Caltech: F-theory at 20

Abelian and Discrete Symmetries in Heterotic/F-theory Duality

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Based on:

M.C., A. Grassi, D. Klevers, M. Poretschkin and P. Song,
``Origin of Abelian Gauge Symmetries in Heterotic/F-theory Duality,''
arXiv:1511.08208 [hep-th]

& work in progress with
A. Grassi and M. Poretschkin

Outline and Summary

- F-theory with $U(1)^n$ Abelian Gauge Symmetry:

  Elliptically fibered Calabi-Yau manifolds with Mordell-Weil (MW) rnk n
  (brief status summary) → focus on rnk 1 ←→ U(1)

- Heterotic/F-theory Duality:

  Stable degeneration limit → focus on 8D/6D
  Employ toric geometry techniques

- Examples with MW rnk 1:

  Highlight three cases with split vector bundle structure groups:
  I. S(U(N-1) xU(1)) IIb. $SU(N(n-1)xZ_2)$ IIc. SU(N)x SU(M)

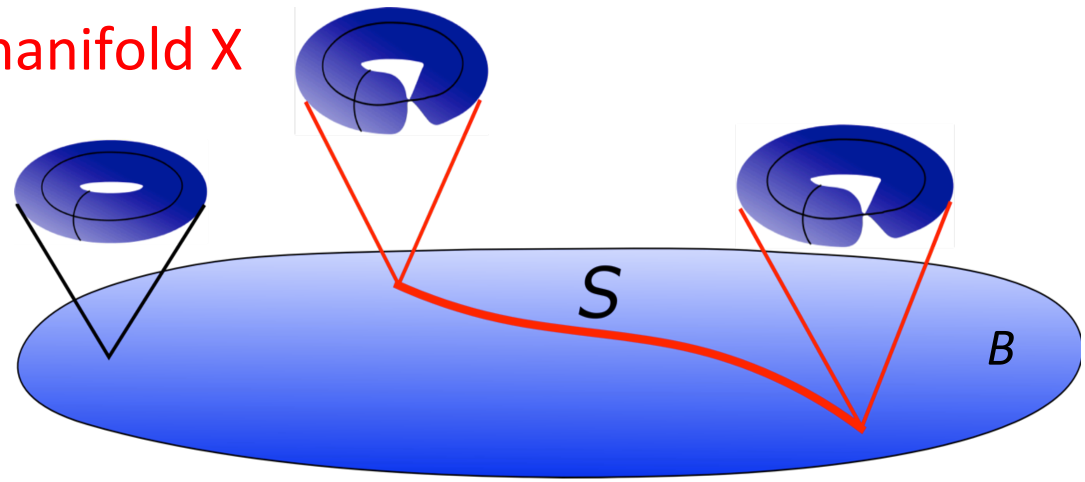
- Heterotic/F-theory Duality & Discrete Symmetry

  Example with SU(U(2)xU(1)) vector bundle on double base cover
  time permitting - work in progress

- Outlook

F-theory Compactification

Elliptically fibered Calabi-Yau manifold X



Weierstrass normal form for elliptic fibration of *X*

$$y^2 = x^3 + fxz^4 + gz^6$$

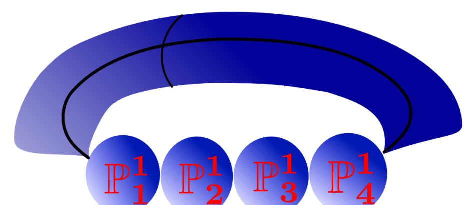
[z:x:y] of $\mathbb{P}^2(1,2,3)$

[Kodaira; Tate; Vafa; Morrison,Vafa;...]

Non-Abelian gauge symmetries

determined by severity of singularity along divisor S in B [$ord_S(f), ord_S(g), ord_S(\Delta)$]

resolved [I_n] singularity ←→[SU(n)] Dynkin diagram



Cartan gauge bosons, supported by (1,1)-forms $\omega_i \leftrightarrow \mathbb{P}^1_i$ on resolved X

Abelian Gauge Symmetries

Different: (1,1) forms ω_m, supporting U(1) gauge bosons, isolated & associated with I_1-fibers, only

[Morrison,Vafa]

(1,1) - form ω_m ⟺ rational section of elliptic fibration

Abelian Gauge Symmetry &Mordell-Weil Group

rational sections of elliptic fibr. ➡ rational points of elliptic curve

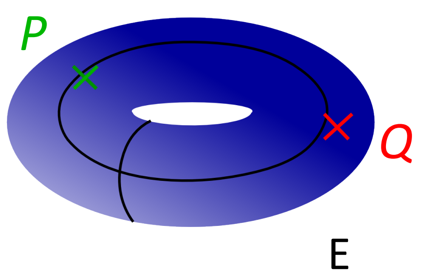
1. Rational point Q on elliptic curve E with zero point P

