Matter Transitions and Heterotic/F-theory Duality

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Work done in collaboration with:

(J. Gray, N. Raghuram, W. Taylor) - arXiv:1512.05791

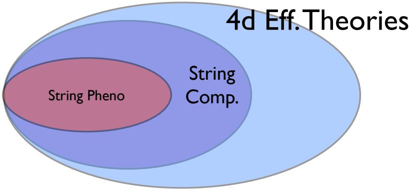
F-theory at 20

February 24th, 2016

Motivation

General questions in F-theory:

- Finiteness and characterization of F-theory vacua?
- Better understanding of range of possible matter structures (and later: Yukawa couplings, etc)
- Improved understanding of string dualities (Sen limits, F-/M-theory, Heterotic/F-theory duality, Flux backgrounds, Non-geometric, etc)



What possible EFTs?

$\Updownarrow$

Which geometries?

Matter in Transition

Today we'll consider one small window into these questions: **Matter transitions**

- Geometric transitions in F-theory characterized by their effect on the spectrum. Three main types:
  1. Blowing-up/down the base $\Rightarrow$ tensionless string transitions ($6D$: change n_T and n_H)
  2. Higgsing/unHiggsing transitions $\Rightarrow$ ($6D$: n_T unchanged, change in n_V, n_H)
  3. More exotic: Matter multiplicities change without changing gauge symmetry ($6D$: n_T and n_V unchanged. Only representation content of matter fields change.) (Morrison, Taylor)
- Let's look at $SU(N)$ examples...

Consistent Transitions: Anomaly Equivalent Matter

Rep.	N	Dimension	A_R	B_R	C_R	g
Adjoint	N	N^2-1	$2N$	$2N$	6	1
	6, 7, 8	**35**, **48**, **63**	12, 14, 18	12, 14, 18	6	1
	N	N	1	1	0	0
	N	$\frac{N(N-1)}{2}$	$N-2$	$N-8$	3	0
	6, 7, 8	**15**, **21**, **28**	4, 5, 6	-2, -1, 0	3	0
	N	$\frac{N(N-1)(N-2)}{6}$	$\frac{N^2-5N+6}{2}$	$\frac{N^2-17N+54}{2}$	$3N-12$	0
	6, 7, 8	**20**[**10**], **35**, **56**	6[3], 10, 15	-6[-3], -8, -9	6, 9, 12	0
	N	$\frac{N(N-1)(N-2)(N-3)}{24}$	$\frac{(N-2)(N-3)(N-4)}{6}$	$\frac{(N-4)(N^2-23N+96)}{6}$	$\frac{3(N^2-9N+20)}{2}$	0
	8	**70**[**35**]	20[10]	-16[-8]	18[9]	0

Anomalies:

$$-a\cdot b = -\frac{1}{6}\left(A_{\text{adj}} - \sum_R x_R A_R\right)$$

$$0 = B^i_{\text{adj}} - \sum_R x_R B_R$$

$$b\cdot b = -\frac{1}{3}\left(C_{\text{adj}} - \sum_R x_R C_R\right)$$

Where a, b are the coefficients of BR^2, BF^2 Green-Schwarz terms. (See talks of Grimm, Klevers).

Examples

- $SU(6)$

$$\mathbf{10}\ \left(\tfrac{1}{2}\begin{array}{c}\Box\\[-4pt]\Box\\[-4pt]\Box\end{array}\right)+\mathbf{6}\,(\Box)\,.\quad\leftrightarrow\quad\mathbf{15}\left(\begin{array}{c}\Box\\[-4pt]\Box\end{array}\right)+\mathbf{1}\,.$$

- $SU(7)$

$$\mathbf{35}\ \left(\begin{array}{c}\Box\\[-4pt]\Box\\[-4pt]\Box\end{array}\right)+5\times\mathbf{7}\,(5\times\Box)\,.\quad\leftrightarrow\quad 3\times\mathbf{21}\left(\begin{array}{c}\Box\\[-4pt]\Box\end{array}\right)+7\times\mathbf{1}\,.$$

