# Stringy Instantons, Geometric Transitions, and Dynamical SUSY Breaking

based on work with:
Ofer Aharony, Eva Silverstein arXiv:0708.0493
Mina Aganagic, Chris Beem arXiv:0709.4277

also building on papers with:
Ofer Aharony arXiv:0707.3126
Bogdan Florea, John McGreevy, Natalia Saulina hep-th/0610102

and partially inspired by:
Michael Dine, Jonathan Feng, Eva Silverstein hep-th/0608159

One of the central mysteries of fundamental physics is the dimensionless ratio:

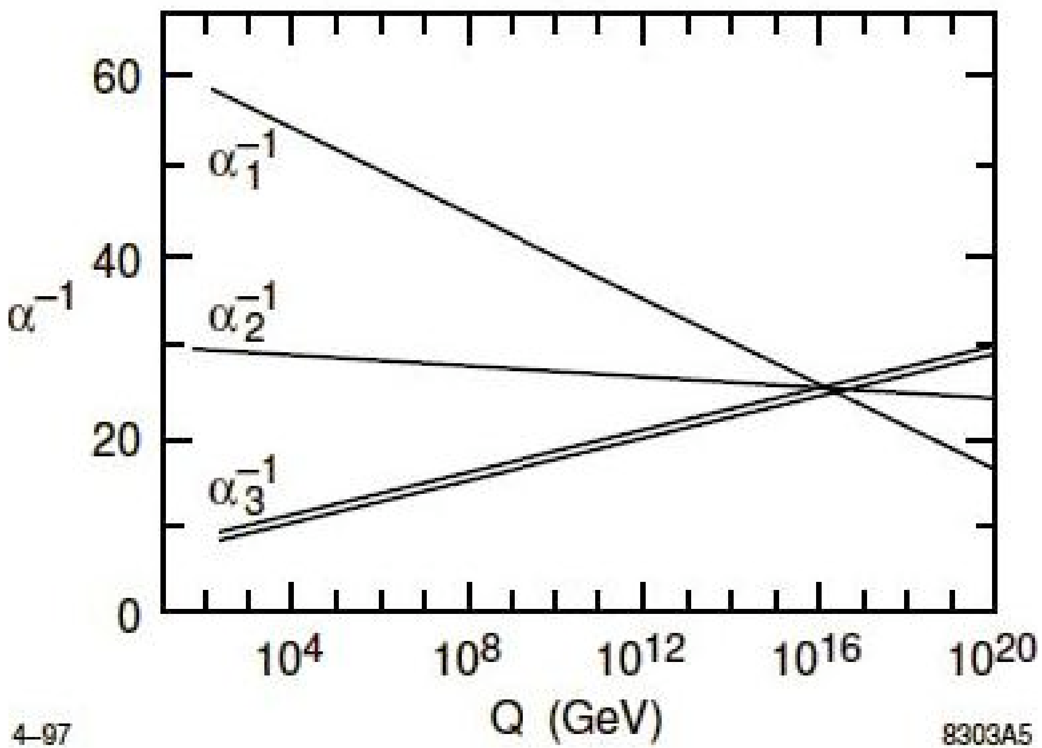
$$\frac{M_W}{M_P} \sim 10^{-16}$$

LHC will turn on roughly one year from now. It will probe the details of electroweak symmetry breaking, and hopefully elucidate the mechanism responsible for protecting this hierarchy.

The front runner, at least in many people's minds, is low-energy supersymmetry.

In addition to stabilizing the hierarchy, SUSY has two additional nice features that come along for free:

1) SUSY grand unified theories correctly predict $\alpha_3$ given the measured values of $\alpha_1, \ \alpha_2$



2) In models with intermediate scale SUSY breaking and R-parity, the freeze-out abundance of the LSP is in the right ballpark to explain why

$$\Omega_{DM} \sim .25$$

Now, although SUSY makes the hierarchy radiatively stable, in order to really explain the small ratio, one needs a natural mechanism whereby a supersymmetric theory decides to break SUSY at an exponentially small scale:

$$\frac{F}{M_P^2} \leq 10^{-16}$$

Witten proposed one such mechanism, dynamical supersymmetry breaking, in 1981.

Imagine a SUSY field theory with chiral multiplets $\Phi_i$

$$\Phi = \phi + \theta^\alpha \psi_\alpha + \theta^2 F_\phi$$

The Lagrangian takes the schematic form

$$L = \int d^4\theta \; K(\Phi_i, \bar{\Phi}_i) \; + \; \left(\int d^2\theta \; W(\Phi_i) + c.c.\right)$$

(I assume gauge fields are present, but have neglected to explicitly describe the gauge supermultiplets, kinetic terms, etc.)

Powerful non-renormalization theorems, proven by supergraph techniques in the late 1970s and explained by holomorphy arguments in the early 1990s, allow one to show that in wide classes of theories, the superpotential is of the form

$$W = W_{tree} + \mathcal{O}(e^{-\frac{1}{g^2}})$$

The scalar potential takes the form:

$$V = \sum_i |F_{\phi_i}|^2$$

with

$$F_{\phi_i} = \frac{\partial W}{\partial \phi_i}$$

So, one can hope to build models where:

* To all orders in perturbation theory, the vacuum (or all vacua) have vanishing F-terms.

