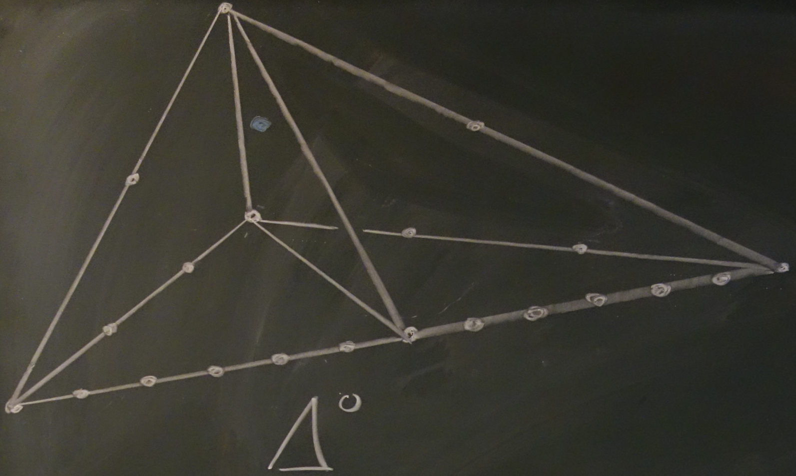
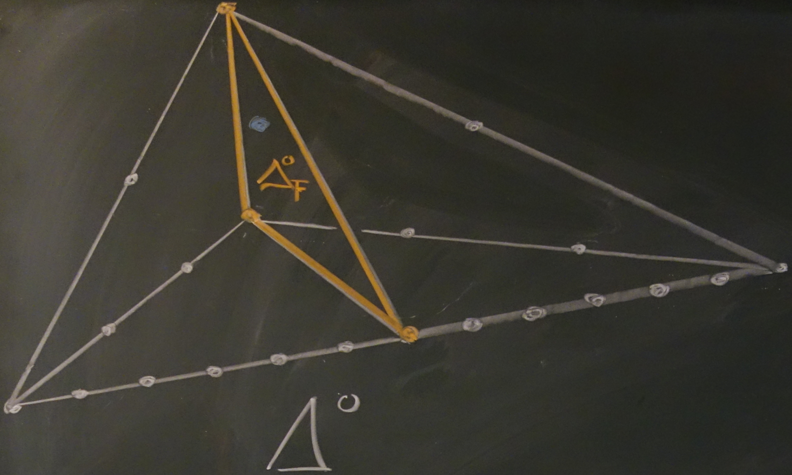
⋄ Tops as Building Blocks for G_2 manifolds ⋄