- $SU(8)$

$$\mathbf{56}\ \left(\begin{array}{c}\Box\\[-4pt]\Box\\[-4pt]\Box\end{array}\right)+9\times\mathbf{8}\,(\times\Box)\,.\quad\leftrightarrow\quad 4\times\mathbf{28}\left(\begin{array}{c}\Box\\[-4pt]\Box\end{array}\right)+16\times\mathbf{1}\,.$$

and

$$\mathbf{35}\ \left(\frac{1}{2}\begin{array}{c}\Box\\[-4pt]\Box\\[-4pt]\Box\\[-4pt]\Box\end{array}\right)+8\times\mathbf{8}\,(\Box)\,.\quad\leftrightarrow\quad 3\times\mathbf{28}\left(\begin{array}{c}\Box\\[-4pt]\Box\end{array}\right)+15\times\mathbf{1}\,.$$

Today:

- Understand matter transitions purely in Field theory, and in F-theory and heterotic compactifications (illustrate in 6D but much structure extends to 4D).
- To do this systematically will require some new tools:
  1. General Weierstrass models for $SU(N)$ $6 \leq N \leq 9$ (see Morrison's talk).
  2. Heterotic/F-theory duality for reducible bundles: $V = V_1 \oplus V_2$ with $c_1(V_1) = -c_1(V_2)$ (see also talks of Cvetic, Grassi)
  3. Het/F pairs with *generic* Green-Schwarz massive $U(1)$ symmetries and the associated stable degeneration limits.
  4. Exotic matter representations (See talks of Klevers, Raghuram)

Possible matter transitions

- Many different gauge symmetries possible: $SU(3)$, $Sp(3)$, $SU(N)$ with $N \geq 6$, $SO(12)$, etc. and assorted product groups $SU(2) \times SU(4)$, etc.
- Transitions somewhat mysterious from the point of view of pure field theory...
- Intriguing questions on Higgsing chains (are the transitions 'inherited'?)
- What types of matter, Weierstrass models? (representations, unique factorization domain questions, etc).
- To explore Het/F dual pairs with such transitions, we'll begin with $SU(N)$ with $6 \leq N \leq 9$...

Non-Tate: $SU(N)$ with $N \geq 6$

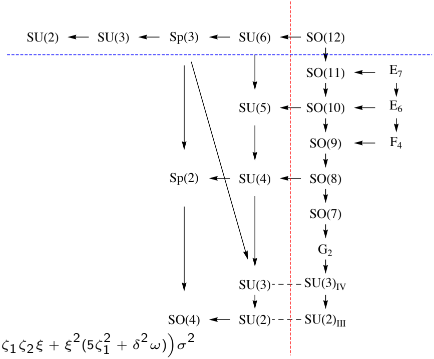
- The Kodaira form of singularities hold, but not necessarily Tate form
- Some previous examples (Morrison, Vafa, Intriligator...)(Morrison, Taylor)
- Previous special cases in the literature lacked either double or triple antisymmetric matter
- We find explicit $SU(N)$ Weierstrass models w/ $N \geq 6$.
- Idea: Step-by-step enhancement $SU(6) \to SU(7) \to SU(8) \to \dots$
- Can verify complete degrees of freedom in 6D (but form valid in general)
- Same techniques for product groups (e.g. $SU(N) \times SU(M)$...)

$$f = F_0 + F_1\sigma(\sigma - \epsilon) + F_2\sigma^2(\sigma - \epsilon)^2 + \dots$$

$$g = G_0 + G_1\sigma(\sigma - \epsilon) + G_1\sigma^2(\sigma - \epsilon)^2 + \dots$$

Leads to $\Delta = \sigma^N(\sigma - \epsilon)^M(\dots)$

- New chains: $SU(9) \to \ldots,$
  $SU(7) \to SU(4) \times SU(3) \to \ldots\ldots$
- E.g. $SU(7)$...



$$f = -\frac{\delta^{12}\xi^4}{48} - \frac{\delta^8\xi^2}{6}\left(\zeta_1\xi + \zeta_2\delta^2\right)\sigma - \frac{\delta^4}{6}\left(2\delta^4\zeta_2^2 + 4\delta^2\zeta_1\zeta_2\xi + \xi^2(5\zeta_1^2 + \delta^2\omega)\right)\sigma^2$$
$$- \frac{1}{6}\left(4\delta^2(3\zeta_1^2\zeta_2 + \delta^2\zeta_2\omega) + \xi(6\zeta_1^3 + \delta^2\zeta_1\omega + 18\delta^4\lambda_1)\right)\sigma^3 - \frac{1}{12}\left(\omega^2 + 72\zeta_1\lambda_1 - 12\delta^2\psi_4\right)\sigma^4 + \mathcal{O}(\sigma^5)$$