$[x_P:y_P:z_P]= [1:1:0]$

- is solution (x_Q, y_Q, z_Q) in field K of Weierstrass form

$$y^2 = x^3 + fxz^4 + gz^6$$

- Rational points form group (addition) on E

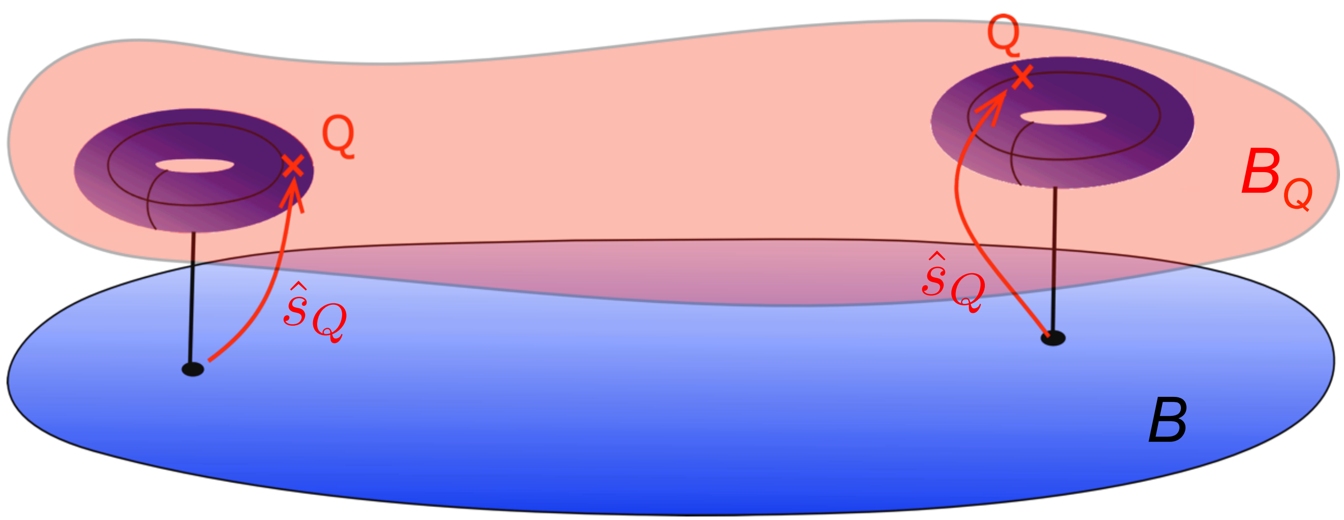


➡ Mordell-Weil group of rational points

U(1)'s-Abelian Symmetry &Mordell-Weil Group

Rational point

Q induces a rational section $\hat{s}_Q : B \to X$ of elliptic fibration



➡ $\hat{s}_Q$ gives rise to a second copy of *B* in *X*:

new divisor B_Q in *X*

➡ (1,1)-form ω_m constructed from divisor B_Q (Shioda map)

indeed (1,1) - form ω_m ⬌ rational section

Explicit constructions with zero & n-rational sections [MW rnk n - $U(1)^n$]

Employ for study of Heterotic/F-theory duality

n=0: with P - generic CY in $\mathbb{P}^2(1,2,3)$ (Tate form)

n=1: with *P, Q* - generic CY in $\mathrm{Bl}_1\mathbb{P}^2(1,1,2)$ [Morrison,Park'12]

n=2: with *P, Q, R* - specific example: generic CY in *dP_2*

[Borchmann,Mayerhofer,Palti,Weigand'13]
[M.C.,Klevers,Piragua 1303.6970,1307.6425]
[M.C.,Grassi,Klevers,Piragua 1306.0236]

generalization: non-generic cubic in $\mathbb{P}^2[u:v:w]$

non-Abelian enchancement (unHiggsing) by merging rational points

[M.C.,Klevers,Piragua,Taylor 1507.05954]
c.f., D. Klevers's talk

n=3: with *P, Q, R, S* - CICY in $\mathrm{Bl}_3\mathbb{P}^3$ [M.C.,Klevers,Piragua,Song 1310.0463]

n=4 determinantal variety in $\mathbb{P}^4$,...

higher *n*, not clear...

Heterotic/F-Theory Duality

[Morrison,Vafa '96; Friedman,Morgan,Witten '97]

Manifest in stable degeneration limit

Basic Duality (8D):