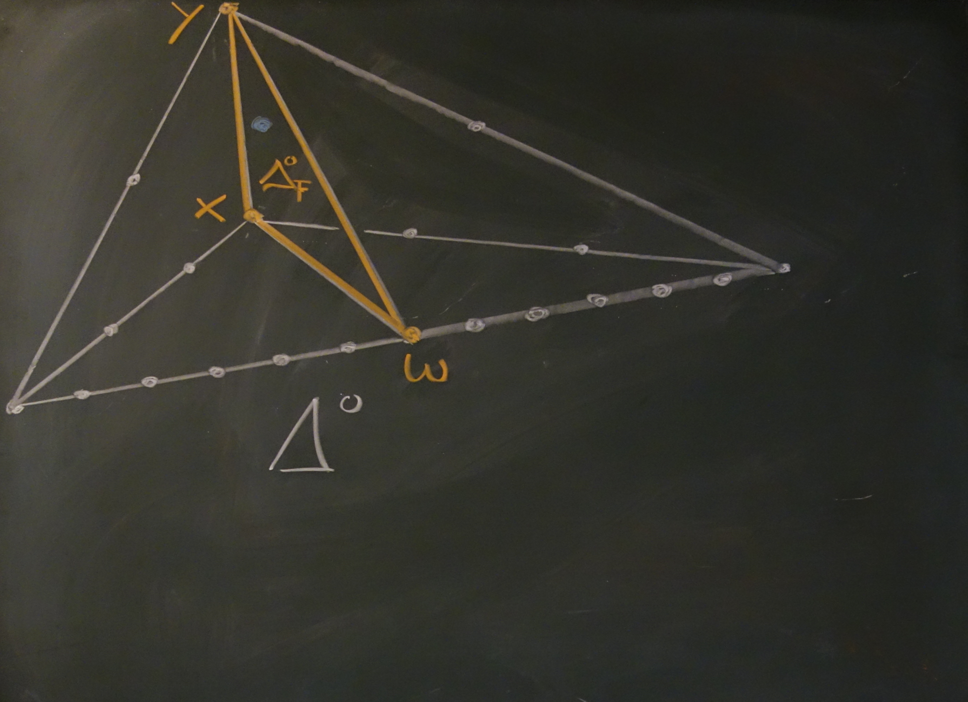
Andreas Braun

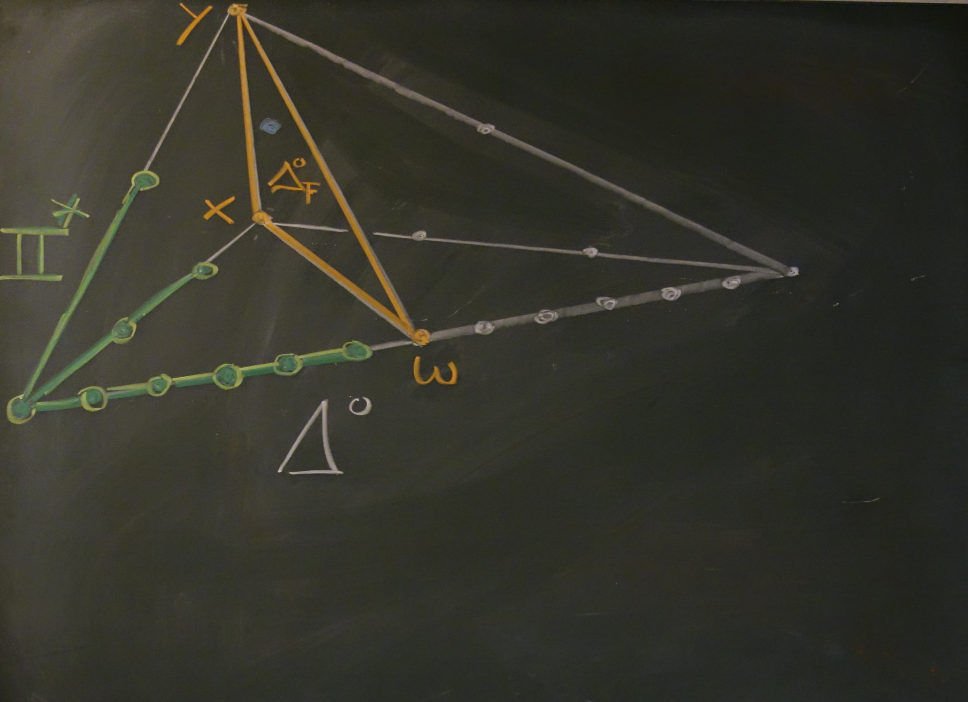
UNIVERSITY OF OXFORD

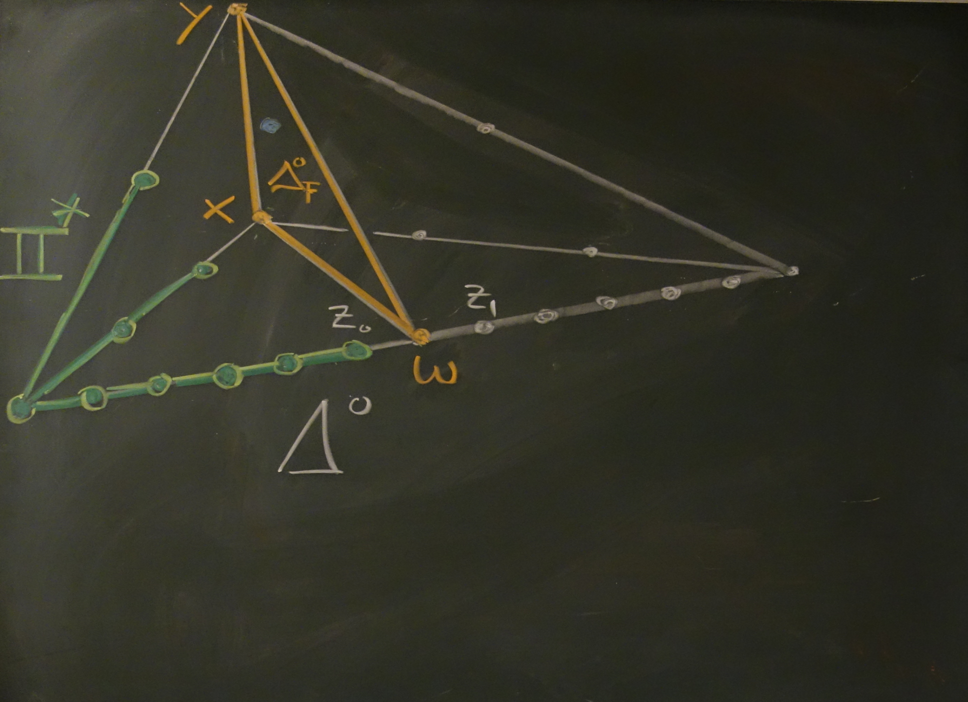
based on [1602.03521]