$$g = \frac{\delta^{18}\xi^6}{864} + \frac{\delta^{14}\xi^4}{72}\left(\zeta_1\xi + \zeta_2\delta^2\right)\sigma + \frac{\delta^{10}\xi^2}{72}\left(4\delta^4\zeta_2^2 + 8\delta^2\xi\zeta_1\zeta_2 + 7\zeta_1^2\xi^2 + \delta^2\xi^2\omega\right)\sigma^2$$
$$+ \frac{\delta^6}{216}\left(16\delta^6\zeta_2^3 + 48\delta^4\xi\zeta_1\zeta_2^2 + 120\delta^2\zeta_1^2\zeta_2\xi^2 + 70\zeta_1^3\xi^3 + 54\delta^4\xi^3\lambda_1 + 24\delta^4\zeta_2\xi^2\omega + 15\delta^2\xi^3\zeta_1\omega\right)\sigma^3$$
$$+ \frac{\delta^2}{144}\left(84\zeta_1^4\xi^2 + \delta^4(96\zeta_1^2\zeta_2^2 + 5\xi^2\omega^2 + 8\zeta_1\xi(27\lambda_1\xi + 5\zeta_2\omega)) + 16\delta^2\zeta_1^2\xi(9\zeta_1\zeta_2 + 2\xi\omega) + 4\delta^6(36\zeta_2\lambda_1\xi - 3\xi^2\psi_4 + 8\zeta_2^2\omega)\right)\sigma^4$$
$$+ \frac{1}{36}\left(2\left(3\zeta_1^2 + \delta^2\omega\right)\left(6\zeta_1^2\zeta_2 - \zeta_1\xi\omega + 2\delta^2(9\lambda_1\xi + \zeta_2\omega)\right) - 3\delta^6\xi^2 f_5 - \delta^2\left(\delta^2\zeta_2 + \zeta_1\xi\right)\left(-72\zeta_1\lambda_1 + 12\delta^2\psi_4 - \omega^2\right)\right)\sigma^5$$
$$- \left(\frac{\omega^3}{108} + \zeta_1\left(\lambda_1\omega + \zeta_1\psi_4\right) + \frac{\delta^2}{3}\left(\psi_4\omega - 27\lambda_1^2 + f_5\nu + \frac{\xi^2\delta^4}{4}f_6\right)\right)\sigma^6 + \mathcal{O}(\sigma^7)$$

Realization of the transitions

- Consider in $6D$ an $SU(6)$ tuned on $+n$ curve (σ):

$$f = -\frac{\alpha^4\beta^4}{48} - \frac{1}{6}\alpha^2\beta^3\nu\sigma - \frac{\beta}{6}\left(\alpha^2\phi_2 + 2\beta\nu^2\right)\sigma^2 - \left(3\beta\lambda + \frac{\nu\phi_2}{3}\right)\sigma^3 + \mathcal{O}(\sigma^4)$$

  and

$$g = \frac{\alpha^6\beta^6}{864} + \frac{\alpha^4\beta^5}{72}\nu\sigma + \frac{\alpha^2\beta^3}{72}\left(4\beta\nu^2 + \alpha^2\phi_2\right)\sigma^2 + \frac{\beta^2}{108}\left(8\beta\nu^3 + 9\alpha^2\nu\phi_2 + 27\alpha^2\beta\lambda\right)\sigma^3$$
$$+ \frac{1}{36}\left(4\beta\nu^2\phi_2 + \alpha^2\phi_2^2 + 36\beta^2\nu\lambda - 3\alpha^2\beta^2 f_4\right)\sigma^4$$
$$+ \frac{1}{12}\left(12\lambda\phi_2 - 4\beta\nu f_4 - \alpha^2\beta^2 f_5\right)\sigma^5 + \mathcal{O}(\sigma^6)$$

- With $\Delta = \alpha^4\beta^3\Delta_6\sigma^6 + \mathcal{O}(\sigma^7)$

- To change matter

  $\frac{1}{2}\mathbf{20}\ \left(\frac{1}{2}\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline \ \\ \hline\end{array}\right) + \mathbf{6}\,(\square) \ \leftrightarrow\ \mathbf{15}\left(\begin{array}{|c|}\hline \ \\ \hline \ \\ \hline\end{array}\right) + \mathbf{1}$

  need to change degree of α ($\mathbf{15}$s), β ($\mathbf{20}$s) and Δ_6 ($\mathbf{6}$s)..