* But non-perturbative corrections to W give rise to one or more vacua with $F \neq 0$

It would then be natural to expect that

$$\langle F \rangle \sim e^{-\frac{1}{g^2}} << M_P^2$$

Then, however this SUSY breaking is mediated to the Standard Model, the parametrically low breaking scale (and hence protection of the Higgs mass) will have been explained.

The search for examples was time consuming and required many non-trivial developments, but they were eventually found (first by Affleck, Dine and Seiberg).

Typical examples are rather complicated (with even the simplest recent models involving multiple small scales that need to be explained in a full theory, like string theory).

Obviously, one way to obtain low-scale supersymmetry breaking in string theory, is to engineer a DSB gauge theory on branes and embed it into a string compactification. Here, I propose an alternative. I describe string models that dynamically break supersymmetry without non-Abelian gauge dynamics.

# ~~SUSY~~ for dummies

The simplest models of supersymmetry breaking that one can imagine are the Polonyi model, the Fayet model, and the O'Raifeartaigh model.

The Polonyi model is the theory of a single chiral superfield X, with:

$$K = X^\dagger X + \cdots$$

$$W = \mu^2 X$$

Naively, this theory breaks supersymmetry with

$$F_X = \mu^2$$

However, the leading approximation to the scalar potential is then:

$$V = |\mu^2|^2$$

In this approximation the theory has a moduli space of degenerate, non-supersymmetric vacua. A priori it is not protected by any symmetry in various UV completions. The fate of the theory then depends on the ... in K.

If the leading correction induced by UV physics at scale M is given by:

$$K = X^\dagger X + c\frac{(X^\dagger X)^2}{M^2} + \ldots$$

then for one sign of c one obtains a stable SUSY breaking vacuum at the origin, while for the other, there is a runaway to large field vevs. Note that if one could justify

$$\mu^2 << M_P^2$$

this would be a perfectly respectable model of SUSY breaking; but the dynamics of this field theory clearly does not generate a small scale via non-perturbative effects.

The Fayet model is only slightly more complicated.

The field content consists of a U(1) gauge multiplet, and two chiral multiplets with opposite charges.

The superpotential is given by

$$W = m\Phi_+\Phi_-$$

The U(1) gauge field can also have a Fayet-Iliopoulos term with coefficient r, so the potential including the D-term takes the form

$$V = |m|^2\left(|\phi_+|^2 + |\phi_-|^2\right) + \frac{1}{2}\left(e|\phi_+|^2 - e|\phi_-|^2 - r\right)^2$$

For any nonzero m & r, this theory breaks supersymmetry.

Unlike the Polonyi model, here the vacuum is stable even before considering radiative corrections.

E.g., for $r^2 >> \frac{m^2}{2e^2}$ the minimum of V is at

$$|\phi_+|^2 = r - \frac{m^2}{2e^2} \sim r, \quad \phi_- = 0$$

and the SUSY breaking order parameter is

$$F_{\phi_-} \simeq m\sqrt{r}$$

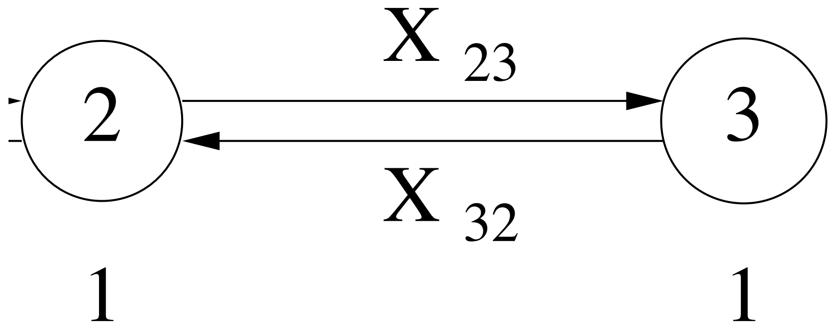
The limit $m \to 0$ restores a non-anomalous axial symmetry, $\Phi_\pm \to e^{i\lambda}\Phi_\pm$ .

Therefore, any model where m is generated by exponentially small effects is natural in the sense of `t Hooft and Wilson.

I now describe simple D-brane constructions where precisely this model is realized, and where an exponentially small supersymmetry breaking scale is obtained by generating m from a stringy instanton effect.

# D-brane construction: The basic idea

Branes at singularities, or intersecting branes, give rise quite generally to quiver gauge theories. The quiver that we need to engineer the field content of the Fayet model is completely trivial:




The numbers in the circles are node numbers, the numbers underneath are the ranks.

Our basic idea is the following. Consider a non-compact Calabi-Yau space which contains two 2-cycles on which space-filling D5 branes are wrapped, and a third two-cycle C which is not wrapped by a 5-brane. There are two chiral multiplets of charges $(\pm 1, \mp 1)$ under the U(1) gauge groups.