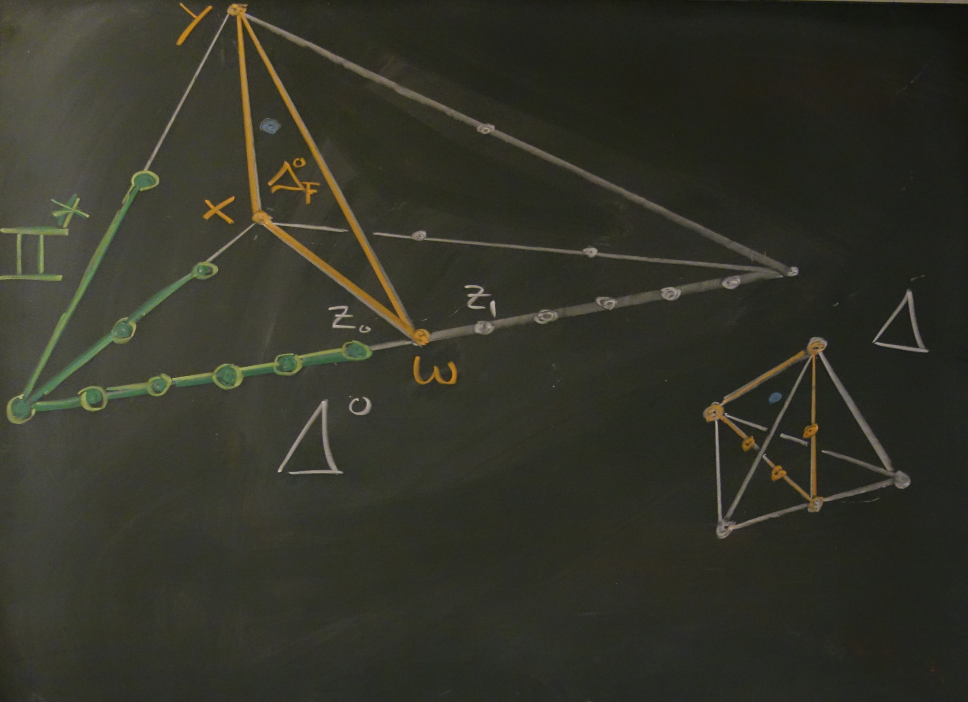


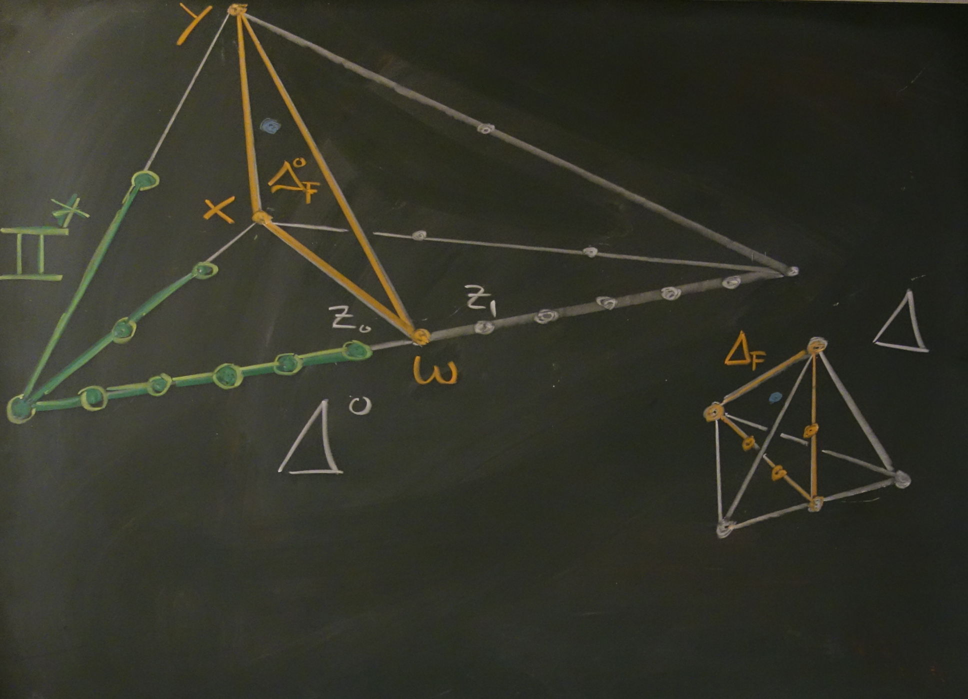

Δ°_{F}
Δ°


Y
Δ^0_F
X
ω
Δ^0


Y
X
Δ^0_F
ω
Π^*
Δ^0


Y
X
Δ^0_F
Z_0
Z_1
ω
Δ^0


Y
Δ^0_F
X
Π^X
Z_0
Z_1
ω
Δ^0
Δ


Y
Δ°_F
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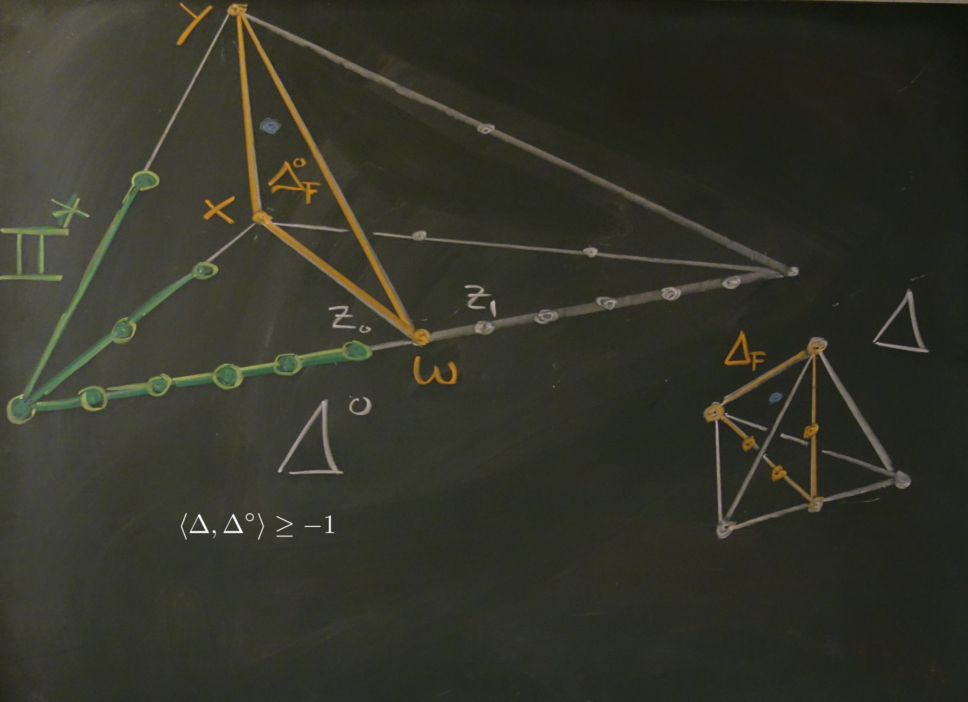
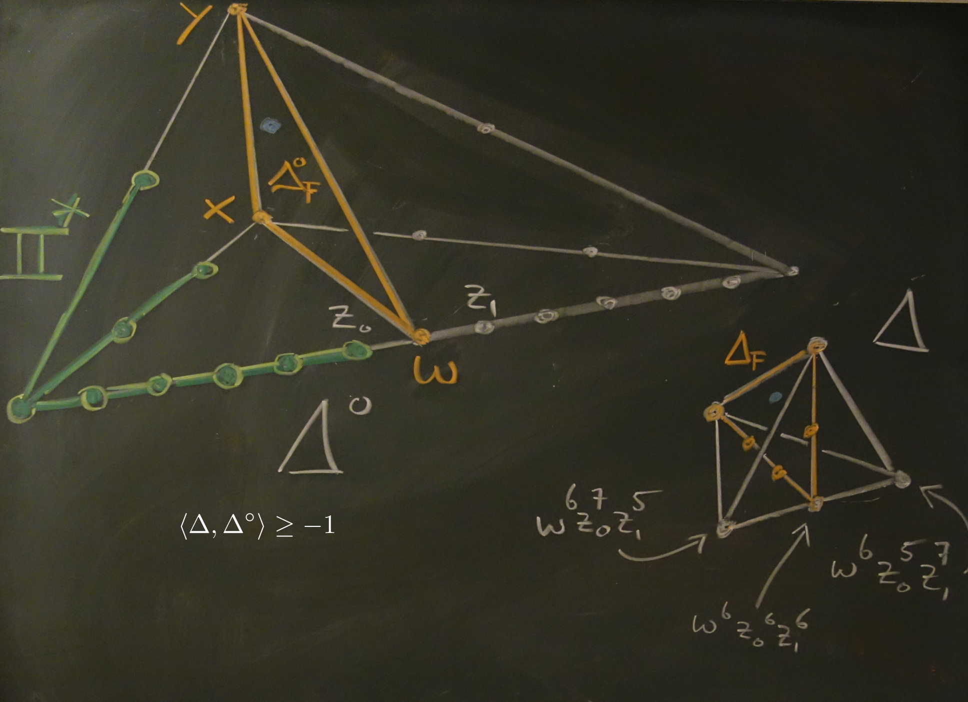
Z_0
Z_1
ω
Δ°
Δ_F
Δ