Matter transitions in $SU(6)$

- $\Delta = \alpha^4 \beta^3 \Delta_6 \sigma^6 + \mathcal{O}(\sigma^7)$. To change multiplicities, first let α, ν and λ develop a common factor

$$\alpha \to a\alpha',$$
$$\nu \to a\nu',$$
$$\lambda \to a\lambda'.$$

- Then reabsorb a into β ϕ_2:

$$a\beta \to \beta',$$
$$a\phi_2 \to \phi_2'.$$

- Important observation: f and g vanish to orders 4 and 6 when $a = \sigma = 0$ $\Rightarrow$ a is a superconformal point.

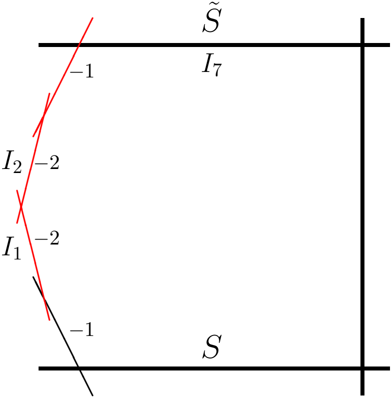
- Can track matter carefully through the transition:
- The discriminant goes to

$$\Delta = a^6 \alpha^4 \beta^3 \Delta_6' \sigma^6 + \mathcal{O}(\sigma^7)$$

- 29 multiplets participate in total $\rightarrow$ expected if a blow up in the base were performed (increase in n_T)

$$\begin{smallmatrix}\square\\\square\end{smallmatrix} + 2 \times \square + 2 \times \mathbf{1} \rightarrow \textbf{Superconformal Matter} \rightarrow \frac{1}{2}\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} + 3 \times \square + \mathbf{1}$$

- Likewise, can consider $SU(7)$...



Start with

$$\xi \to a^3 \xi$$

$$\zeta_2 \to a^4 \zeta_2$$

$$\lambda_1 \to a \lambda_1$$

$$\psi_4 \to a^2 \psi_4.$$

Then go to

$$a\delta \to \delta$$

$$a\zeta_1 \to \zeta_1.$$

$$\Delta_{SU(7)} = \delta^8 \xi^4 \Big(-\frac{1}{8} \zeta_1^7 \xi + \mathcal{O}(\delta^2) \Big) \sigma^7 + \mathcal{O}(\sigma^8)$$

$\delta \sim \begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} + \square$, $\xi \sim \begin{smallmatrix}\square\\\square\end{smallmatrix}$

$(3 \times \begin{smallmatrix}\square\\\square\end{smallmatrix} + 3 \times \square + 8 \times \mathbf{1}) \to$

Superconformal Matter $\to$

$(\begin{smallmatrix}\square\\\square\\\square\end{smallmatrix} + 8 \times \square + \mathbf{1})$

92 multiplets participate here: Corresponds to 3 tensor multiplets and a strongly coupled $SU(2)$ gauge symmetry (See talks of Heckman, Rudelius)

- Almost all matter transitions involve such superconformal loci (we'll come back to exceptions)
- Intriguingly, transitions for $SU(8)$ and higher, superconformal points may not be resolvable by blowing up the base (no tensor branch?) (See Tachikawa's talk).
- Let's turn now to the transitions and superconformal geometry in a dual heterotic theory...

Heterotic Geometry: $SU(6)$

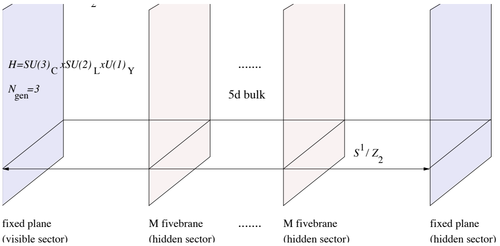
- The commutant of $SU(6) \subset E_8$ is $SU(3) \times SU(2) \rightarrow V = V_2 \oplus V_3$ with $c_1(V_2) = c_1(V_3) = 0$
- $c_2(V_2) + c_2(V_3) + c_2(V_{hidden}) = c_2(TX_3)$
- Transition moves "pieces" of $c_2(V_2) \leftrightarrow c_2(V_3)$ (within bounds)
- $6D$ illustration (Bershadsky, et al):