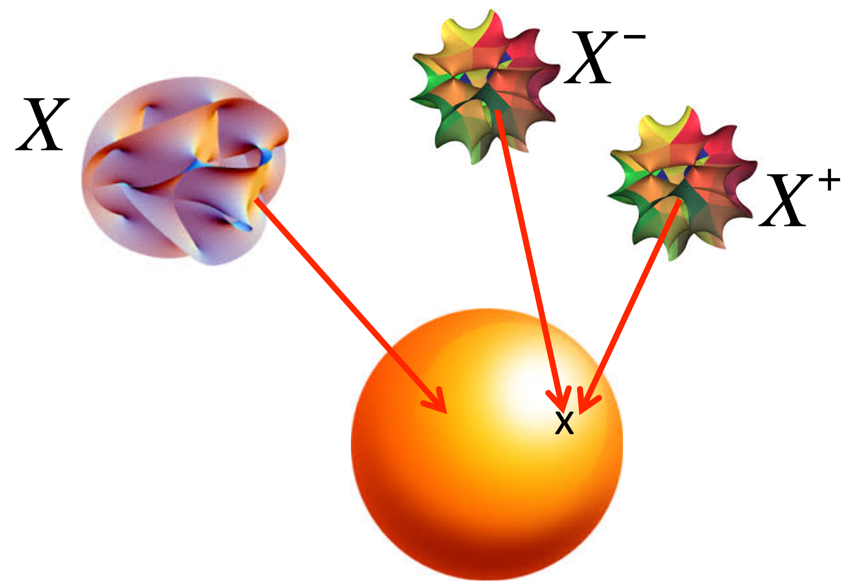
Heterotic E_8 x E_8 String on T^2

⬍ dual to

F-Theory on elliptically fibered K3 surface X

Stable degeneration limit:

K3 surface X splits into two half-K3 surfaces X^+ and X^-



K3-fibration over P^1 (moduli)

Dictionary:

- X^+ and X^- → background bundles V_1 and V_2
- Heterotic gauge group $G = G_1 \times G_2$ $G_i = [E_8, V_i]$
- The Heterotic geometry T^2 : at intersection of X^+ and X^-

Toric Construction of K3 Surfaces (with MW rnk 1)

& Stable Degeneration Limit

c.f., A. Grassi's talk

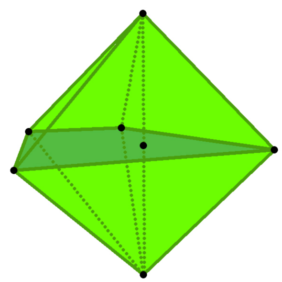
- Construct K3 surface as section of $O(-K_{P^1 \times Bl_1 P^{(1,1,2)}})$

$$\chi : s_1 x_1^4 x_4^3 x_5^2 + s_2 x_1^3 x_2 x_4^2 x_5^2 + s_3 x_1^2 x_2^2 x_4 x_5 + s_4 x_1 x_2^3 x_5^2$$
$$+ s_5 x_1^2 x_3 x_4^2 x_5 + s_6 x_1 x_2 x_3 x_4 x_5 + s_7 x_2^2 x_3 x_5 + s_8 x_3^2 x_4 = 0$$

$$s_i = s_{i1} U^2 + s_{i2} UV + s_{i3} V^2$$
$$[U : V] \in P^1$$
$$[x_1, x_2, x_3, x_4, x_5] \in Bl_1 P^{(1,1,2)}$$

Toric polytope:

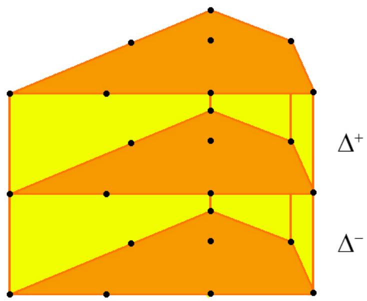
Δ^0



specifies the ambient space $P^1 x Bl_1\, P^{(1,1,2)}$

Dual polytope:

Δ



specifies the elements of $O(-K_{P^1 \times Bl_1 P^{(1,1,2)}})$

- Six-dimensional set-up: fiber this construction over another P^1

Decomposing the F-Theory Geometry

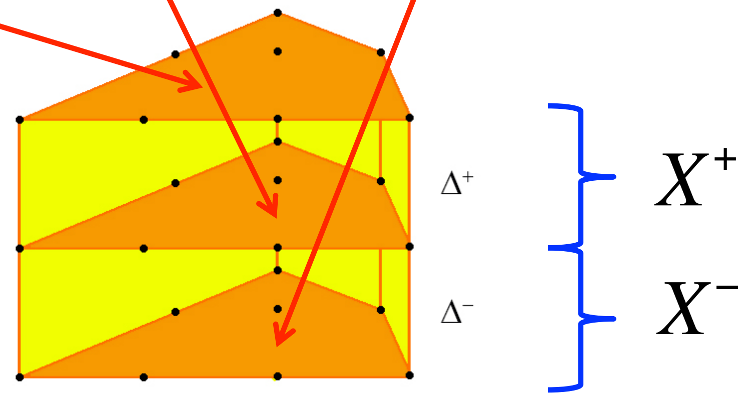
[Morrison,Vafa '96], [Berglund,Mayr '98]

