- Impose e.g.  $\mathbb{Z}_2$  symmetry:  $\mathcal{O} = T^2/P, \quad P = \{1, \theta\}$

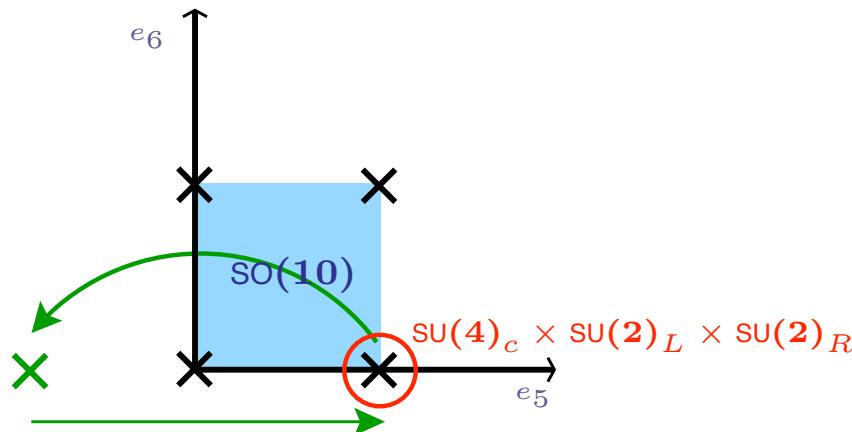


# Breaking the Symmetry



- Rotation + translation in spacetime  $\sim$  phase in gauge d.o.f.
  - ↪ Convenient in field theory orbifolds
  - ↪ Mandatory in string orbifolds
- Consider fixed  $f_1$  point in origin
$$\theta f_1 = f_1 \quad \sim \quad |\alpha_i\rangle \rightarrow e^{2\pi i V \cdot \alpha_i} |\alpha_i\rangle$$
State survives if and only if “parity” is +1 !!!
- In this model,  $V \equiv 0$   $\sim$  Full symmetry survives !!!

# Breaking the Symmetry



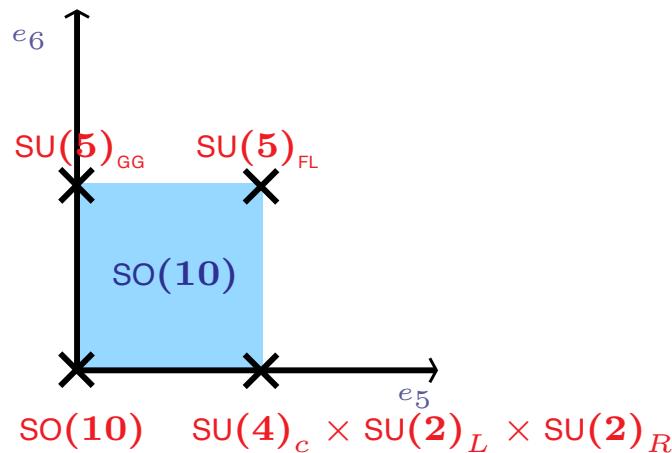
- Consider fixed  $f_2$  point in origin

$$\theta f_2 + e_5 = f_2 \quad \sim \quad |\alpha_i\rangle \rightarrow e^{2\pi i(V+A_5)\cdot\alpha_i} |\alpha_i\rangle$$

- Choose  $A_5 = (\ast \ast \ast \ast \ast)$ :

$$A_5 \cdot \alpha_i \stackrel{!}{=} 0 \quad \rightarrow \quad 21 \text{ } \alpha_i \text{ survive} \quad \sim \quad \text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R$$

# Breaking the Symmetry



- 4 gauge groups at fixed points in extra dimension
- In 4 dimensions, we see intersection of these gauge symmetries:
$$SO(10) \cap SU(5) \cap PS = SU(3)_c \times SU(2)_L \times U(1)_Y$$
- Judicious placement of fields  $\leadsto$  Semi-realistic fermion masses
- Doublet-triplet splitting  $\leadsto$  proton stability

# Taking the Step to 10 Dimensions

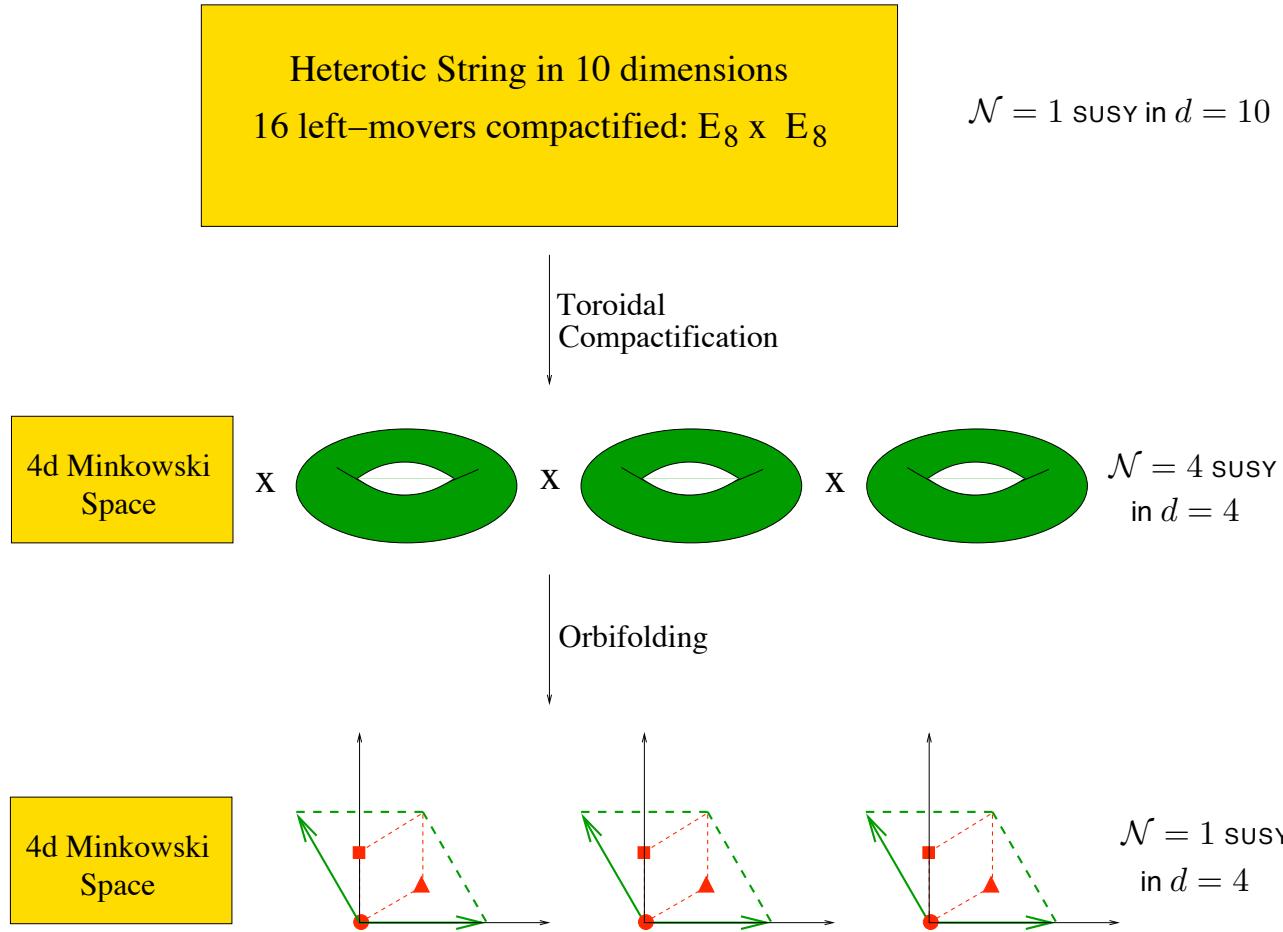
Some shortcomings of field theory orbifolds:

- Non-renormalizable and as such UV-divergent
- Number of dimensions arbitrary
- Symmetry breaking pattern arbitrary
- Placement of fields arbitrary
- In short: No organizing principle

Advantages of stringy orbifolds:

- Finite and does not need renormalization
- Predicts 10 spacetime dimensions
- Symmetry in 10 dimensions (“bulk”) is  $E_8 \times E_8$
- Spectrum and localization of particles predicted
- Includes quantum theory of gravity

# Orbifolds in 10 Dimensions



# How to choose the Compactification Lattice

- Symmetry must be automorphism of lattice defining  $T^6$
- In 3 dimensions: 219 crystallographic space groups
- In 6 dimensions: 28,927,922 crystallographic space groups
- Demand  $P \subset \text{SU}(3) \subset \text{SO}(6)$  for  $\mathcal{N} = 1$  SUSY
- Restrict to abelian  $P$  for simplicity of construction

# How to choose the Compactification Lattice

Point Group	Twist $v$
$\mathbb{Z}_3$	(1/3, 1/3, -2/3)
$\mathbb{Z}_4$	(1/4, 1/4, -1/2)
$\mathbb{Z}_{6-\text{I}}$	(1/6, 1/6, -1/3)
$\mathbb{Z}_{6-\text{II}}$	(1/6, 1/3, -1/2)
$\mathbb{Z}_7$	(1/7, 2/7, -3/7)
$\mathbb{Z}_{8-\text{I}}$	(1/8, 1/4, -3/8)
$\mathbb{Z}_{8-\text{II}}$	(1/8, 3/8, -1/2)
$\mathbb{Z}_{12-\text{I}}$	(1/12, 1/3, -5/12)
$\mathbb{Z}_{12-\text{II}}$	(1/12, 5/12, -1/2)

Point Group	Twist $v_1$	Twist $v_2$
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1/2, 0, -1/2)	(0, 1/2, -1/2)
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1/3, 0, -1/3)	(0, 1/3, -1/3)
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1/2, 0, -1/2)	(0, 1/4, -1/4)
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1/4, 0, -1/4)	(0, 1/4, -1/4)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{I}}$	(1/2, 0, -1/2)	(0, 1/6, -1/6)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{II}}$	(1/2, 0, -1/2)	(1/6, 1/6, -1/3)
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1/3, 0, -1/3)	(0, 1/6, -1/6)
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1/6, 0, -1/6)	(0, 1/6, -1/6)

Abelian point groups with  $\mathcal{N} = 1$  SUSY in 4 dimensions

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$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1/3, 0, -1/3)	(0, 1/6, -1/6)
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1/6, 0, -1/6)	(0, 1/6, -1/6)

Abelian point groups with  $\mathcal{N} = 1$  SUSY in 4 dimensions

Ibanez / Kim / Nilles / Quevedo (1987)  
and many others

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$\mathbb{Z}_3$	(1/3, 1/3, -2/3)
$\mathbb{Z}_4$	(1/4, 1/4, -1/2)
$\mathbb{Z}_{6-\text{I}}$	(1/6, 1/6, -1/3)
$\mathbb{Z}_{6-\text{II}}$	(1/6, 1/3, -1/2)
$\mathbb{Z}_7$	(1/7, 2/7, -3/7)
$\mathbb{Z}_{8-\text{I}}$	(1/8, 1/4, -3/8)
$\mathbb{Z}_{8-\text{II}}$	(1/8, 3/8, -1/2)
$\mathbb{Z}_{12-\text{I}}$	(1/12, 1/3, -5/12)
$\mathbb{Z}_{12-\text{II}}$	(1/12, 5/12, -1/2)

Point Group	Twist $v_1$	Twist $v_2$
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1/2, 0, -1/2)	(0, 1/2, -1/2)
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1/3, 0, -1/3)	(0, 1/3, -1/3)
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1/2, 0, -1/2)	(0, 1/4, -1/4)
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1/4, 0, -1/4)	(0, 1/4, -1/4)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{I}}$	(1/2, 0, -1/2)	(0, 1/6, -1/6)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{II}}$	(1/2, 0, -1/2)	(1/6, 1/6, -1/3)
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1/3, 0, -1/3)	(0, 1/6, -1/6)
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1/6, 0, -1/6)	(0, 1/6, -1/6)

Abelian point groups with  $\mathcal{N} = 1$  SUSY in 4 dimensions

K.S. Choi / Groot Nibbelink / Trapletti (2004)  
 Nilles / Ramos-Sanchez / Vaudrevange / A. W. (2006)

# How to choose the Compactification Lattice

Point Group	Twist $v$
$\mathbb{Z}_3$	(1/3, 1/3, -2/3)
$\mathbb{Z}_4$	(1/4, 1/4, -1/2)
$\mathbb{Z}_{6-\text{I}}$	(1/6, 1/6, -1/3)
$\mathbb{Z}_{6-\text{II}}$	(1/6, 1/3, -1/2)
$\mathbb{Z}_7$	(1/7, 2/7, -3/7)
$\mathbb{Z}_{8-\text{I}}$	(1/8, 1/4, -3/8)
$\mathbb{Z}_{8-\text{II}}$	(1/8, 3/8, -1/2)
$\mathbb{Z}_{12-\text{I}}$	(1/12, 1/3, -5/12)
$\mathbb{Z}_{12-\text{II}}$	(1/12, 5/12, -1/2)