$$\langle \Delta, \Delta^\circ \rangle \geq -1$$



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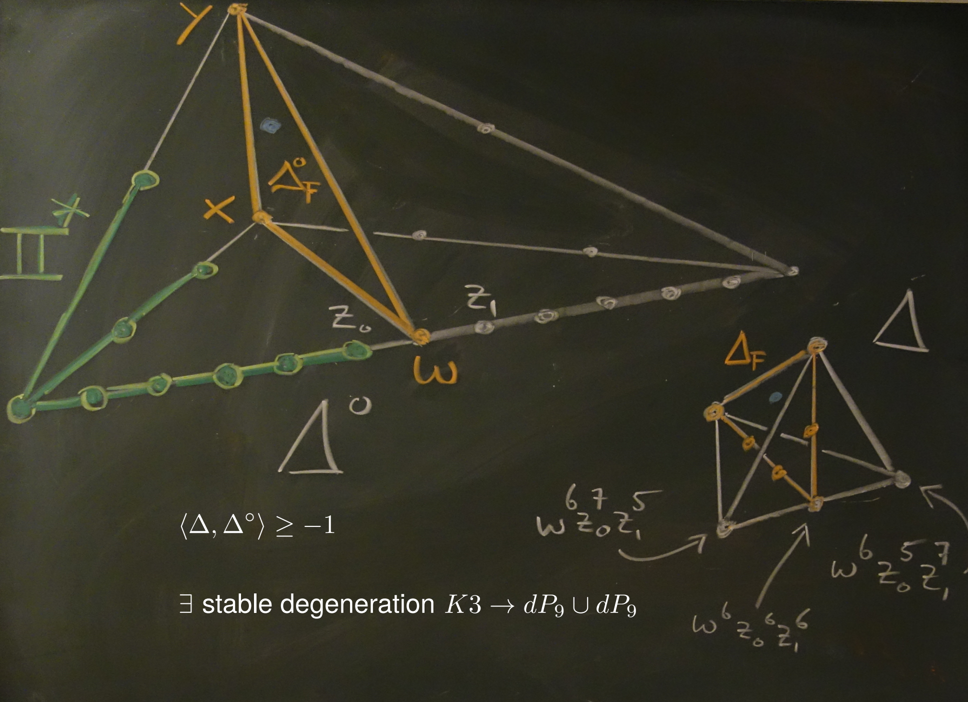

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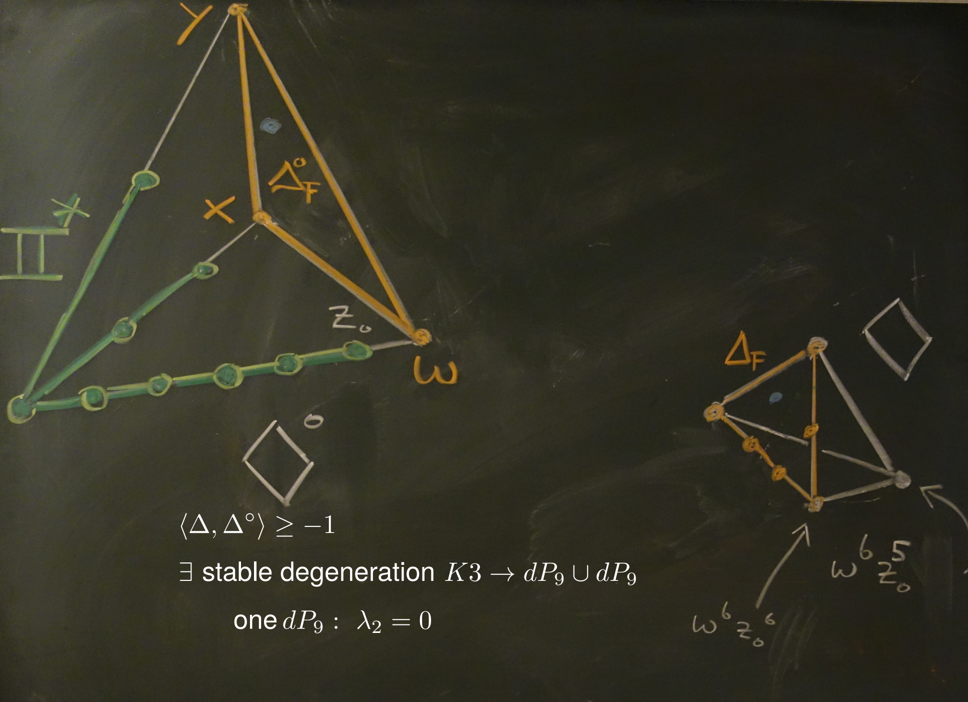
$\exists$ stable degeneration $K3 \to dP_9 \cup dP_9$



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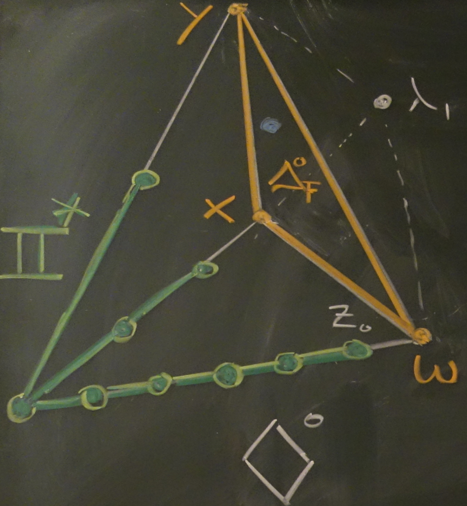

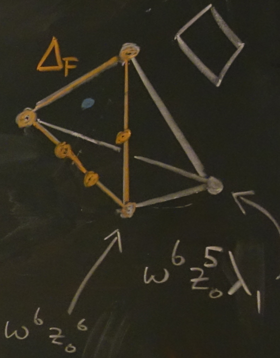

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We can formalize this construction and apply it in any dimension for 'projecting' tops: $Im(\Diamond^\circ \to F) \subseteq \Delta_F^\circ$