$$\chi : p^{+}(s_{ij}, x_k)U^2 + p_0(s_{ij}, x_k)UV + p^{-}(s_{ij}, x_k)V^2$$

p^+ specifies spectral cover C^+

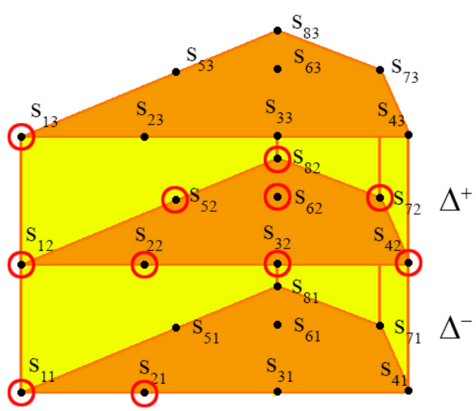
p_0 specifies elliptic curve E

p^- specifies spectral cover C^-

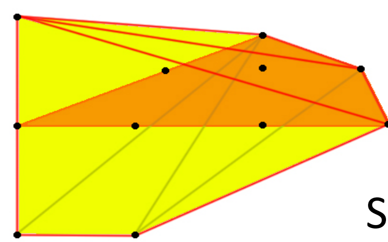


- Spectral cover defines (together with a line bundle) a SU(N) vector bundle on E

- Specialize to large gauge groups to keep spectral cover under control [N-small]



Example of specialization to E_7 x E_6 x U(1) gauge symmetry



Specialization corresponds to change of ambient space

Split Spectral Covers (6D)

[Friedman,Morgan,Witten '97],[Donagi'97],[Berglund,Mayr'98]

The isomorphism class of an SU(N) vector bundle on an elliptically fibered K3 surface

$$y^2 = x^3 + fx + g \qquad f \in O_{P^1}(8), g \in O_{P^1}(12)$$

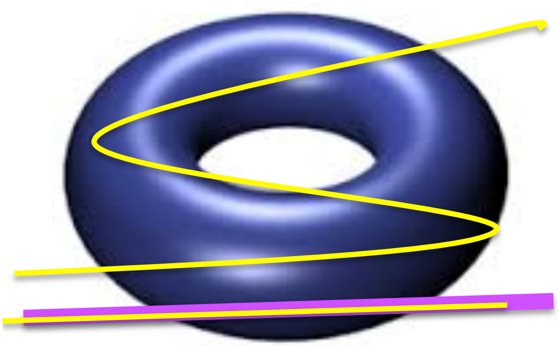
Spectral cover C:

$$w = c_0 + c_1 x + c_2 y + c_3 x^2 + \ldots + \begin{cases} c_N x^{\frac{N}{2}}; N - even \\ c_N x^{\frac{N-3}{2}} y; N - odd \end{cases}$$