Point Group	Twist $v_1$	Twist $v_2$
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1/2, 0, -1/2)	(0, 1/2, -1/2)
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1/3, 0, -1/3)	(0, 1/3, -1/3)
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1/2, 0, -1/2)	(0, 1/4, -1/4)
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1/4, 0, -1/4)	(0, 1/4, -1/4)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{I}}$	(1/2, 0, -1/2)	(0, 1/6, -1/6)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{II}}$	(1/2, 0, -1/2)	(1/6, 1/6, -1/3)
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1/3, 0, -1/3)	(0, 1/6, -1/6)
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1/6, 0, -1/6)	(0, 1/6, -1/6)

Abelian point groups with  $\mathcal{N} = 1$  SUSY in 4 dimensions

Kobayashi / Raby / Zhang (2004)

Buchmuller / Hamaguchi / Lebedev / Ratz (2005/6)

Lebedev / Nilles / Raby / Ramos-Sanchez / Ratz / Vaudrevange / A.W. (2006/7)

# How to choose the Compactification Lattice

Point Group	Twist $v$
$\mathbb{Z}_3$	(1/3, 1/3, -2/3)
$\mathbb{Z}_4$	(1/4, 1/4, -1/2)
$\mathbb{Z}_{6-\text{I}}$	(1/6, 1/6, -1/3)
$\mathbb{Z}_{6-\text{II}}$	(1/6, 1/3, -1/2)
$\mathbb{Z}_7$	(1/7, 2/7, -3/7)
$\mathbb{Z}_{8-\text{I}}$	(1/8, 1/4, -3/8)
$\mathbb{Z}_{8-\text{II}}$	(1/8, 3/8, -1/2)
$\mathbb{Z}_{12-\text{I}}$	(1/12, 1/3, -5/12)
$\mathbb{Z}_{12-\text{II}}$	(1/12, 5/12, -1/2)

Point Group	Twist $v_1$	Twist $v_2$
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1/2, 0, -1/2)	(0, 1/2, -1/2)
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1/3, 0, -1/3)	(0, 1/3, -1/3)
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1/2, 0, -1/2)	(0, 1/4, -1/4)
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1/4, 0, -1/4)	(0, 1/4, -1/4)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{I}}$	(1/2, 0, -1/2)	(0, 1/6, -1/6)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{II}}$	(1/2, 0, -1/2)	(1/6, 1/6, -1/3)
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1/3, 0, -1/3)	(0, 1/6, -1/6)
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1/6, 0, -1/6)	(0, 1/6, -1/6)

Abelian point groups with  $\mathcal{N} = 1$  SUSY in 4 dimensions

Förste / Nilles/ Vaudrevange / A. W. (2004)

# How to choose the Compactification Lattice

Point Group	Twist $v$
$\mathbb{Z}_3$	(1/3, 1/3, -2/3)
$\mathbb{Z}_4$	(1/4, 1/4, -1/2)
$\mathbb{Z}_{6-\text{I}}$	(1/6, 1/6, -1/3)
$\mathbb{Z}_{6-\text{II}}$	(1/6, 1/3, -1/2)
$\mathbb{Z}_7$	(1/7, 2/7, -3/7)
$\mathbb{Z}_{8-\text{I}}$	(1/8, 1/4, -3/8)
$\mathbb{Z}_{8-\text{II}}$	(1/8, 3/8, -1/2)
$\mathbb{Z}_{12-\text{I}}$	(1/12, 1/3, -5/12)
$\mathbb{Z}_{12-\text{II}}$	(1/12, 5/12, -1/2)

Point Group	Twist $v_1$	Twist $v_2$
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1/2, 0, -1/2)	(0, 1/2, -1/2)
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1/3, 0, -1/3)	(0, 1/3, -1/3)
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1/2, 0, -1/2)	(0, 1/4, -1/4)
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1/4, 0, -1/4)	(0, 1/4, -1/4)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{I}}$	(1/2, 0, -1/2)	(0, 1/6, -1/6)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{II}}$	(1/2, 0, -1/2)	(1/6, 1/6, -1/3)
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1/3, 0, -1/3)	(0, 1/6, -1/6)
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1/6, 0, -1/6)	(0, 1/6, -1/6)

Abelian point groups with  $\mathcal{N} = 1$  SUSY in 4 dimensions

Raby / A. W. work in progress

# How to choose the Compactification Lattice

Point Group	Twist $v$
$\mathbb{Z}_3$	(1/3, 1/3, -2/3)
$\mathbb{Z}_4$	(1/4, 1/4, -1/2)
$\mathbb{Z}_6\text{-I}$	(1/6, 1/6, -1/3)
$\mathbb{Z}_6\text{-II}$	(1/6, 1/3, -1/2)
$\mathbb{Z}_7$	(1/7, 2/7, -3/7)
$\mathbb{Z}_8\text{-I}$	(1/8, 1/4, -3/8)
$\mathbb{Z}_8\text{-II}$	(1/8, 3/8, -1/2)
$\mathbb{Z}_{12}\text{-I}$	(1/12, 1/3, -5/12)
$\mathbb{Z}_{12}\text{-II}$	(1/12, 5/12, -1/2)

Point Group	Twist $v_1$	Twist $v_2$
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1/2, 0, -1/2)	(0, 1/2, -1/2)
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1/3, 0, -1/3)	(0, 1/3, -1/3)
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1/2, 0, -1/2)	(0, 1/4, -1/4)
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1/4, 0, -1/4)	(0, 1/4, -1/4)
$\mathbb{Z}_2 \times \mathbb{Z}_6\text{-I}$	(1/2, 0, -1/2)	(0, 1/6, -1/6)
$\mathbb{Z}_2 \times \mathbb{Z}_6\text{-II}$	(1/2, 0, -1/2)	(1/6, 1/6, -1/3)
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1/3, 0, -1/3)	(0, 1/6, -1/6)
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1/6, 0, -1/6)	(0, 1/6, -1/6)

Abelian point groups with  $\mathcal{N} = 1$  SUSY in 4 dimensions

B. Kye / J. E. Kim et al. (2006)

# How to choose the Compactification Lattice

Point Group	Twist $v$
$\mathbb{Z}_3$	(1/3, 1/3, -2/3)
$\mathbb{Z}_4$	(1/4, 1/4, -1/2)
$\mathbb{Z}_{6-\text{I}}$	(1/6, 1/6, -1/3)
$\mathbb{Z}_{6-\text{II}}$	(1/6, 1/3, -1/2)
$\mathbb{Z}_7$	(1/7, 2/7, -3/7)
$\mathbb{Z}_{8-\text{I}}$	(1/8, 1/4, -3/8)
$\mathbb{Z}_{8-\text{II}}$	(1/8, 3/8, -1/2)
$\mathbb{Z}_{12-\text{I}}$	(1/12, 1/3, -5/12)
$\mathbb{Z}_{12-\text{II}}$	(1/12, 5/12, -1/2)