The superpotential is zero perturbatively. A Euclidean D1-brane wrapped on C contributes an instanton effect with precisely the right zero-mode structure to generate

$$W \sim \Phi_+ \Phi_-$$

This cannot be interpreted as an ordinary field-theoretic instanton. There is no field theory associated with the cycle C, and no non-Abelian gauge dynamics is required for the effect.

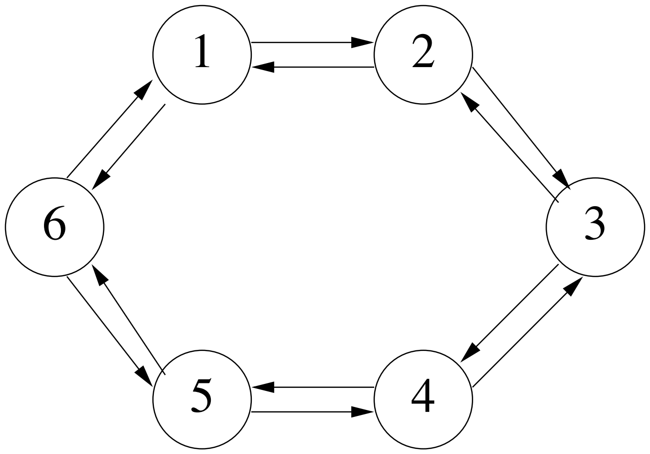
Note also that if one can engineer such a gauge theory, m and r are fixed parameters at the level of the non-compact system; they arise from non-normalizable modes in the geometry.

How to find this arising at a singularity?

A simple class of non-chiral quivers arises at the singularities

$$(xy)^n = zw$$

The gauge theories on D3 branes and fractional D5 branes in type IIB string theory at such a singularity, are captured by the quiver (e.g. for n=3):




Uranga

The bi-fundamentals connecting the 2n nodes are governed by a tree-level superpotential:

$$W = h \sum_{i=1}^{2n} (-1)^i \; X_{i,i+1} X_{i+1,i+2} X_{i+2,i+1} X_{i+1,i} \; .$$

Since the theories are non-chiral, any occupation numbers $r_i$ are allowed at the various nodes.

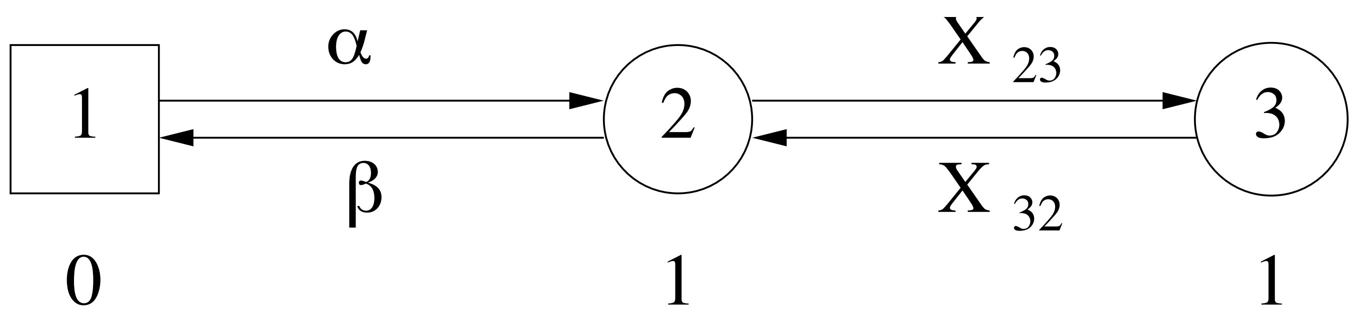
Choosing

$$r_2 = r_3 = 1$$

with other occupation numbers vanishing, gives us a gauge theory whose field content reproduces the Fayet model (up to a completely decoupled U(1)).

The Euclidean D1-brane wrapping node 1 is of potential interest. It breaks half of the supersymmetry. It has massless Ramond sector open strings connecting it to the (fractional) D5 brane on node 2.

The relevant extended quiver diagram including these Ganor strings which stretch to/from the instanton, is:



# Zero mode subtleties

Such stringy instantons, which arise from Euclidean branes on unoccupied quiver nodes, have recently been studied by several groups.

Florea, S.K. , McGreevy, Saulina;
Ibanez, Uranga;
Blumenhagen, Cvetic, Weigand;
.............

There are two important points about the integral over the Ramond sector collective coordinates of the instanton.

1) The Ganor strings have a coupling

$$S = \cdots + \alpha X_{23} X_{32} \beta$$

in their worldvolume action.

Performing the integral over $\alpha, \beta$ then generates a superpotential term which is proportional to

$$\Delta W \simeq X_{23} X_{32} \ \mathrm{Exp}(-\mathrm{Area})$$

2) Although naively the instanton breaks half of the N=1 supersymmetry preserved by the space-filling D-branes in a Calabi-Yau, locally in its ED1-ED1 open string sector, "it thinks" it is breaking half of the N=2 supersymmetry of the Calabi-Yau model.