Arbitrary line bundle on P^1

↓

$c_i \in M \otimes O_{P^1}(2i)$



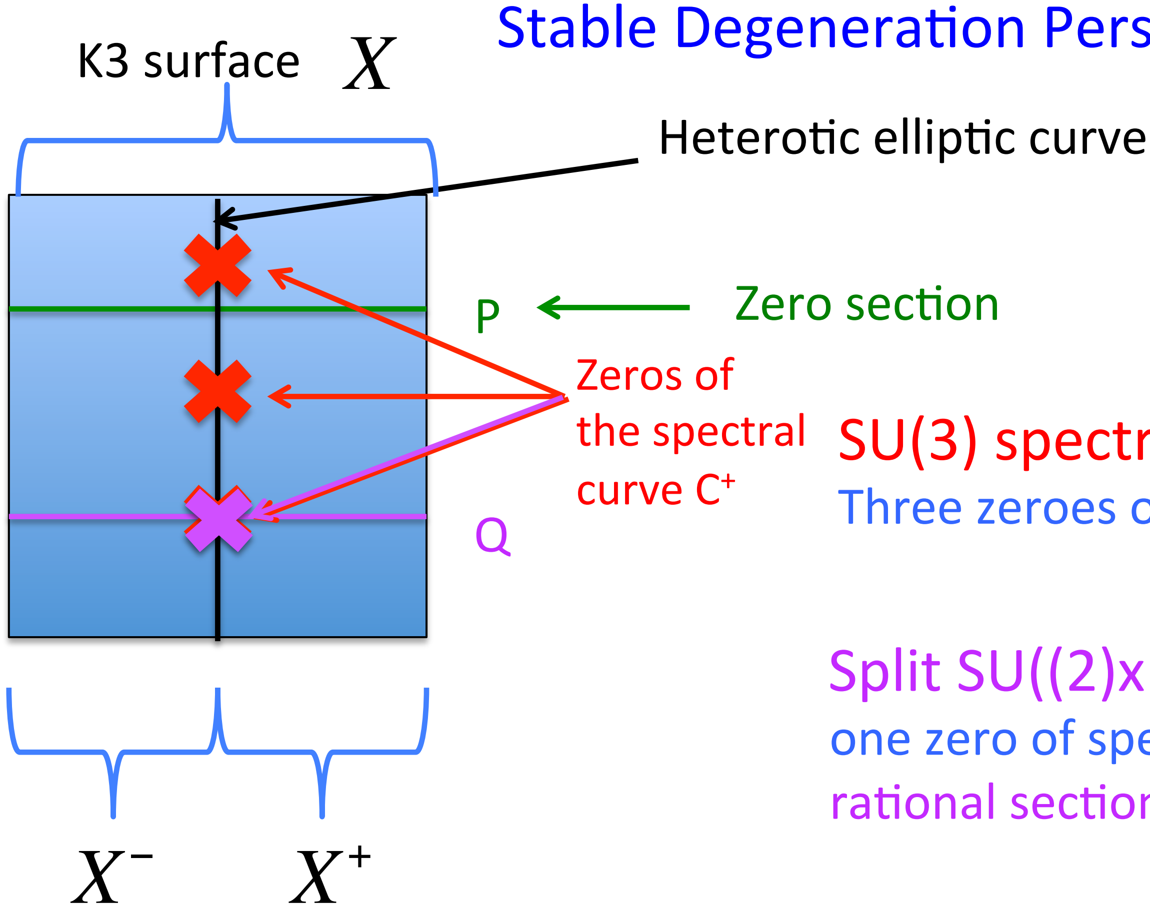
Q

- C defines an N-section of the elliptically fibered Heterotic K3 surface [SU(N)-structure group]
- If one of the leaves is globally well-defined section Q, the structure group splits to S(U(N-1)xU(1))

[Marsano,Saulina,Schäfer-Nameki '09],[Blumenhagen,Grimm,Jurke,Weigand '10], [Choi, Hayashi '12]

Example: [Split] Spectral Cover SU(3)[SU((2)xU(1))]

Stable Degeneration Perspective (8D)



SU(3) spectral cover:

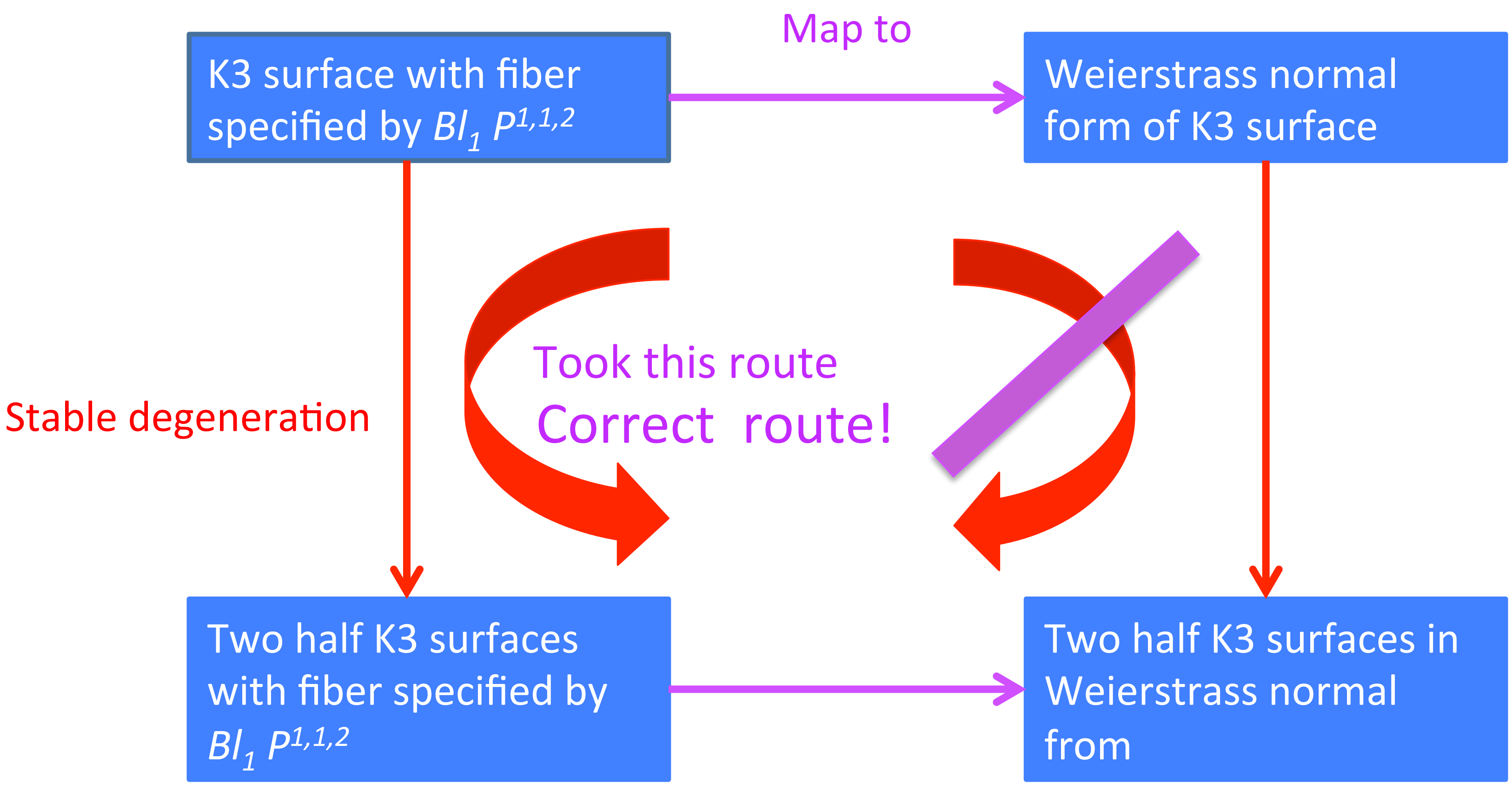
Three zeroes of spectral cover C^+

Split SU((2)xU(1)) spectral cover:

one zero of spectral cover becomes rational section Q

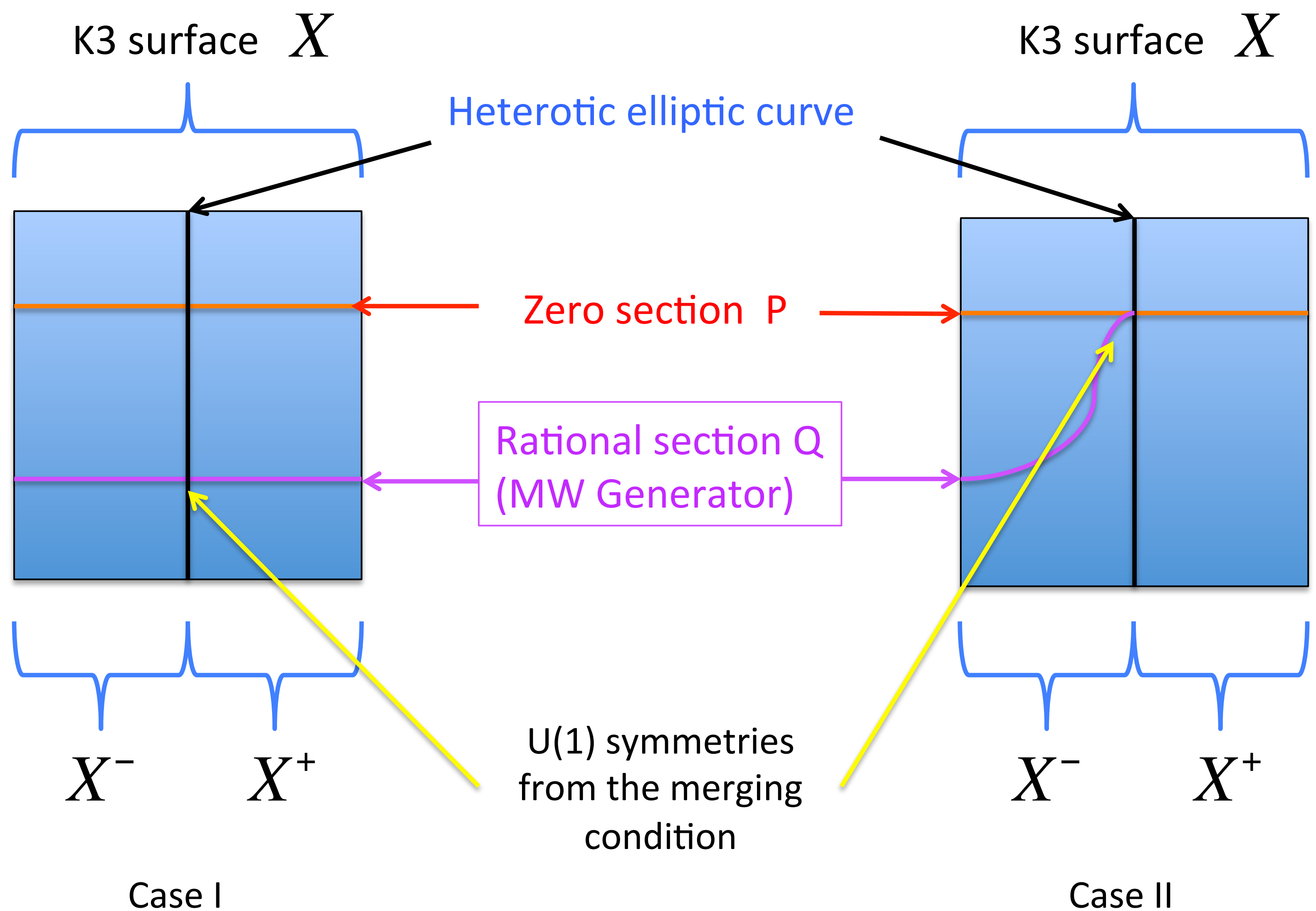
c.f.,: [Witten '85], [Blumenhagen, Honecker, Weigand '05] [Aspinwall '05]

Weierstrass form and stable degeneration (with MW)