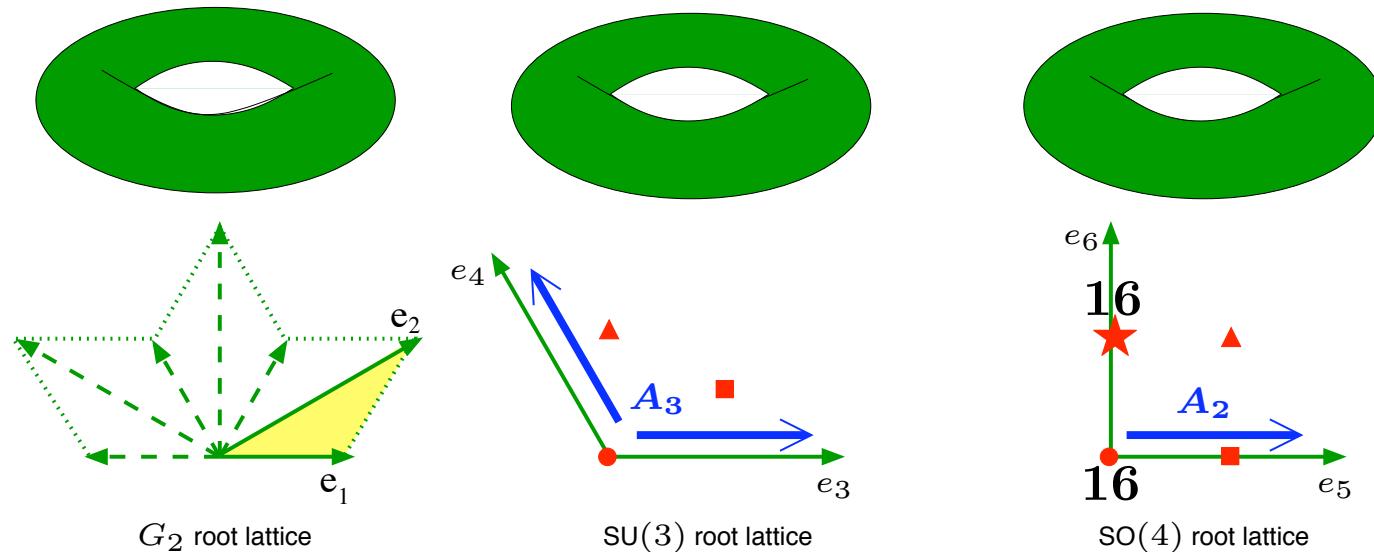
Point Group	Twist $v_1$	Twist $v_2$
$\mathbb{Z}_2 \times \mathbb{Z}_2$	(1/2, 0, -1/2)	(0, 1/2, -1/2)
$\mathbb{Z}_3 \times \mathbb{Z}_3$	(1/3, 0, -1/3)	(0, 1/3, -1/3)
$\mathbb{Z}_2 \times \mathbb{Z}_4$	(1/2, 0, -1/2)	(0, 1/4, -1/4)
$\mathbb{Z}_4 \times \mathbb{Z}_4$	(1/4, 0, -1/4)	(0, 1/4, -1/4)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{I}}$	(1/2, 0, -1/2)	(0, 1/6, -1/6)
$\mathbb{Z}_2 \times \mathbb{Z}_{6-\text{II}}$	(1/2, 0, -1/2)	(1/6, 1/6, -1/3)
$\mathbb{Z}_3 \times \mathbb{Z}_6$	(1/3, 0, -1/3)	(0, 1/6, -1/6)
$\mathbb{Z}_6 \times \mathbb{Z}_6$	(1/6, 0, -1/6)	(0, 1/6, -1/6)

Abelian point groups with  $\mathcal{N} = 1$  SUSY in 4 dimensions

~ We will work with this orbifold

# The $\mathbb{Z}_6$ -II Orbifold

The geometry of compact space



Gauge embedding

- “Shift”  $V$  : 61 choices
  - “Wilson lines”  $A_3$  and  $A_2$  : Thousands of choices
- ↪ Need guidelines

# Adding Phenomenological Priors

- Grand Unified Group in intermediate step

$$E_8 \xrightarrow{V} SO(10) \times \dots \xrightarrow{A_3} \dots \xrightarrow{A_2} SU(3) \times SU(2) \times U(1)$$

~ leaves us with 15 shifts

- One family in complete multiplet of  $SO(10)$

$$16 \xrightarrow{A_3} \dots \xrightarrow{A_5} (3,2)_{1/3} + (\bar{3},1)_{-4/3} + (1,1)_{-2} + (\bar{3},1)_{2/3} + (1,2)_{-1} + (1,1)_0 \quad \checkmark$$

~ leaves us with 2 shifts :  $V_{22}$  and  $V_{56}$

# Recent Orbifold Publications

## Shift $V_{56}$

T. Kobayashi, S. Raby, R.-J. Zhang, *Phys. Lett.* **B593** (2004)

T. Kobayashi, S. Raby, R.-J. Zhang, *Nucl. Phys.* **B704** (2005)

W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, *Nucl. Phys.* **B712** (2005)

## Shift $V_{22}$

W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, *Phys. Rev. Lett.* **96** (2006)

W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, [hep-th/0606187](#)

Lebedev, Nilles, Raby, Ratz, Ramos-Sanchez, Vaudrevange, A.W., *Phys. Lett.* **B645** (2007)

Lebedev, Nilles, Raby, Ratz, Ramos-Sanchez, Vaudrevange, A.W., *Phys. Rev. Lett.* **98**, 181602 (2007)

S. Raby, A.W., *Phys. Rev. Lett.* **99**, 051802 (2007)

S. Raby, A.W., *Phys. Rev.* **D76**, 086006 (2007)

Lebedev, Nilles, Raby, Ratz, Ramos-Sanchez, Vaudrevange, A.W., [arXiv:0708.2691](#)

# A Landscape of Heterotic Orbifold Models

Criterion	$V^{\text{SO}(10),1}$	$V^{\text{SO}(10),2}$	$V^{E_6,1}$	$V^{E_6,2}$
① Inequivalent models with 2 Wilson lines	22,000	7,800	680	1,700
② SM gauge group $\subset \text{SU}(5) \subset \text{SO}(10)$ (or $E_6$ )	3563	1163	27	63
③ 3 net ( <b>3, 2</b> )	1170	492	3	32
④ Non-anomalous $U(1)_Y \subset \text{SU}(5)$	528	234	3	22
⑤ Spectrum = 3 generations + vector-like	127	90	3	2
⑥ Vector-like exotics decouple	106	85		
⑦ “Heavy” top	55	32		
⑧ Suitable B-L exists	34	5		
⑨ SM singlets w/ B-L =even/odd	85	8		
⑩ All $U(1)$ 's except $U(1)_Y$ broken	42	0		
Decoupling along new set of singlets	15	0		