  $c_2(V) = 12 + n$, $c_2(V_2) = 4 + r$, $c_2(V_3) = 16 + 2n + r$
- Spectra a function of integers (r, n):

$$\frac{r}{2}\mathbf{20} + (16 + r + 2n)\mathbf{6} + (2 + n - r)\mathbf{15}$$

- Matter transition: $V_2 \oplus V_3 \rightarrow V_2' \oplus V_3' \oplus \mathcal{I}_{SM}$ **(superconf.)** $\rightarrow V_2'' \oplus V_3''$

Dual Interpretation: Heterotic M-theory



- Higgsing $\to$ Deforming V/Y_{n+1}
- Blowing-up/down the base $\to$ small instanton transitions across $S^1/\mathbb{Z}_2$
- Exotic transitions $\to$ Small instanton transitions on the *same* fixed plane
- Deformation/Resolution of superconformal loci
- Straightforward to classify which symmetries admit these matter transitions. Almost all coupled to superconformal loci (exceptions: Duals of $SO(32)$ heterotic theories)
- Relevant to recent developments in superconformal matter

$SU(N)$ Matter Transitions

- What about the heterotic duals of $SU(7)$,$SU(8)$,$SU(9)$ F-theory models?
- Here, unlike other cases, the commutant inside of E_8 takes the generic (and special form): $S[U(m_1) \times U(m_2)]$
- These bundles do **not** generically satisfy HYM eqns. Polystability $\Rightarrow$ non-trivial D-term conditions constraining the EFT and restricting moduli. (bundle stability)
- Split ($U(n)$) spectral covers have been studied in many e.g.s (Hayashi, Choi, Watari, Braun, Mayrhofer, Palti, Weigand...)
- Here the special feature is that this splitting is required/generic in the complete moduli space (leads to new higgsing chains)
- Requires much more careful analysis of stable degeneration limits $Y \rightarrow Y_1 \cup_X Y_2$

Illustration: $SU(7)$

- The commutant of $SU(7) \subset E_8$ is $SU(2) \times U(1)$ and takes the form:

$$V = U_2 \oplus L$$

with $c_1(V)$, but $c_1(U_2) = -c_1(L) \neq 0$

- Tuning from $SU(6)$: $V_2 \to L \oplus L^\vee$, $V_3 \to U_2 \oplus L$
- Non-trivial constraint, $\mu(L) = 0$
- $U(1)$ factor is self-commuting in E_8 and Green-Schwarz massive

Representation	Cohomology	$6D$ Multiplicity
1	$H^1(\mathrm{End}(U_2))$	$4c_2(U_2) - c + 1(L)^2 - 6$
7	$H^1({U_2}^\vee \otimes L) \oplus H^1(L^{\vee 2})$	$(c_2(U_1) - \frac{5}{2}c_1(L)^2 - 4) + (-2c_1(L)^2 - 2)$
$\bar{7}$	$H^1(U_2 \otimes L^\vee) \oplus H^1(L^2)$	$(c_2(U_2) - \frac{5}{2}c_1(L)^2 - 4) + (-2c_1(L)^2 - 2)$
$\overline{35}$	$H^1(L)$	$-\frac{1}{2}c_1(L)^2 - 2$
21	$H^1({V_2}^\vee)$	$(c_2(V_2) - c_1(L)^2 - 4)$

Het/F Duality: Reducible bundles and spectral covers

- How to match general F-theory tuning $SU(6) \to SU(7)$ to bundle geometry?
- D.o.f matching through spectral covers
- At $SU(6)$, $SU(2) \times SU(3)$ bundle:

$$S_V = (\phi_2 Z^2 + \beta X)(\lambda Z^3 + \nu XZ + \alpha Y) = 0$$

  How does this change under $SU(6) \to SU(7)$ tuning? Does it match $V_2 \to L \oplus L^{\vee}$, $V_3 \to U_2 \oplus L$?

- $SU(2)$ piece must factor into sum of line bundle and its dual.
- If good spectral cover $\Rightarrow$ Heterotic geometry must develop an additional section (alternative: T-branes)

Spectral Covers: $SU(7)$ illustration

- At $SU(6)$, roots of spectral cover $(p_1 \boxplus p_2) = 0$ $(\leftrightarrow SU(2))$ $(q_1 \boxplus q_2 \boxplus q_3) = 0$ $(\leftrightarrow SU(3))$
- At $SU(7)$ expect one p root to overlap q's and $SU(2)$ component to become reducible. $U(1)$'s $\Rightarrow$ additional section to heterotic ell. fibration
- In stable degeneration limit:

$$f_4 \rightarrow -6\zeta_1\lambda_1 - \frac{1}{12}\omega^2 + \psi_4\delta^2$$

$$g_6 \rightarrow \frac{1}{108}\left(972\delta^2{\lambda_1}^2 - 108\zeta_1^2\psi_4 - 108\zeta_1\lambda_1\omega - 36\delta^2\psi_4\omega - \omega^3\right)$$