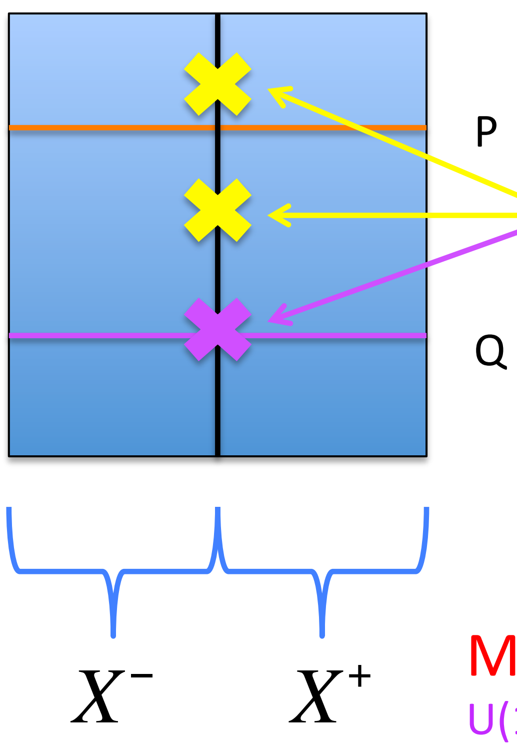
do not commute!

Tracing the U(1)'s through the Duality



Case I - Split Spectral Cover [SU(N-1)xU(1)]

Example: $E_7 x E_6 x U(1)$ gauge symmetry



Findings:

- X^- - U(1) background bundle with $E_7 x U(1)_{massive}$ gauge symmetry
- X^+ - S(U(2)xU(1)) background bundle with $E_6 x\ U(1)_{massive}$ gauge symmetry
- One linear combination of two $U(1)_{massive}$'s is a *massless* U(1)

c.f., [Aspinwall'05]

Massive U(1) gauge symmetry:

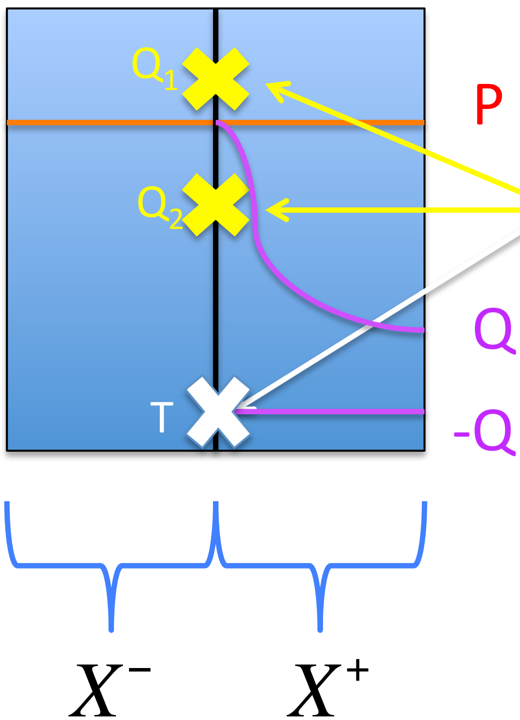
U(1) factors within the background bundle generate a mass term for the physical U(1): stemming from the Chern-Simons terms in the generalized 10D field strength of the Kalb-Ramond field

$$H = dB_2 - \frac{\alpha}{4}\left(\omega_{3Y}(A) - \omega_{3L}(\Omega)\right)$$

Related work: [Anderson,Gray,Raghuram,Taylor 1512.05791]

Case IIa - Spectral Cover with ``Torsion''