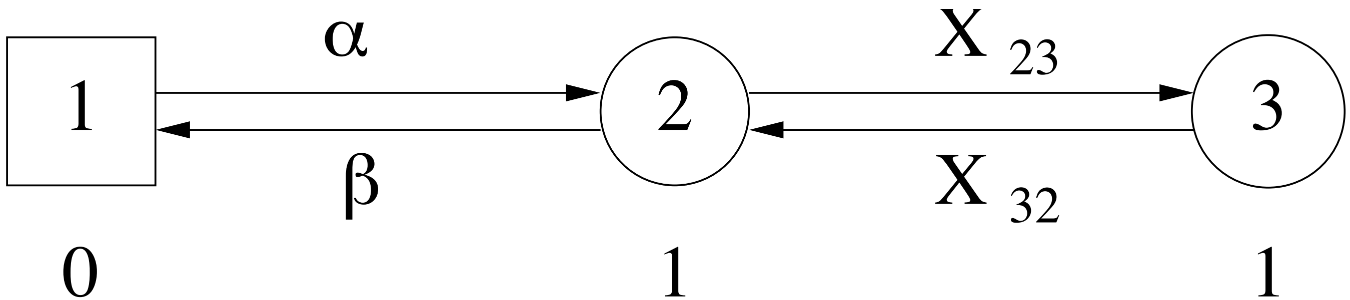
So there is a danger that there will be four fermion zero modes in the ED1-ED1 Ramond sector, 2 too many for this instanton to correct the space-time superpotential!

Perhaps, in some cases, these extra modes are lifted by interactions with background flux. However, a trivial and explicit way to get the correct zero mode counting for a superpotential correction, is to put an orientifold plane on the node wrapped by the instanton. This halves the number of ED1-ED1 Ramond sector zero modes to the 2 required for a correction to W.

Such orientifolds of the 2n-node quivers are easy to construct; they leave our gauge theory untouched while allowing the Euclidean D1-brane on C to contribute.

Argurio, Bertolini, Franco, S.K.;
Franco, Hanany, Krefl, Park, Uranga, Vegh

The end result is that the simple quiver:



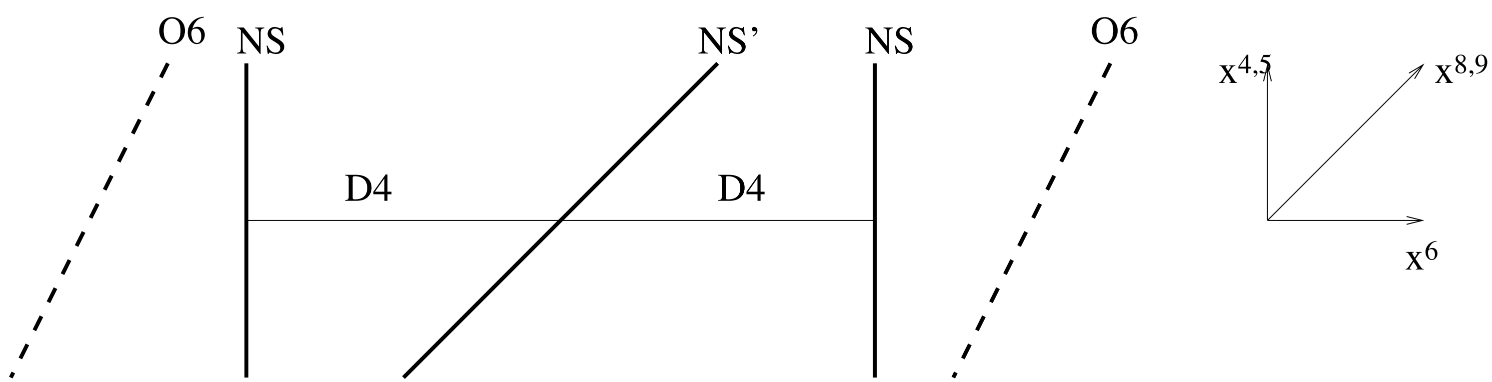

with the square node denoting now a node that would give rise to a symplectic group if wrapped by a fractional D5, generates the Fayet model with exponentially small SUSY breaking scale:

$$V = |m|^2 \left(|\phi_+|^2 + |\phi_-|^2\right) + \frac{1}{2}\left(e|\phi_+|^2 - e|\phi_-|^2 - r\right)^2$$

with $m \sim \mathrm{Exp}(-\mathrm{Area})$ , so $F \sim m\sqrt{r} << M_P$ .

In other words, string theory in the appropriate geometries ``retrofits'' (in the parlance of Dine, Feng, Silverstein) one of the old classic SUSY breaking toy models, to make it a model of dynamical SUSY breaking! There is no need here for intricate non-Abelian gauge dynamics.

Of course there are dual type IIA views of the same geometry. Here is one. (Others with intersecting D6-branes undoubtedly exist).