- Novel feature: For $SU(7), SU(8)$, multiple paths to stable degeneration (see also talks of Cvetic, Grassi, Song)
- MW rank 1: New section at

$$[X, Y, Z] = \left[-\frac{1}{3}\left(3{\zeta_1}^2 + \delta^2\omega\right), i\left({\zeta_1}^3 + \frac{1}{2}\zeta_1\delta^2\omega - 3\lambda_1\delta^4\right), -i\delta\right]$$

SU(7) Spectral Cover

- MW rank 1: New section at

$$[X, Y, Z] = \left[-\frac{1}{3}\left(3{\zeta_1}^2 + \delta^2\omega\right), i\left({\zeta_1}^3 + \frac{1}{2}\zeta_1\delta^2\omega - 3\lambda_1\delta^4\right), -i\delta\right]$$

- $SU(7)$ tuning leads to reduced $S[U(1) \times U(1)] \times S[U(2) \times U(1)]$ spectral cover

$$\left((3\zeta_1^2 + \delta^2\omega)Z^2 - 3\delta^2 X)\right)\left(-3\left(\frac{1}{3}{\zeta_1}^2\zeta_2 - \frac{1}{18}\zeta_1\xi\omega + \frac{1}{9}\delta^2\zeta_2\omega + \lambda_1\delta^2\xi\right)Z^3 + (\zeta_2\delta^2 + \zeta_1\xi)XZ - \delta\xi Y\right) = 0$$

- $SU(2)$: $(p_1 \boxplus (-p_1))$ with p_1 new section **and** $(p_1 \boxplus q_2 \boxplus q_3) = 0$ $(\leftrightarrow SU(3))$
- Perfect agreement with bundle geometry: $L \oplus L^\vee$ and $L \oplus U_2$
- $U(1)$ only arises in "half" of the stable degeneration geometry $\rightarrow$ massive
- Can explicitly verify D-terms and Kähler axions transforming under $U(1)$ via shifts: $\delta\chi^i = -c_1^i(L)\eta^a$

$SU(7), SU(8), SU(9)$ and higher

- Want to contrast the questions what is possible in 6D (resp. 4D) EFT? vs. What is possible in F-theory? and heterotic compactifications?
- We find that at $SU(7)$ both heterotic/F-theory geometries sweep out full range of EFT and transitions
- $SU(8)$ Most transitions realizable in F-theory (except Λ^4). Only special class in perturbative heterotic
- $SU(9)$ Most general EFT not realizable in F-theory. No $SU(9)$ solns in perturbative heterotic.
- $SU(10)$ and triple anti-symmetric reps ruled out in F-theory and in EFT (by anomaly cancellation)

$G \subset SO(32)$ and Matter Transitions

- Bundle geometry of $SO(32)$ Small Instanton transitions identical to $E_8 \times E_8$ (ADHM construction, etc.)
- However, physics very different: $SO(32)$ - EFT, purely Higgsing transitions. $E_8 \times E_8$ non-critical tensionless strings
- How to see the difference in F-theory?
- $SO(32)$ duals involve no superconformal pts.
- Example: $(SO(6) \times SU(2)) \times SO(22) \times SU(2) \subset SO(32)$
- Small Instanton transitions,

  $V_{SO(6)} \oplus V_{SU(2)} \to V'_{SO(6)} \oplus V'_{SU(2)} \oplus \mathcal{I} \to V''_{SO(6)} \oplus V''_{SU(2)}$
- Transition point: $SO(22) \times SU(2) \to SO(22) \times SU(2) \times SU(2)$ (ordinary higgsing)

Summary and Conclusions

- Matter transitions provide a useful playground to explore Venn diagram of EFT vs. F-theory vs. Heterotic
- Found general form of $SU(N)$ Weierstrass models with $6 \leq N \leq 9$ and novel types of matter (see talk of Raghuram).
- In dual pairs, exotic matter transitions in F-theory are linked to heterotic small instanton transitions and can be classified.
- There are new Higgsing chains of geometries which provide an explicit/calculable arena to explore GS massive $U(1)$s and their generic effects.
- Novel forms of the stable degeneration limit