# ① Inequivalent Models

Criterion	$V^{\text{SO(10),1}}$	$V^{\text{SO(10),2}}$	$V^{E_6,1}$	$V^{E_6,2}$
① Inequivalent models with 2 Wilson lines	22,000	7,800	680	1,700
② SM gauge group $\subset \text{SU}(5) \subset \text{SO}(10)$ (or $E_6$ )	3563	1163	27	63
③ 3 net ( <b>3, 2</b> )	1170	492	3	32
④ Non-anomalous $U(1)_Y \subset \text{SU}(5)$	528	234	3	22
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⑩ All $U(1)$ 's except $U(1)_Y$ broken	42	0		
Decoupling along new set of singlets	15	0		

# ① Inequivalent Models

- Grand Unified Group in intermediate step

$$\begin{aligned} E_8 \times E_8' &\xrightarrow{V_{22}} SO(10) \times SU(2)^2 \times U(1) \times SO(14)' \times U(1)' \\ &\xrightarrow{A_3} \dots \dots \dots \\ &\xrightarrow{A_2} \dots \dots \dots \end{aligned}$$

~ 22,000 models

- Notion of inequivalence must be defined:

Basically, we call 2 models equivalent, if their spectra coincide. There are some subtleties, e.g. spectrum may be complex conjugated columnwise, and gauge group factors may be permuted.

## ② Standard Model Gauge Group

Criterion	$V^{\text{SO}(10),1}$	$V^{\text{SO}(10),2}$	$V^{E_6,1}$	$V^{E_6,2}$
① Inequivalent models with 2 Wilson lines	22,000	7,800	680	1,700
② SM gauge group $\subset \text{SU}(5) \subset \text{SO}(10)$ (or $E_6$ )	3563	1163	27	63
③ 3 net ( <b>3, 2</b> )	1170	492	3	32
④ Non-anomalous $U(1)_Y \subset \text{SU}(5)$	528	234	3	22
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⑨ SM singlets w/ B-L =even/odd	85	8		
⑩ All $U(1)$ 's except $U(1)_Y$ broken	42	0		
Decoupling along new set of singlets	15	0		

## ② Standard Model Gauge Group

- 1 shift, all Wilson lines:

$$\begin{aligned} E_8 \times E_8' &\xrightarrow{V_{22}} SO(10) \times SU(2)^2 \times U(1) \times SO(14)' \times U(1)' \\ &\xrightarrow{A_3} \dots\dots\dots \\ &\xrightarrow{A_2} SU(3) \times SU(2) \times \text{anything} \end{aligned}$$

~ 3,563 models

- Note that we require

$$SU(3) \times SU(2) \subset SU(5) \subset SO(10) \subset E_8$$

### ③ Spectrum is 3-2 Standard Model

Criterion	$V^{\text{SO}(10),1}$	$V^{\text{SO}(10),2}$	$V^{E_6,1}$	$V^{E_6,2}$
① Inequivalent models with 2 Wilson lines	22,000	7,800	680	1,700
② SM gauge group $\subset \text{SU}(5) \subset \text{SO}(10)$ (or $E_6$ )	3563	1163	27	63
③ 3 net (3, 2)	1170	492	3	32
④ Non-anomalous $U(1)_Y \subset \text{SU}(5)$	528	234	3	22
⑤ Spectrum = 3 generations + vector-like	127	90	3	2
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⑦ “Heavy” top	55	32		
⑧ Suitable B-L exists	34	5		
⑨ SM singlets w/ B-L =even/odd	85	8		
⑩ All $U(1)$ 's except $U(1)_Y$ broken	42	0		
Decoupling along new set of singlets	15	0		

# ③ Spectrum is 3-2 Standard Model

$$SU(3) \times SU(2) \times U(1)^5 \times SU(5)' \times U(1)'^4$$

$3 \times (\mathbf{3}, \mathbf{2}, \mathbf{1})$	$12 \times (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1})$	$29 \times (\mathbf{1}, \mathbf{2}, \mathbf{1})$	$8 \times (\mathbf{1}, \mathbf{1}, \mathbf{5})$
	$6 \times (\mathbf{3}, \mathbf{1}, \mathbf{1})$	$136 \times (\mathbf{1}, \mathbf{1}, \mathbf{1})$	$8 \times (\mathbf{1}, \mathbf{1}, \overline{\mathbf{5}})$

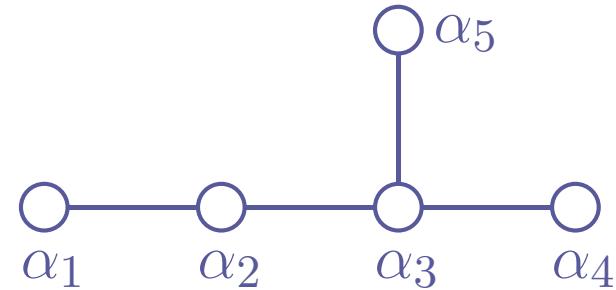
- Spectrum "factorizes" into SM and hidden sector:  
Particles that transform under SM gauge group do not transform under hidden gauge group.
- Pair up exotic particles:

$3 \times (\mathbf{3}, \mathbf{2}, \mathbf{1})$	$6 \times (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1})$	$3 \times (\mathbf{1}, \mathbf{2}, \mathbf{1})$	$8 \times (\mathbf{1}, \mathbf{1}, \mathbf{5})$
	$6 \times (\overline{\mathbf{3}}, \mathbf{1}, \mathbf{1})$	$1 \times (\mathbf{1}, \mathbf{2}, \mathbf{1})$	$8 \times (\mathbf{1}, \mathbf{1}, \overline{\mathbf{5}})$
	$6 \times (\mathbf{3}, \mathbf{1}, \mathbf{1})$	$1 \times (\mathbf{1}, \mathbf{2}, \mathbf{1})$	
		$24 \times (\mathbf{1}, \mathbf{2}, \mathbf{1})$	
		$136 \times (\mathbf{1}, \mathbf{1}, \mathbf{1})$	