It is natural to ask: what happens if the instanton effect generates additional field theory operators?

Simple dimensional analysis shows that if higher powers of

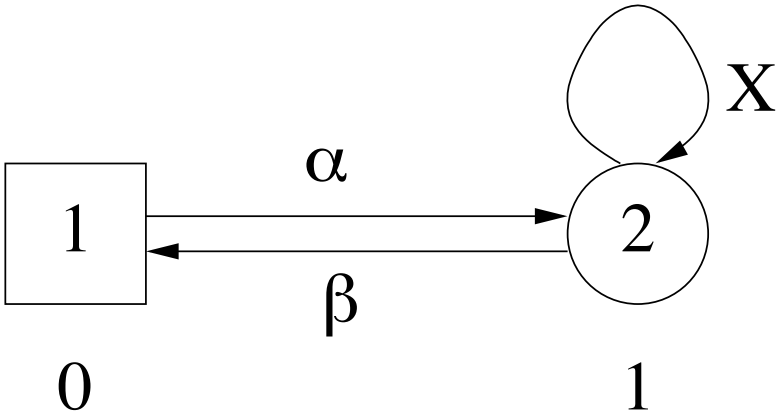
$\Phi_+\Phi_-$ are generated, they appear with additional powers of $M_{\mathrm{string}}$ suppression:

$$(\Delta W) = \left(\frac{1}{M_{\mathrm{string}}}\right)^{2N-3}(\Phi_+\Phi_-)^N \mathrm{Exp}(-\mathrm{Area})$$

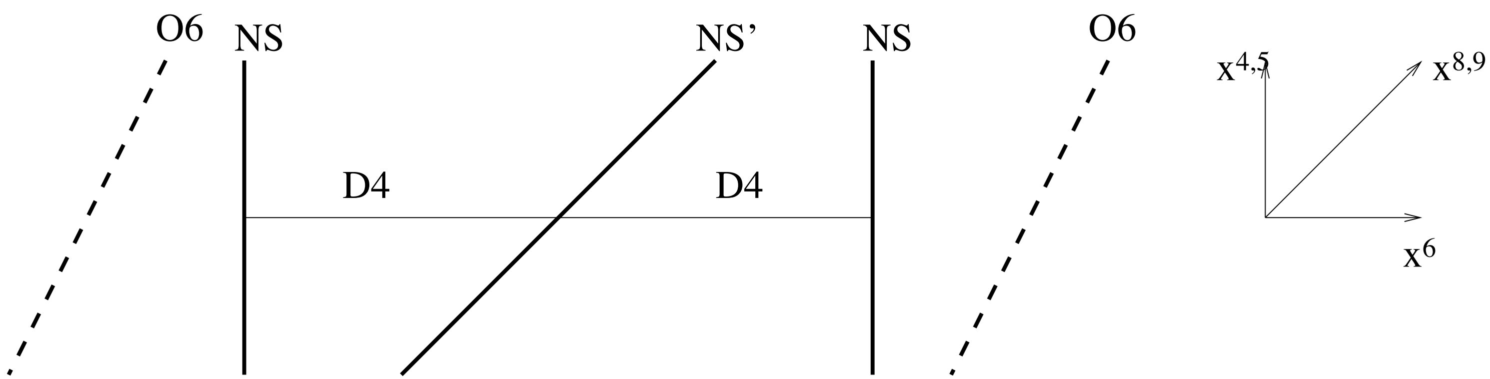
Such potential corrections would be negligible in the vicinity of our SUSY breaking vacuum.

# The Polonyi Model redux

Now, it is easy to see quivers that would generate other classic SUSY breaking toy models with very modest field content, automatically retrofitted by stringy instantons. For instance, to get the Polonyi model, consider:



In fact, it is trivial to engineer this quiver by starting with our IIA brane configuration for the Fayet model:




Consider moving the NS' brane in e.g. the $x^6$ direction, so it swaps placed with one of the NS branes.

Then one is left with two parallel NS 5 branes with a D4 stretched between them (and the neighboring O-plane).

This engineers precisely the desired Polonyi quiver; the stringy instanton stretching between the O6-plane and the NS 5 brane, generates a non-perturbatively small potential linear in the adjoint field X describing the position of the D4 brane in the $x^4 - x^5$ plane.

Alternatively one can obtain the Polonyi model (with a stable vacuum) as a limit of the Fayet model. In the limit where one takes:

$$r \to \infty, \quad m\sqrt{r} \equiv \mu^2 \quad \text{fixed}$$

the Fayet model reduces to a Polonyi model with linear superpotential $\mu^2 X$

The U(1) under which $\Phi_\pm$ are charged becomes very massive, along with $\Phi_+$. This leaves a free U(1) theory with a singlet $X = \Phi_-$ that has a linear superpotential. In the limit there is a small stabilizing mass at the origin:

$$m^2 \sim \frac{\mu^2}{\sqrt{r}} \; .$$

In the string construction, we should keep r smaller than the string scale to avoid introducing new degrees of freedom. This still leaves a regime where the low energy effective theory is a Polonyi model with a stable minimum and a dynamically generated, exponentially small SUSY breaking scale.