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Z is fibred over $\mathbb{P}_1$ with Calabi-Yau fibre

$$S = X_{(\Delta_F,\Delta_F^\circ)} = [z_0]$$

$$\Delta_F = \Diamond \cap F \qquad \Delta_F^\circ = \Diamond^\circ \cap F \qquad F = \nu_0^\perp = m_0^\perp$$

and

$$c_1(Z_{(\Diamond,\Diamond^\circ)}) = [S_{(\Delta_F,\Delta_F^\circ)}]$$

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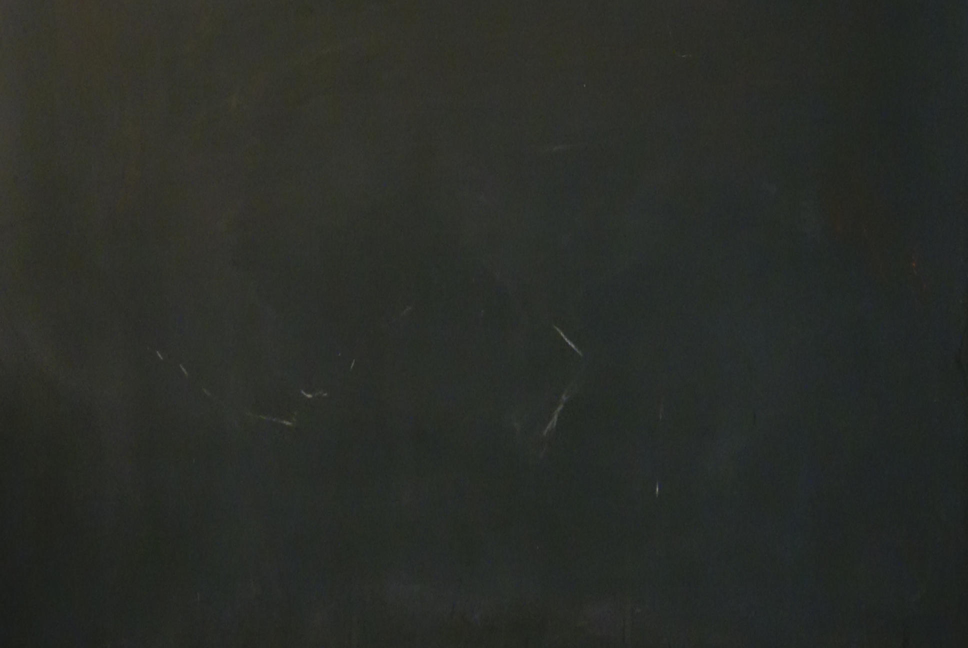
As $Z \supset \mathbb{P}_\Sigma$ originates from normal fan $\Sigma_n(\Diamond)$, can use stratification technique to find combinatorial formulas for

$$h^{i,j}(Z_{(\Diamond,\Diamond^\circ)})$$

Similar to [Batyrev] using [Danilov,Kohvanskii], see paper for details !

Why is this interesting ?



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If we take $\Diamond$ to be four-dimensional, Z is a K3 fibred threefold !

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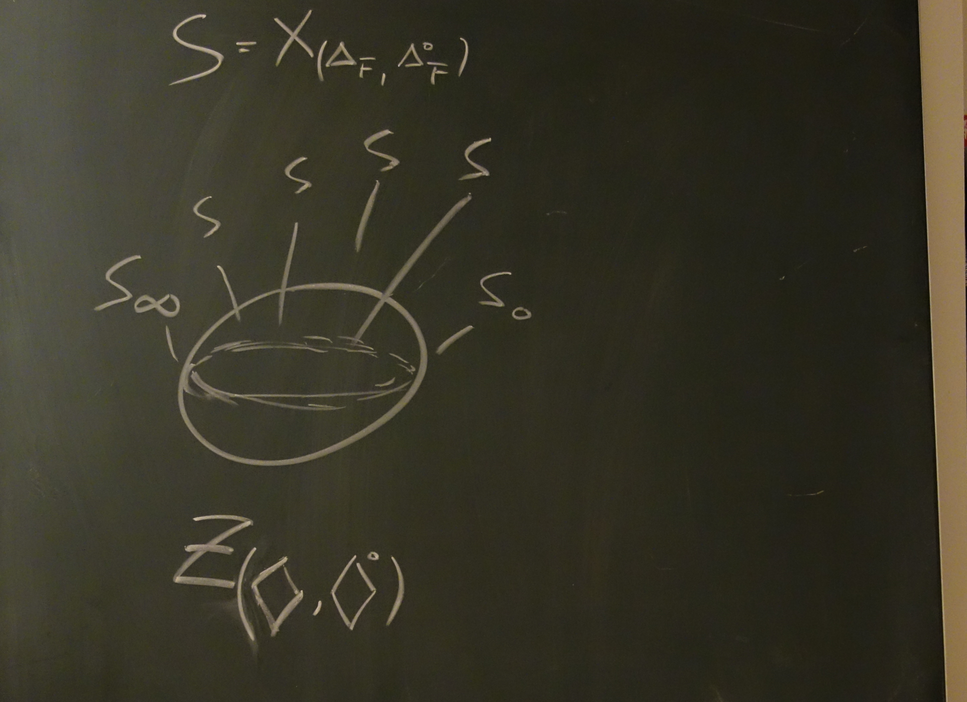
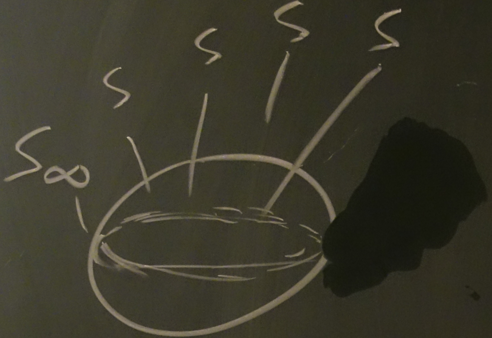
Can be used to construct G_2 manifolds as twisted connected sums à la [Donaldson; Kovalev; Corti, Haskins, Nordström, Pacini] !

These are interesting to get more examples of M-Theory compactifications to 4D.


$S = X_{(\Delta_F, \Delta_F^\circ)}$
S
S
S
S
S_∞
S_0
$Z(\diamond, \diamond^\circ)$

$$S = X_{(\Delta_F,\, \Delta_F^\circ)}$$



$$Z(\Diamond, \Diamond^\circ) \setminus S_0$$

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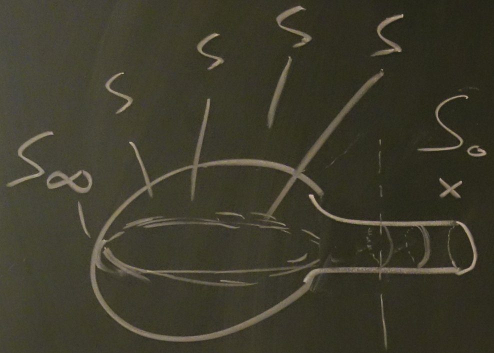