# ④ Construct Hypercharge from SU(5)

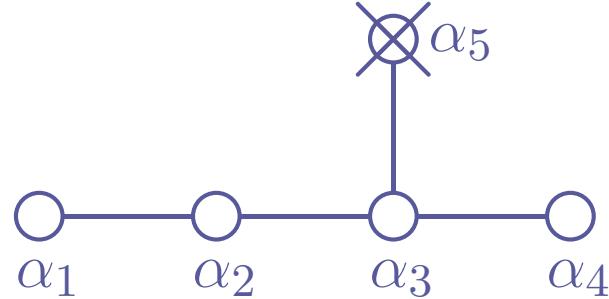
Criterion	$V^{\text{SO}(10),1}$	$V^{\text{SO}(10),2}$	$V^{E_6,1}$	$V^{E_6,2}$
① Inequivalent models with 2 Wilson lines	22,000	7,800	680	1,700
② SM gauge group $\subset \text{SU}(5) \subset \text{SO}(10)$ (or $E_6$ )	3563	1163	27	63
③ 3 net (3, 2)	1170	492	3	32
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⑤ Spectrum = 3 generations + vector-like	127	90	3	2
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⑧ Suitable B-L exists	34	5		
⑨ SM singlets w/ B-L =even/odd	85	8		
⑩ All $U(1)$ 's except $U(1)_Y$ broken	42	0		
Decoupling along new set of singlets	15	0		

## ④ Construct Hypercharge from SU(5)



SO(10)

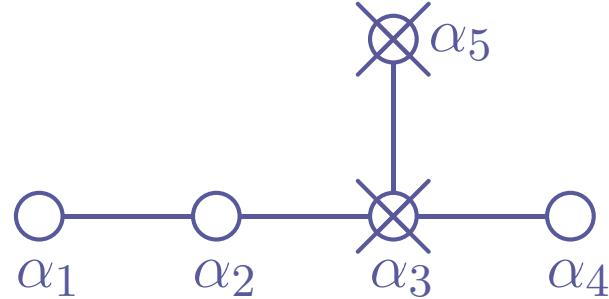
## ④ Construct Hypercharge from SU(5)



$$SO(10) \rightarrow SU(5) \times U(1)_X$$

$$U(1)_X = 4\alpha_5^* = 4 \sum_{j=1}^5 (A_{SO(10)}^{-1})_{5j} \alpha_j = (2\alpha_1 + 4\alpha_2 + 6\alpha_3 + 3\alpha_4 + 5\alpha_5)$$

## ④ Construct Hypercharge from SU(5)

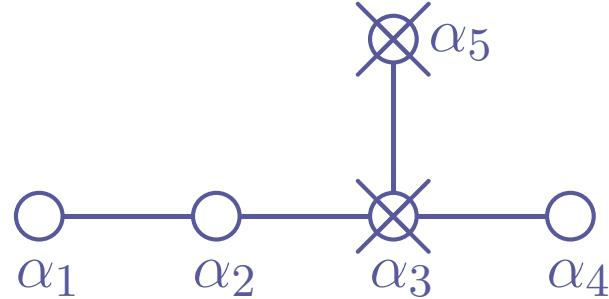


$$\text{SO}(10) \rightarrow \text{SU}(5) \times \text{U}(1)_X \rightarrow \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_X$$

$$\text{U}(1)_X = 4\alpha_5^* = 4 \sum_{j=1}^5 (A_{\text{SO}(10)}^{-1})_{5j} \alpha_j = (2\alpha_1 + 4\alpha_2 + 6\alpha_3 + 3\alpha_4 + 5\alpha_5)$$

$$\text{U}(1)_Y = \frac{5}{3}\alpha_3^* = \frac{5}{3} \sum_{j=1}^4 (A_{\text{SU}(5)}^{-1})_{3j} \alpha_j = \frac{1}{3}(2\alpha_1 + 4\alpha_2 + 6\alpha_3 + 3\alpha_4)$$

## ④ Construct Hypercharge from SU(5)



$$\text{SO}(10) \rightarrow \text{SU}(5) \times \text{U}(1)_X \rightarrow \text{SU}(3)_c \times \text{SU}(2)_L \times \text{U}(1)_Y \times \text{U}(1)_X$$

$$\text{U}(1)_X = 4\alpha_5^* = 4 \sum_{j=1}^5 (A_{\text{SO}(10)}^{-1})_{5j} \alpha_j = (2\alpha_1 + 4\alpha_2 + 6\alpha_3 + 3\alpha_4 + 5\alpha_5)$$

$$\text{U}(1)_Y = \frac{5}{3}\alpha_3^* = \frac{5}{3} \sum_{j=1}^4 (A_{\text{SU}(5)}^{-1})_{3j} \alpha_j = \frac{1}{3}(2\alpha_1 + 4\alpha_2 + 6\alpha_3 + 3\alpha_4)$$

$\text{U}(1)_Y \perp \text{U}(1)_A ?$

# ⑤ Spectrum is 3-2-1 Standard Model

Criterion	$V^{\text{SO}(10),1}$	$V^{\text{SO}(10),2}$	$V^{E_6,1}$	$V^{E_6,2}$
① Inequivalent models with 2 Wilson lines	22,000	7,800	680	1,700
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⑧ Suitable B-L exists	34	5		
⑨ SM singlets w/ B-L =even/odd	85	8		
⑩ All $U(1)$ 's except $U(1)_Y$ broken	42	0		
Decoupling along new set of singlets	15	0		

## ⑤ Spectrum is 3-2-1 Standard Model

$$\mathbf{SU(3) \times SU(2) \times U(1)^5 \times SU(5)' \times U(1)'^4}$$

$3 \times (3, 2, 1)$	$12 \times (\bar{3}, 1, 1)$	$29 \times (1, 2, 1)$	$8 \times (1, 1, 5)$
	$6 \times (3, 1, 1)$	$136 \times (1, 1, 1)$	$8 \times (1, 1, \bar{5})$

# ⑤ Spectrum is 3-2-1 Standard Model

$SU(3) \times SU(2) \times \text{something}$

$3 \times (\mathbf{3}, \mathbf{2})$	$12 \times (\bar{\mathbf{3}}, \mathbf{1})$	$29 \times (\mathbf{1}, \mathbf{2})$
	$6 \times (\mathbf{3}, \mathbf{1})$	$216 \times (\mathbf{1}, \mathbf{1})$

# ⑤ Spectrum is 3-2-1 Standard Model

$SU(3) \times SU(2) \times \text{something}$

$3 \times (\mathbf{3}, \mathbf{2})$	$12 \times (\bar{\mathbf{3}}, \mathbf{1})$	$29 \times (\mathbf{1}, \mathbf{2})$
	$6 \times (\mathbf{3}, \mathbf{1})$	$216 \times (\mathbf{1}, \mathbf{1})$

- Particles w/hypercharge

$3 \times (\mathbf{3}, \mathbf{2})_{1/3}$	$5 \times (\bar{\mathbf{3}}, \mathbf{1})_{-4/3}$	$7 \times (\bar{\mathbf{3}}, \mathbf{1})_{2/3}$	$16 \times (\mathbf{1}, \mathbf{2})_{-1}$	$45 \times (\mathbf{1}, \mathbf{1})_2$	$129 \times (\mathbf{1}, \mathbf{1})_0$
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- Vectorlike?