# The O'Raifeartaigh Model

Not to discriminate against the third classic SUSY breaking model...

A simple O'Raifeartaigh model is described by the theory with U(1) gauge group and four chiral multiplets, two with equal and opposite charges $\Phi, \tilde{\Phi}$ and two which are gauge neutral. The superpotential is:

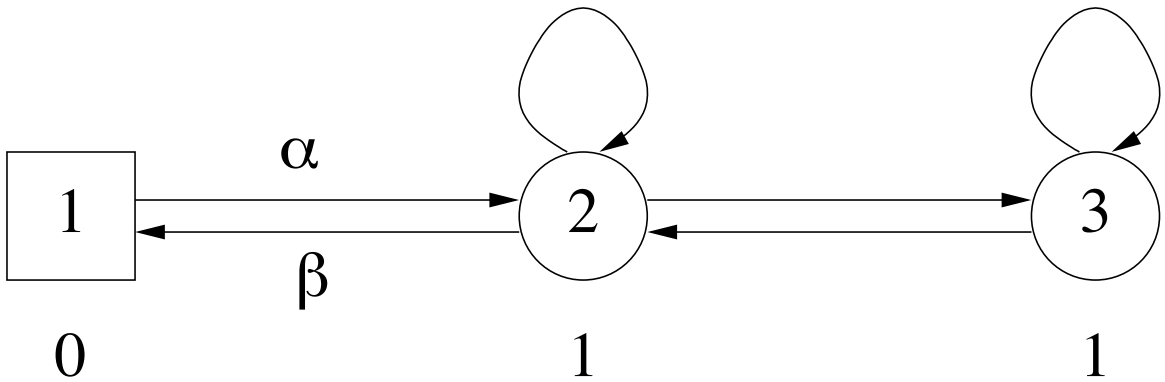
$$W = X\tilde{\Phi}\Phi + \tilde{X}(\tilde{\Phi}\Phi + \mu^2)$$

In addition, there is a D-term constraint for SUSY vacua:

$$|\phi|^2 - |\tilde{\phi}|^2 = r$$

In the limit $\mu^2 \to 0$ there would be a SUSY vacuum: one can set one of the charged fields to zero, and saturate the D-term constraint with the other.

So one can design a stringy retrofitted O'Raifeartaigh model by considering the following quiver (which again arises from an appropriate configuration of two D-branes):




The two "adjoints" of U(1) x U(1) play the role of the $X, \tilde{X}$ fields. The Ganor strings have

$$S = \ldots + \alpha \tilde{X} \beta$$

and the D-instanton at node 1 generates an exponentially small $\mu^2$.

# A classical description via geometric transitions

Geometric transitions can recast the instanton generated superpotential as a classical flux superpotential

$$W = \int_M (F - \tau H) \wedge \Omega$$

Gukov, Vafa, Witten

As a bonus, this allows us to compute all multi-instanton contributions at once.

We look at a general family of geometries given by A_r fibrations over the x-plane:

$$uv = \Pi_{i=1}^{r+1} (z - z_i(x))$$

As described by Cachazo, Katz and Vafa, this geometry contains r $\mathbf{P}^1$s whose volumes can be independently dialed. A given $S_i^2$ arises by blowing up the locus

$$z_i(x) = z_{i+1}(x) = z$$

If we wrap D5-branes on these spheres, the gauge theory we find has a field content given by adjoints $\Phi_i$ and bi-fundamentals between adjacent spheres $Q_{i,i+1},\ Q_{i+1,i}$ governed by a superpotential

$$W = \sum_i \left(W_i(\Phi_i) + Q_{i,i+1}\Phi_i Q_{i+1,i} - Q_{i,i+1}\Phi_{i+1}Q_{i+1,i}\right)$$

The adjoints just parametrize the positions of the D5s on the x-plane.

The superpotential for each adjoint $W(\Phi_i)$ can be computed by using the formula due to Witten:

$$W_i = \int_{C_i} \Omega, \quad \partial C_i = S_i^2$$

For the class of geometries at hand, one can prove that this simplifies to just:

$$W_i = \int dx \, (z_i(x) - z_{i+1}(x))$$

So consider, for instance, the geometry:

$$uv = (z - mx)(z + mx)(z - mx)(z + m(x - 2a))$$

Wrap e.g. one brane on each of the three two-spheres (more general cases also work, but this is the most modest choice; wrapping an O-plane on node 3 would also work and be directly analogous to what we just did).

The superpotential is now

$$W = \sum_i W_i(\Phi_i) + Q_{12}\Phi_2 Q_{21} - Q_{21}\Phi_1 Q_{12} + Q_{23}\Phi_3 Q_{32} - Q_{32}\Phi_2 Q_{23}$$

The $W_i$ are easily computed from the geometry, yielding:

$$W_1(\Phi_1) = m\Phi_1^2, \quad W_2(\Phi_2) = -m\Phi_2^2, \quad W_3(\Phi_3) = m(\Phi_3 - a)^2$$

The important point is that the third node will be localized away from the other two nodes on the x-plane. The quarks stretching to it from node 2 will be massive as is its adjoint, so node 3 is completely massive.