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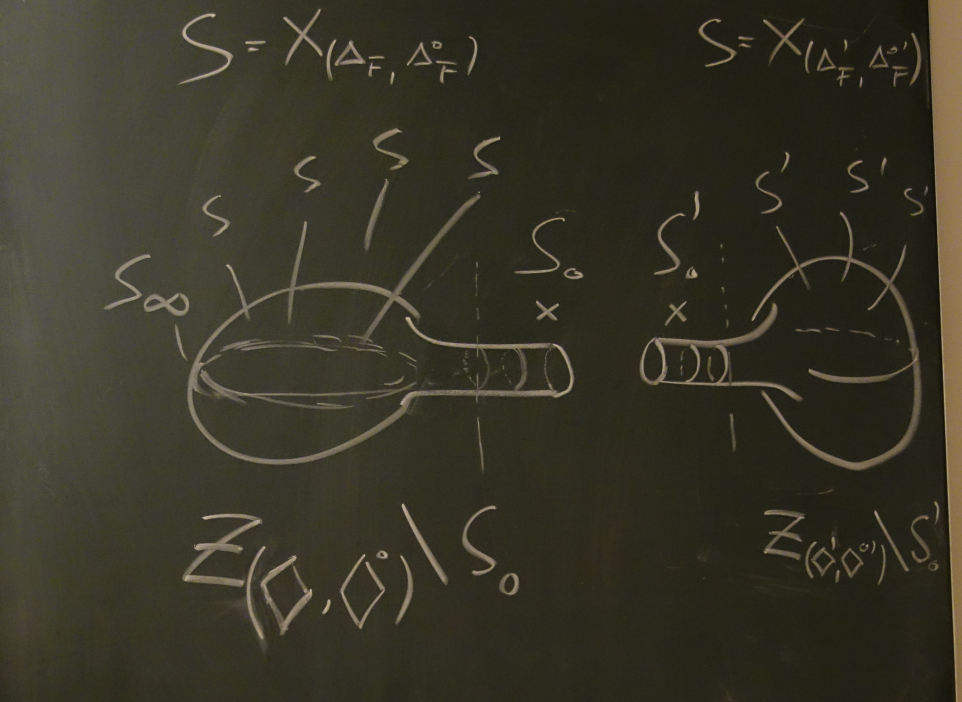
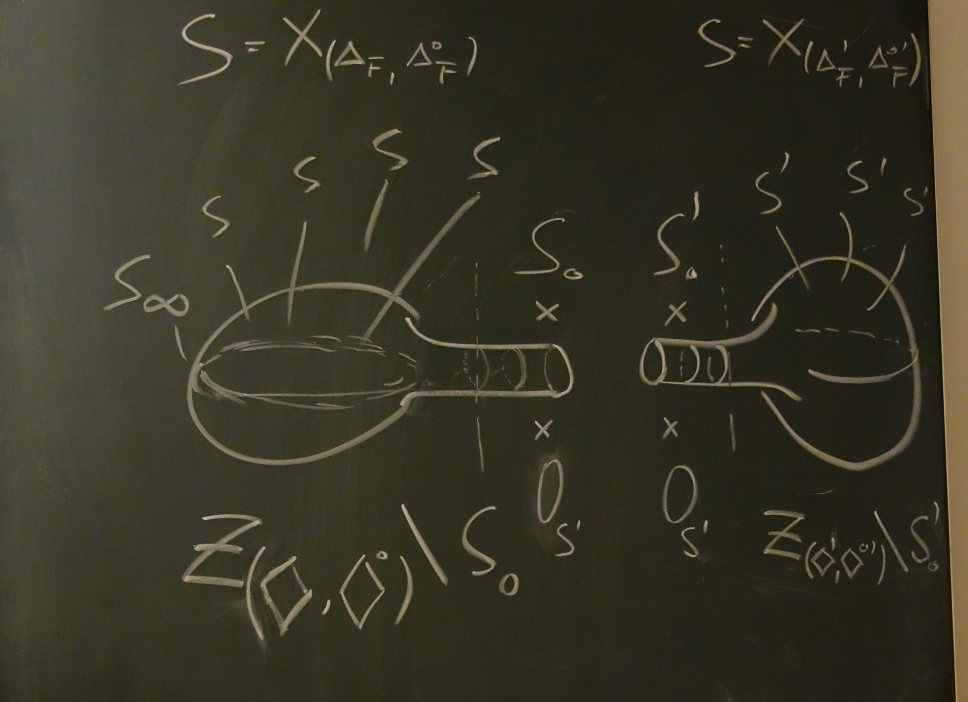
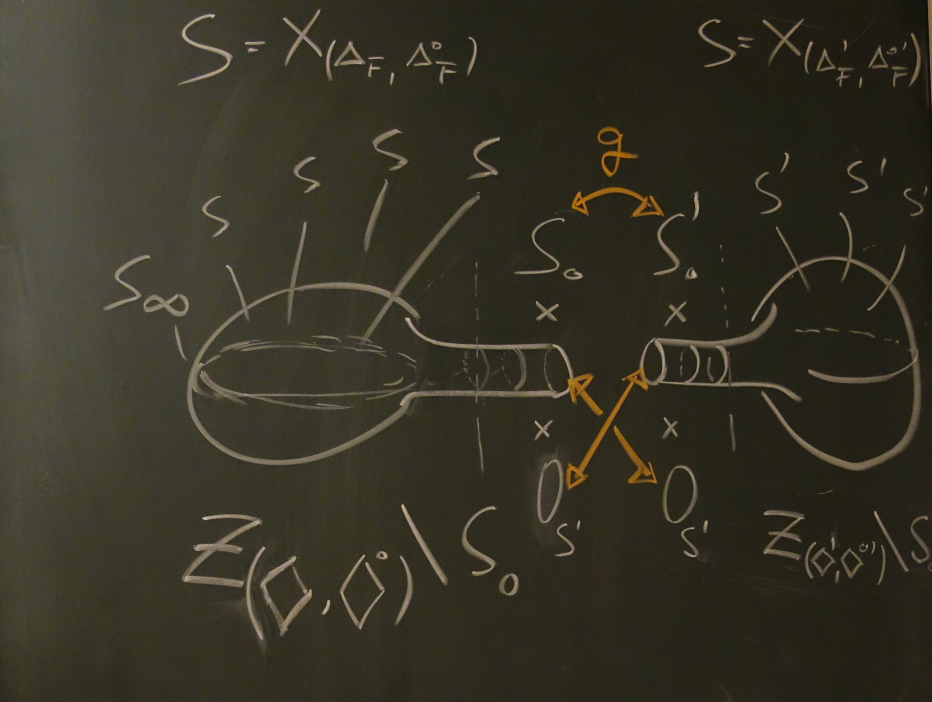