$3 \times (\mathbf{3}, \mathbf{2})_{1/3}$	$3 \times (\overline{\mathbf{3}}, \mathbf{1})_{-4/3}$	$3 \times (\overline{\mathbf{3}}, \mathbf{1})_{2/3}$	$3 \times (\mathbf{1}, \mathbf{2})_{-1}$	$3 \times (\mathbf{1}, \mathbf{1})_2$	$3 \times (\mathbf{1}, \mathbf{1})_0$
	$2 \times (\overline{\mathbf{3}}, \mathbf{1})_{-4/3}$	$4 \times (\overline{\mathbf{3}}, \mathbf{1})_{2/3}$	$1 \times (\mathbf{1}, \mathbf{2})_{-1}$	$42 \times (\mathbf{1}, \mathbf{1})_2$	$126 \times (\mathbf{1}, \mathbf{1})_0$
	$2 \times (\mathbf{3}, \mathbf{1})_{4/3}$	$4 \times (\mathbf{3}, \mathbf{1})_{-2/3}$	$1 \times (\mathbf{1}, \mathbf{2})_1$	$42 \times (\mathbf{1}, \mathbf{1})_{-2}$	
			$12 \times (\mathbf{1}, \mathbf{2})_1$		
			$12 \times (\mathbf{1}, \mathbf{2})_{-1}$		

# ⑥ Exotics Decouple

Criterion	$V^{\text{SO(10),1}}$	$V^{\text{SO(10),2}}$	$V^{E_6,1}$	$V^{E_6,2}$
① Inequivalent models with 2 Wilson lines	22,000	7,800	680	1,700
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## ⑥ Exotics Decouple

- Decoupling: Mass term?

$$M\phi\bar{\phi}$$

- Generic situation: Interaction term w/ singlets

$$\phi\bar{\phi}\tilde{s}_1\tilde{s}_2\dots\tilde{s}_n$$

Singlet fields acquire a vev  $\sim$  Effective mass term

$$\phi\bar{\phi}\langle\tilde{s}_1\tilde{s}_2\dots\tilde{s}_n\rangle$$

- Couplings must be allowed by the string selection rules !
- Allowing for all singlets may break Standard Model symmetries:  
 $s_i^+ = (1, 1; 1, 1)_{(1/2, *)}$ ,  $\langle s_i^+ \rangle \neq 0$  breaks  $U(1)_Y$  **X**
- Find subset of singlet fields which decouple exotics and preserve Standard Model symmetries  $\sim$  Highly non-trivial, superpotential has 286,781 terms

# ⑦ Heavy Top

Criterion	$V^{\text{SO(10),1}}$	$V^{\text{SO(10),2}}$	$V^{E_6,1}$	$V^{E_6,2}$
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## ⑦ Heavy Top

- Demand existence of Yukawa coupling

$$\bar{u}H_uQ \leftrightarrow (\bar{\mathbf{3}}, \mathbf{1})_{-4/3}(\mathbf{1}, \mathbf{2})_1(\mathbf{3}, \mathbf{2})_{1/3}$$

at trilinear level, i.e. w/o singlets

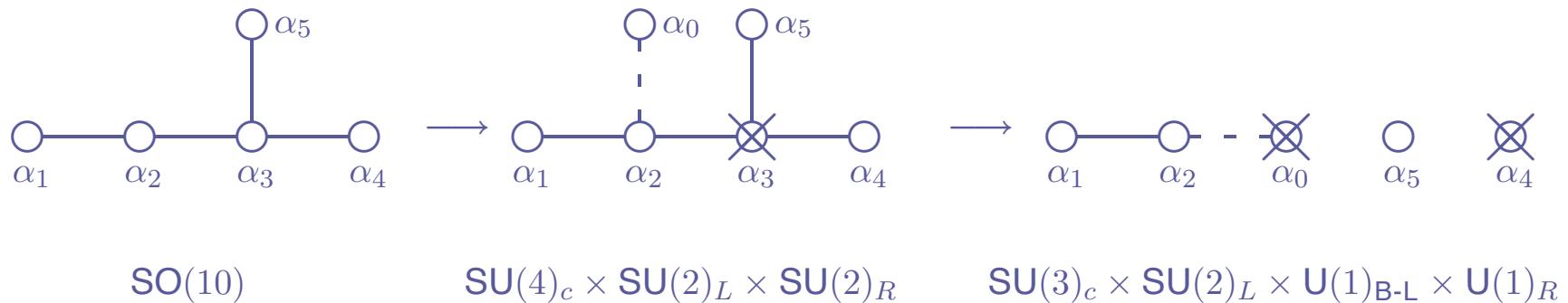
- Singlets  $\rightarrow$  interactions suppressed

$$\bar{u}H_uQ\tilde{s}_1\dots\tilde{s}_n \leftrightarrow \bar{u}H_uQ\frac{\langle\tilde{s}_1\dots\tilde{s}_n\rangle}{M^n}$$

## ⑧ B-L exists

Criterion	$V^{\text{SO(10),1}}$	$V^{\text{SO(10),2}}$	$V^{E_6,1}$	$V^{E_6,2}$
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## ⑧ B-L exists



- Default candidate for B-L in  $\text{SO}(10)$ :

$$\text{SO}(10) \supset \text{SU}(4)_c \times \text{SU}(2)_L \times \text{SU}(2)_R \rightarrow \text{SU}(3)_c \times \text{U}(1)_{\text{B-L}} \times \text{SU}(2)_L \times \text{U}(1)_R$$

- Too restrictive, need to construct more general  $B - L$  generator:  
 $\leadsto$  S. Raby, A.W., arXiv:0706.0217

# One Model in Detail

$$\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{SU}(4)' \times \mathrm{SU}(2)' \times \mathrm{U}(1)^9$$