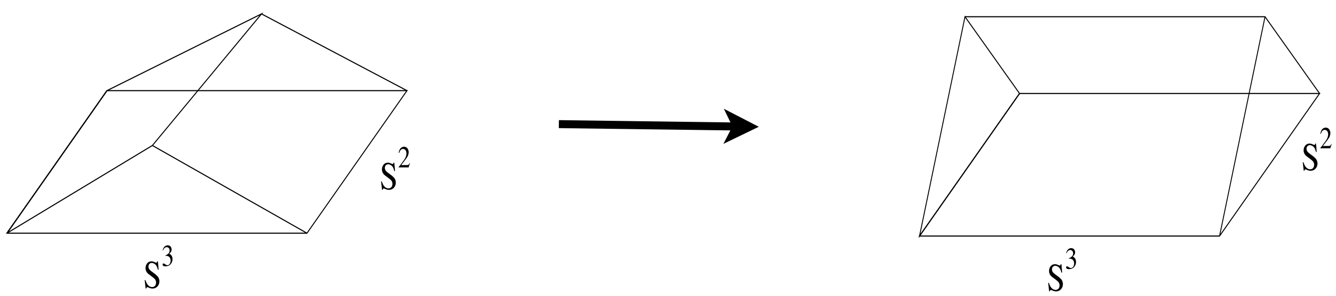
In this situation, we are free to perform a geometric transition on node 3.

Vafa;
Klebanov, Strassler

This results in a deformed geometry:

$$uv = (z - mx)(z + mx)((z - mx)(z + m(x - 2a)) - s)$$

s is the local deformation parameter of a conifold-like singularity on the x-plane. In addition, there should be flux through the new "A-cycle" three-sphere created by the deformation, as well as its B-cycle dual:



$$\int_A F_3 = 1, \quad \int_B H_3 = -t$$

The periods of the holomorphic three-form in this geometry are:

$$\int_A \Omega = S, \quad \int_B \Omega = S\left(\log(\tfrac{S}{\Delta^3}) - 1\right)$$

where $$S = s/m$$

After the transition, the third D5 brane is gone. But there is a flux superpotential, and $W_2(\Phi)$ has changed.

The total superpotential is now:

$$W = W_1(\Phi_1) + \tilde{W}_2(\Phi_2, S) + Q_{12}\Phi_2 Q_{21} - Q_{21}\Phi_1 Q_{12} + W_{flux}(S)$$

The flux superpotential is:

$$W_{flux}(S) = \frac{t}{g_s} S + S\left(\log(\frac{S}{\Delta^3}) - 1\right)$$

while

$$\tilde{W}_2(x) = \int (z_2(x) - \tilde{z}_3(x))$$

Here, we've defined

$$(z - \tilde{z}_3(x))(z - \tilde{z}_4(x)) = (z - z_3(x))(z - z_4(x)) - s$$

where one chooses for $\tilde{z}_3$ the branch which asymptotically looks like z_3 at large x.

As a result:

$$\tilde{W}_2(x) = \int_{\Delta}^{x} \left( -m(x' + a) - \sqrt{m^2(x' - a)^2 + s} \right) dx'$$

This superpotential sums up the instanton effects due to Euclidean D1-branes wrapping node 3!

We can now solve for S and for the location of the D5-brane vacuum in the x-plane.

The result is:

$$S = \Delta^3 \exp(-\frac{\tilde{t}}{g_s})$$

$$\tilde{t} = t - \frac{1}{2} g_s \log(\frac{a}{\Delta})$$

and

$$\Phi_1 = -\frac{1}{2m} Q_{12} Q_{21}$$

$$\Phi_2 = \frac{1}{2m} Q_{21} Q_{12} + \frac{1}{4ma} S + \ldots$$

The omitted terms are higher order in

$$\frac{Q_{21}Q_{12}}{ma}, \quad \exp(-\frac{\tilde{t}}{g_s})$$

Then the low-energy effective superpotential is:

$$W = \frac{1}{m}Q_{12}Q_{21}Q_{12}Q_{21} - \frac{S}{4ma}Q_{12}Q_{21} + \ldots$$

Up to irrelevant terms, this is again the Fayet model "retrofitted" by the geometric transition to have exponentially small SUSY breaking scale. In this case, however, we can classically and explicitly compute the full series of instanton corrections! As promised, the higher corrections do nothing to the SUSY breaking vacuum.

We could have done the same thing with an (Sp-type) O5-plane wrapping node 3. After the geometric transition, this still generates a flux superpotential, and the story is much the same. The absence of the instanton effect if instead we had an empty, un-orientifolded node, is also clear from the transition picture. So, the transition picture and the stringy instanton picture seem to be in complete agreement.