$S = X_{(\Delta_F, \Delta_F^\circ)}$
$S = X_{(\Delta'_F, \Delta_F^{\circ\prime})}$
S_∞
S
S_0
S'
S'_0
$Z_{(\Diamond, \Diamond^\circ)} \setminus S_0$
$Z_{(\Diamond', \Diamond^{\circ\prime})} \setminus S'_0$



$S = X_{(\Delta_F, \Delta_F^\circ)}$
$S = X_{(\Delta'_F, \Delta_F^{\circ\prime})}$
S
S
S
S
S_∞
S_0
S'_0
S'
S'
S'
S'
$O_{S'}$
$O_{S'}$
$Z_{(\Diamond, \Diamond^\circ)} \setminus S_0$
$Z_{(\Diamond', \Diamond^{\circ\prime})} \setminus S'_0$


S = X(Δ_F, Δ°_F)
S = X(Δ'_F, Δ°'_F)
S∞
S
S
S
S
S_0
g
S'_0
S'
S'
S'
O_s'
O_s'
Z(◇, ◇°) \ S_0
Z(◇', ◇°') \ S'_0

Interesting data:

$$\rho : H^2(Z, \mathbb{Z}) \to H^2(S, \mathbb{Z})$$

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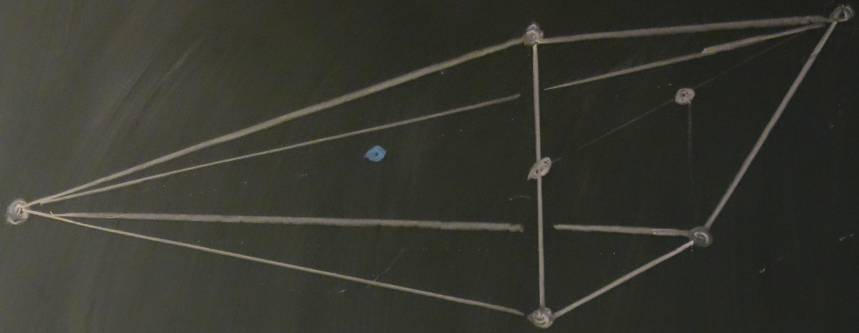
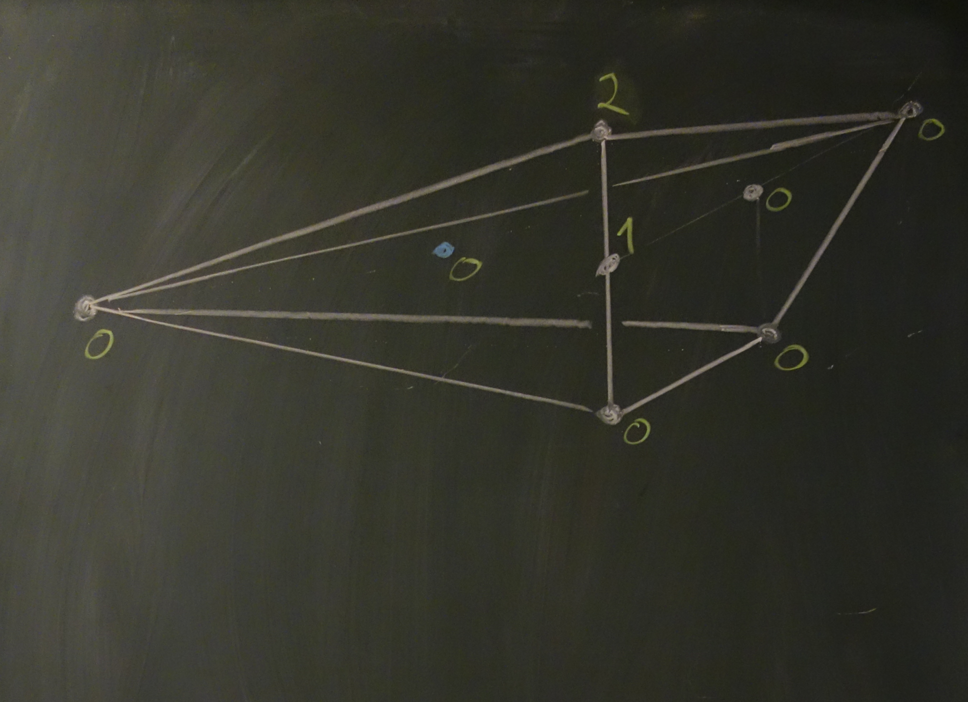
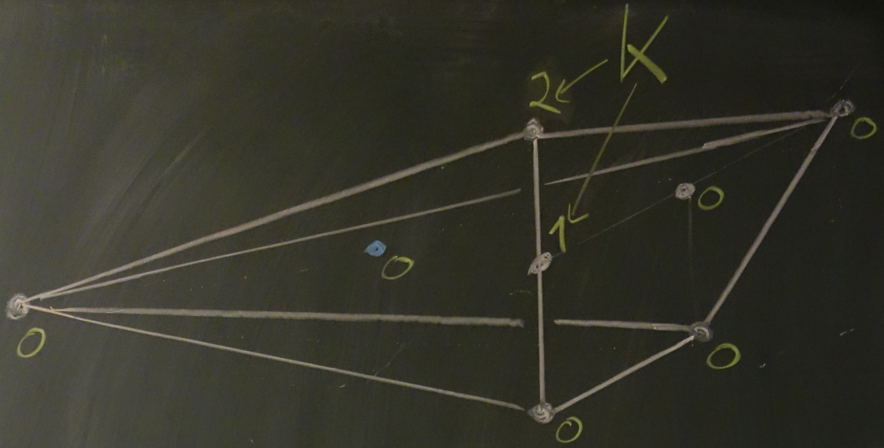
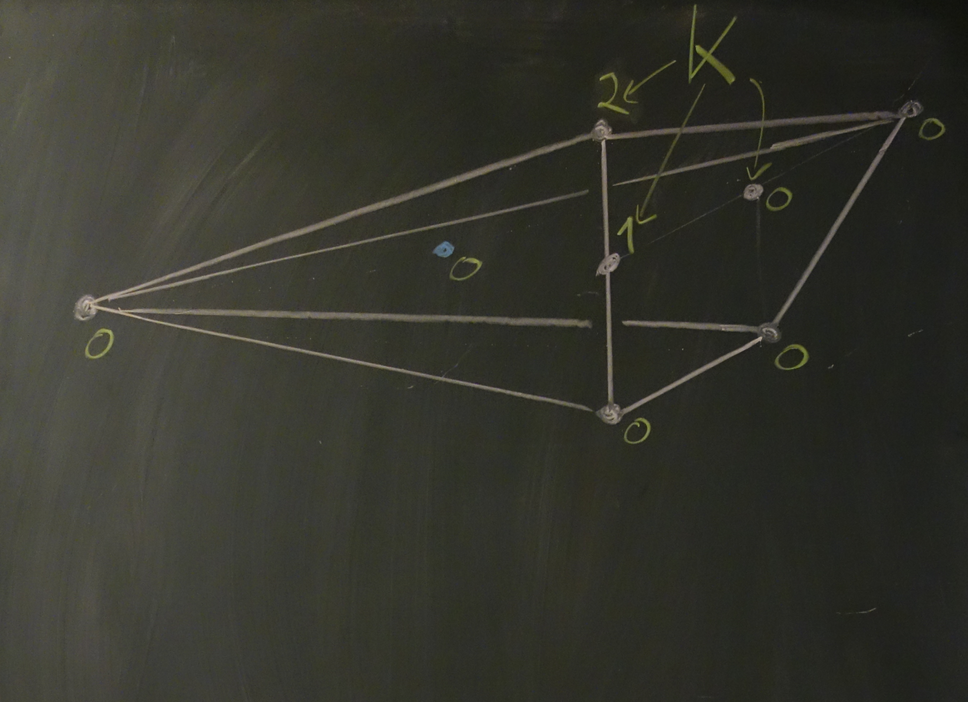
Together with the glueing map g these determine $H^*(M, \mathbb{Z})$.
In particular, $U(1)s$ come from:

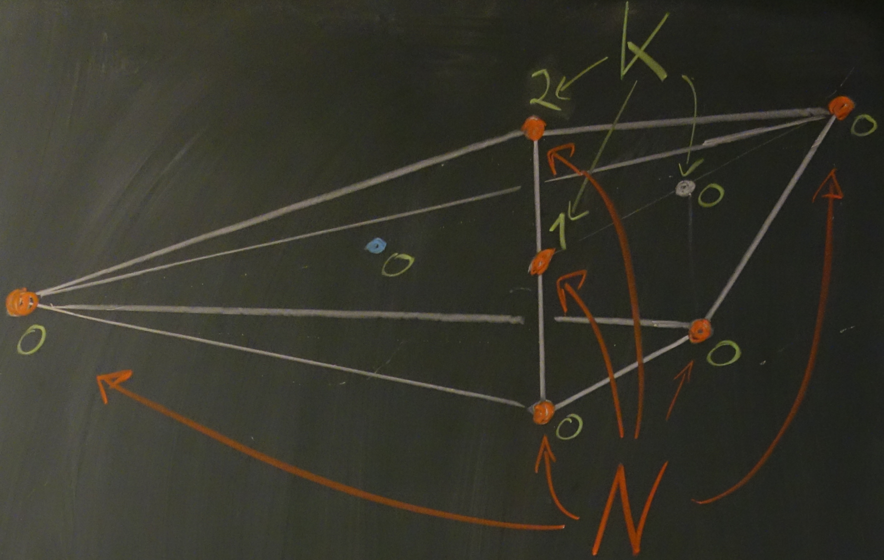
$$H^2(M, \mathbb{Z}) = N \cap N' \oplus K \oplus K'$$

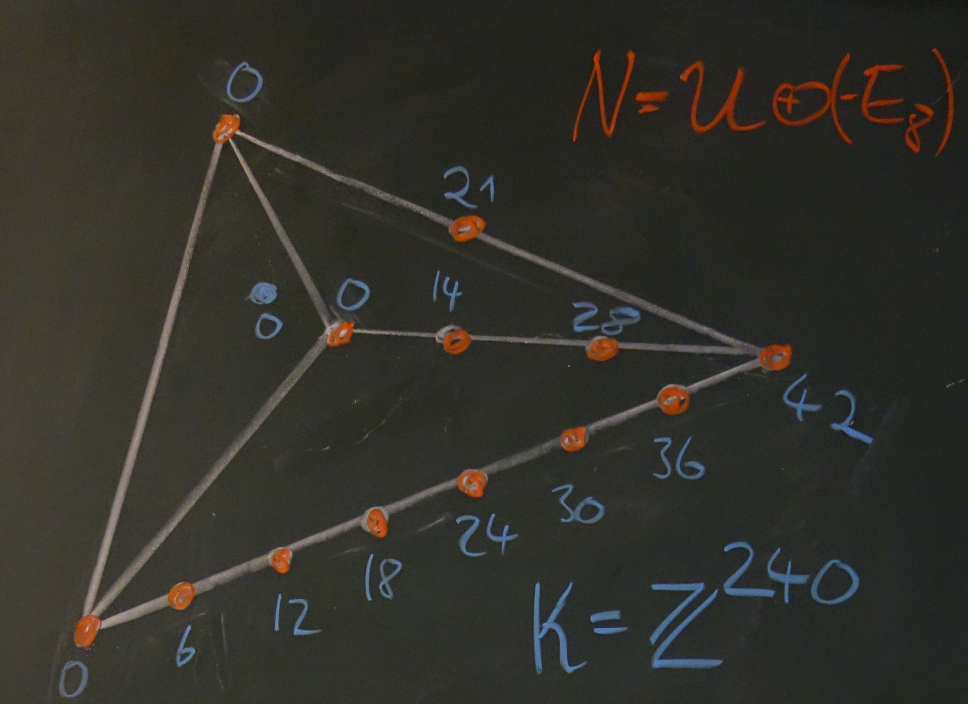



2
0
0
1
0
0
0
0


k
2
0
0
1
0
0
0
0


K
2
1
0
0
0
0
0
0


K
2
1
0
0
0
0
0
0
0
N



N = U ⊕ (-E8)
0
21
0
0
14
28
42
36
30
24
18
12
6
0
K = Z^240

Questions/Challenges/Potential Benefits:

- Singularities/Singular Transitions ?
- Identification of (co)associative submanifolds ?
  See also work by [Halverson,Morrison]
- Moduli Spaces of Z, Z' vs. that of M ?
- Classification of glueings g ?