Example: $E_8 x E_6 x U(1)$ gauge symmetry



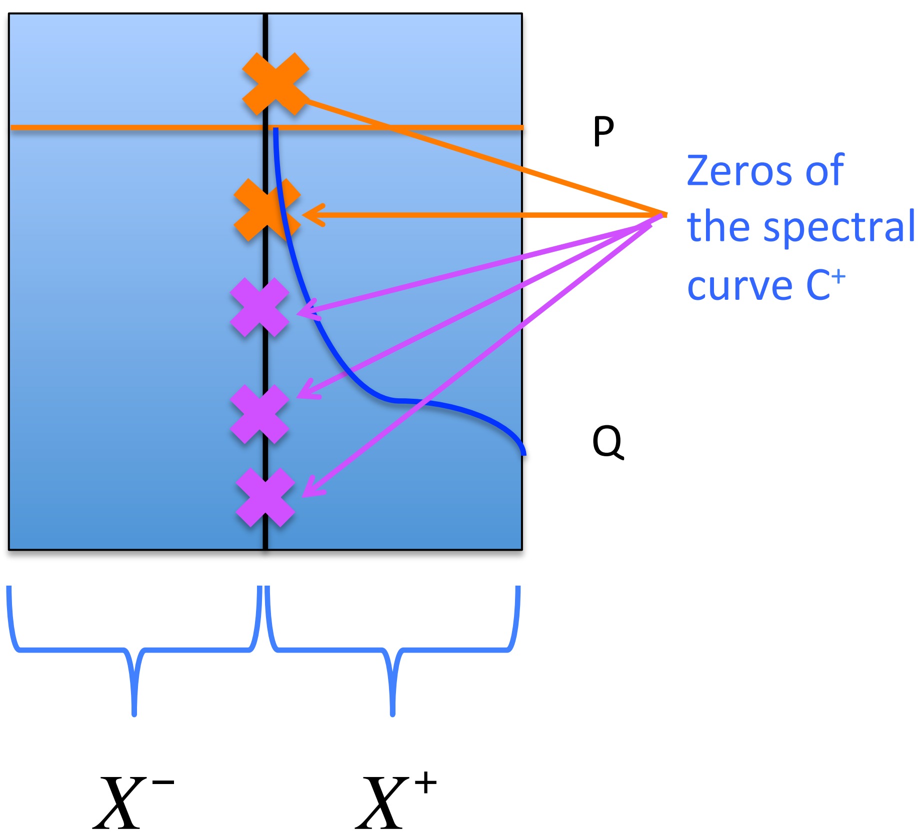
Zeros of the spectral curve C^+

Findings:

- X^- - a trivial background bundle; E_8 gauge symmetry
- X^+ - $S(U(2)xZ_2)$ spectral cover, i.e. two irrational points Q_1,Q_2 with with T being a torsional point of order 2
- No U(1) background bundle → U(1) gauge symmetry massless

Case IIIb -non-Abelian Split Spectral Cover SU(N)xSU(M)

Example: E_8xSO(7)xU(1) gauge symmetry



Findings:

- X- a trivial background bundle
  E_8 gauge symmetry
- X^+ - SU(2) x SU(3) spectral cover
- Embedding of SU(2) x SU(3) into E_8 with SO(7) non-Abelian gauge symmetry necessarily produces a U(1) gauge symmetry
- No background U(1) bundle - U(1) gauge symmetry massless

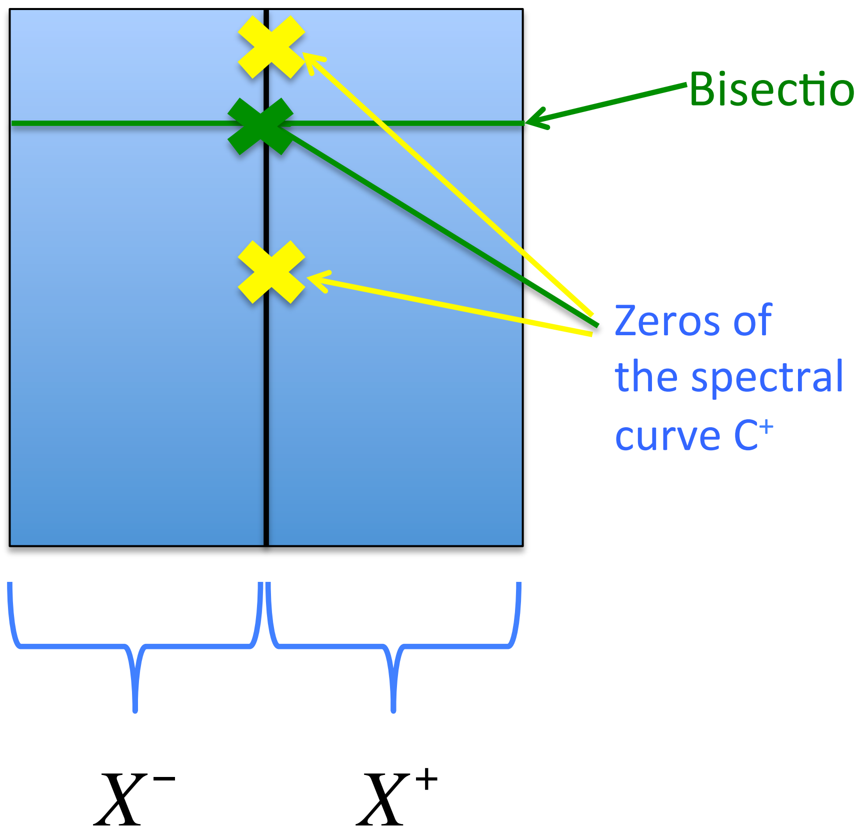
Further highlights: Peng Song's talk on Friday

Discrete Symmetries and Stable Degeneration

work in progress, with A. Grassi and M. Poretschkin

Developed toric stable degeneration limit also applies to different fiber types with bi-section (examples with the ambient space $P^{(1,1,2)}$)

Example: $E_7 \times E_6 \times Z_2$ gauge symmetry



Findings:

- X^- - a trivial background bundle E_7 gauge symmetry and a bi-section
- X^+ - SU(3) spectral cover which signals the bisection
- Double cover of the base: Spectral cover becomes split SU(3) -> S(U(2) x U(1))

Further investigation

Outlook-Further Investigations

- Investigate examples with several U(1)s; matter spectra;4D
- Further studies of discrete symmetries
- Study of geometrical transitions c.f., L. Anderson's talk
- Beyond SU(N) bundles: of BCDE type with split structure group

Building on the work of [Morrison,Vafa], [Berglund,Mayr],.