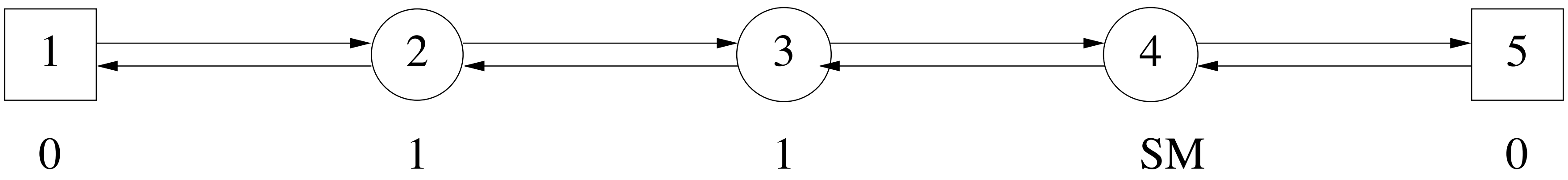
Intriligator, Kraus, Ryzhov, Shigemori, Vafa

# Applications

What one would really like to do, is to find a way to use these very simple models in combination with a realization of the supersymmetric Standard Model. Then the natural question to ask is: how is the SUSY breaking transmitted to the (M)SSM?

If one is not afraid of the SUSY flavor problem, one can use ``gravity mediation" and leave it at that. But I am afraid of the SUSY flavor problem.

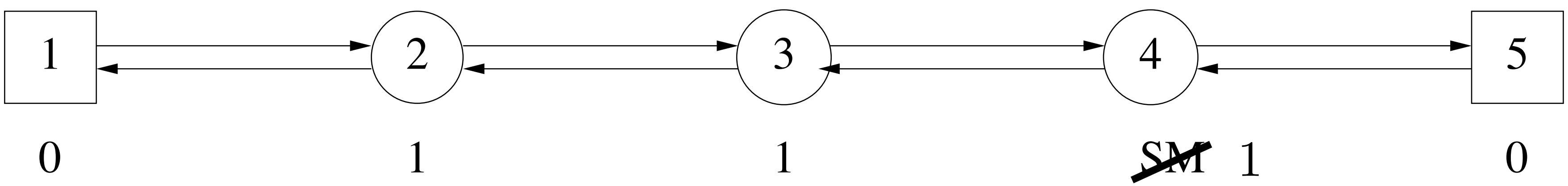
Probably the most elegant idea would be to use gauge mediation. Very roughly, one can imagine a quiver like:




Then, the instanton on node 1 generates the small mass of the Fayet model. The strings between nodes 3 and 4 are messengers. (As long as their mass is large compared to the SUSY breaking scale, they do not change the SUSY breaking dynamics).

c.f. Kawano, Ooguri, Ookouchi

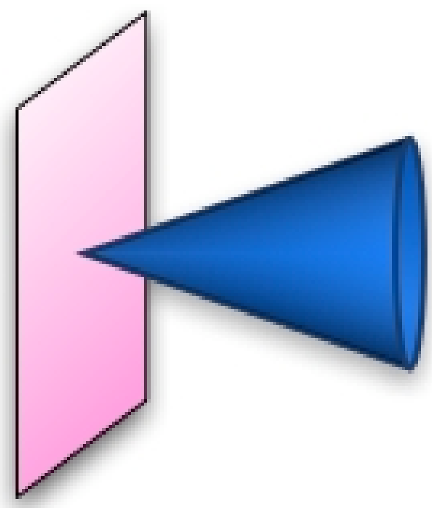
In fact, in this quiver, with the SM replaced by a U(1) ( so this could literally arise from the n=5 case of our singular geometries):




one would obtain dynamically generated messenger masses, from the stringy instanton at node 5.

This kind of mechanism could explain the very small messenger masses needed for low-scale gauge mediation, in a U(1) extension of the SM.

An idea which is perhaps less elegant, would be to embed our quiver theories at the end of a renormalization group cascade with its associated (gravity dual) throat geometry.



In closely related theories, we have explicitly constructed appropriate RG cascades and seen how the stringy instanton effect is generated (in a dual view) by dynamical gauge theory effects.

Aharony, S.K.

Since SUSY breaking at the end of such warped throats is ``sequestered” in at least some of the simplest cases (like the warped, deformed conifold case with SUSY breaking by anti-branes), this could be a way to make models of anomaly mediation.

S.K. , McAllister, Sundrum

One would need to incorporate a cure for the tachyonic sleptons, about which I have nothing to say here.

# Directions for Further Work

* These models require only one or two D-branes, and an appropriate cycle to be wrapped by the Euclidean brane. There should be many simple avatars, some simpler than the singularities I mentioned. Kumar

* For model building, one should extend this idea to make simple SUSY breaking sectors that break R-symmetry.

* A general understanding of Euclidean branes in quivers that arise at Calabi-Yau singularities may be within reach.

* It would be worthwhile to design similar theories in corners of the landscape that naturally accomodate gauge coupling unification.