#	irrep	label		#	irrep	label
3	$(\mathbf{3}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(1/6, 1/3)}$	$Q_i$		3	$(\overline{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-2/3, -1/3)}$	$\bar{u}_i$
3	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1, 1)}$	$\bar{e}_i$		8	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(0, *)}$	$m_i$
4	$(\overline{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1/3, -1/3)}$	$\bar{d}_i$		1	$(\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-1/3, 1/3)}$	$d_i$
4	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(-1/2, -1)}$	$L_i$		1	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(1/2, 1)}$	$\bar{L}_i$
1	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(-1/2, 0)}$	$H_i$		1	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{1})_{(1/2, 0)}$	$\bar{H}_i$
6	$(\overline{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1/3, 2/3)}$	$\bar{\delta}_i$		6	$(\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-1/3, -2/3)}$	$\delta_i$
14	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1/2, *)}$	$s_i^+$		14	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-1/2, *)}$	$s_i^-$
16	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(0, 1)}$	$\bar{\nu}_i$		13	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(0, -1)}$	$\nu_i$
5	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2})_{(0, 1)}$	$\bar{\eta}_i$		5	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2})_{(0, -1)}$	$\eta_i$
10	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{2})_{(0, 0)}$	$h_i$		2	$(\mathbf{1}, \mathbf{2}; \mathbf{1}, \mathbf{2})_{(0, 0)}$	$y_i$
6	$(\mathbf{1}, \mathbf{1}; \mathbf{4}, \mathbf{1})_{(0, *)}$	$f_i$		6	$(\mathbf{1}, \mathbf{1}; \overline{\mathbf{4}}, \mathbf{1})_{(0, *)}$	$\bar{f}_i$
2	$(\mathbf{1}, \mathbf{1}; \mathbf{4}, \mathbf{1})_{(-1/2, -1)}$	$f_i^-$		2	$(\mathbf{1}, \mathbf{1}; \overline{\mathbf{4}}, \mathbf{1})_{(1/2, 1)}$	$\bar{f}_i^+$
4	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(0, \pm 2)}$	$\chi_i$		32	$(\mathbf{1}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(0, 0)}$	$s_i^0$
2	$(\overline{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(-1/6, 2/3)}$	$\bar{v}_i$		2	$(\mathbf{3}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{(1/6, -2/3)}$	$v_i$

Attention: Hypercharge is half the conventional value!!!

# ⑨ Break B-L to R-Parity

Criterion	$V^{\text{SO(10),1}}$	$V^{\text{SO(10),2}}$	$V^{E_6,1}$	$V^{E_6,2}$
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## ⑨ Break B-L to R-Parity

- As before, giving vevs to arbitrary fields may break desirable symmetries...

$$\langle \bar{v}_i \rangle = \langle (\bar{\mathbf{3}}, \mathbf{1}; \mathbf{1}, \mathbf{1})_{-1/6, 2/3} \rangle \neq 0 \quad \sim \quad \cancel{\text{SU(3)}_c} \quad \cancel{\text{U(1)}_Y} \quad \cancel{\text{U(1)}_{\text{B-L}}}$$

- Take "special" fields:

$$\langle \chi_i \rangle = \langle (1, 1; \mathbf{1}, \mathbf{1})_0, \pm 2 \rangle \neq 0 \quad \sim \quad \text{U(1)}_{\text{B-L}} \rightarrow \mathbb{Z}_2$$

- R-parity  $\sim$  Proton stability, etc.
- Note: This restricts the choice of singlet fields we may use to decouple exotics !!!

# ⑩ Break all spurious $U(1)$ 's

Criterion	$V^{SO(10),1}$	$V^{SO(10),2}$	$V^{E_6,1}$	$V^{E_6,2}$
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## ⑩ Break all spurious $U(1)$ 's

- At low energies, we only observe  $U(1)_Y$
- More than one  $U(1)$   $\leadsto$  Many  $Z'$  bosons
- Break these symmetries as before

$$\langle (1, 1; 1, 1)_{0, \pm 2, *, *, \dots *} \rangle \neq 0 \quad \leadsto \quad U(1)_Y \quad \cancel{U(1)_{B-L}^{\xrightarrow{\mathbb{Z}_2}}} \quad \cancel{U(1)} \dots$$

- Note: Again, this restricts the choice of singlet fields we may use to decouple exotics !!!

# Decoupling Revisited

Criterion	$V^{\text{SO(10),1}}$	$V^{\text{SO(10),2}}$	$V^{E_6,1}$	$V^{E_6,2}$
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# Decoupling Revisited

- Choice of vacuum:

$$\{\tilde{s}_i\} = \{\chi_1, \chi_2, \chi_3, \chi_4, h_1, h_2, h_3, h_4, h_5, h_6, h_9, h_{10}, s_1^0, s_4^0, s_5^0, s_6^0, s_9^0, s_{11}^0, s_{13}^0, s_{15}^0, s_{16}^0, s_{17}^0, s_{18}^0, s_{20}^0, s_{21}^0, s_{22}^0, s_{23}^0, s_{25}^0, s_{26}^0, s_{27}^0, s_{30}^0, s_{31}^0\}$$

- Terms of arbitrary order appear in superpotential, one example is:

$$\phi \bar{\phi} \tilde{s}_1 \tilde{s}_2 \dots \tilde{s}_n$$

Consider terms up to order 6 in singlets, i.e.  $n = 6$ : **286,781 terms**

- Check that exotics decouple
- Mass matrices: Collect terms quadratic in field  $\times$  singlets

$$Y_u = \begin{pmatrix} \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^5 \\ \tilde{s}^5 & \tilde{s}^5 & \tilde{s}^6 \\ \tilde{s}^6 & \tilde{s}^6 & 1 \end{pmatrix}, \quad Y_d = \begin{pmatrix} 0 & \tilde{s}^5 & 0 \\ \tilde{s}^5 & 0 & 0 \\ 0 & \tilde{s}^6 & 0 \end{pmatrix}, \quad Y_e = \begin{pmatrix} 0 & \tilde{s}^5 & \tilde{s}^6 \\ \tilde{s}^5 & 0 & 0 \\ \tilde{s}^6 & \tilde{s}^6 & 0 \end{pmatrix}.$$

# *F*- and *D*-Flatness

- Theory supersymmetric  $\leftrightarrow$  Minimum of potential is 0

$$V = \sum F_i^* F_i + \sum D^a D^a$$

$\leadsto$  Unbroken supersymmetry is tantamount to  $F_i = D^a = 0$

- *D*-terms for U(1)'s

$$D^a = \sum \phi_i^* T^a \phi_i = \sum q_i |\phi_i|^2$$

- *F*-terms

$$F_{\phi_i} = \frac{\partial W}{\partial \phi_i} \Big|_{\phi_i = \langle \phi_i \rangle}$$

Under certain conditions,  $F = 0$  implies  $D = 0$

- Superpotential in singlets has 124 terms, there are  $32 \times \tilde{s}$  fields  
 $\leadsto$  32 polynomial equations up to order 5

Works ✓

# Conclusions

- Standard Model leaves many questions unanswered
- SUSY GUTs promising candidates for physics beyond the Standard Model
- Extra dimensions may solve many problems from which ordinary GUTs suffer
- String theory is natural candidate for UV completion of 5- and 6 dimensional models, and includes quantized version of gravity
- Models presented come closer to MSSM than any other string construction so far (compare to D-brane models, Calabi-Yaus, Gepner-type models, . . . )
- Still a far way to go to reproduce other key features of MSSM