Gauge theories on circles and the decoding of F-theory geometries

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Mainly based on works with Andreas Kapfer and Denis Klevers
arXiv: 1502.05398 (AK) 1510.04281 (AK & DK)

Using results with Federico Bonetti, Stefan Hohenegger, Andreas Kapfer, Jan Keitel
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Introduction / Motivation

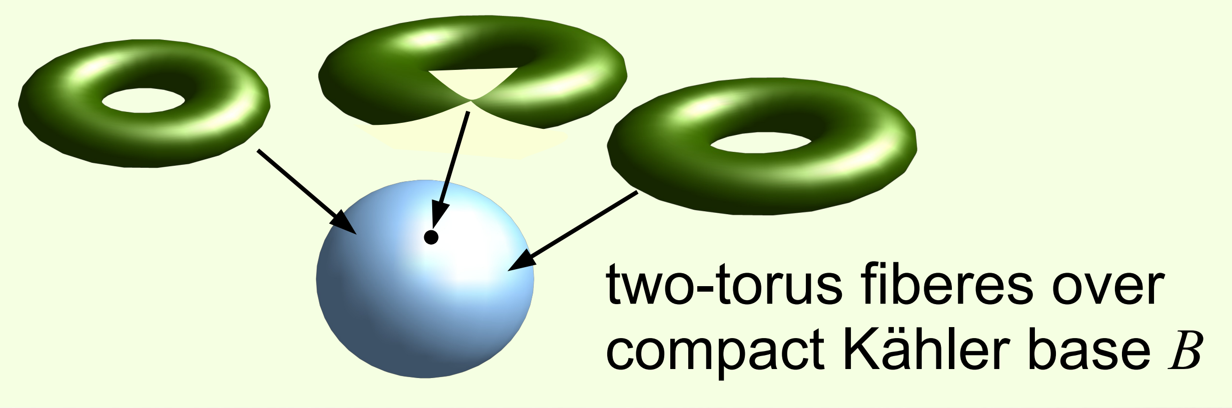
Effective actions of F-theory

- F-theory compactifications arguably provide the richest class of string theory effective actions
  - original excitement about GUTs in F-theory - not realized in pert. strings
    → e.g. talk of Weigand
  - numerous examples with exotic matter spectra → e.g. talks of Anderson, Klevers
  - dualities to heterotic and Type I compactifications → e.g. talks of Mayrhofer, Lüst
  - new 'exotic' theories, for example, 4D N=3 theories of [García-Etxebarria,Regalado]
    → e.g. talk Iñaki

- <u>Two questions arise:</u>
  - How can we infer reliably information about the 2D/4D/6D effective actions of F-theory?
  - Is there a classification in sight? What do F-theory geometries classify?

Some background material

- Well-known slogan:
  F-theory compactifications on elliptically-fibered Calabi-Yau threefolds yield 6D theories with minimal N=(1,0) supersymmetry



  - pinching of two-torus indicates location of seven-branes
  - brane and bulk physics encoded by singular complex geometry

- Six-dimensional theories are perfect to answer the above questions
  - have a rich structure - there are many topologically distinct CY threefolds
  - are strongly constraint by anomalies - fermions in 'all' N=1 multiplets can contribute to anomalies

Goals of this talk

- (1) Argue that the complete information in the F-theory geometries actually describe gauge/sugra theories on a circle.
    - This is not unexpected since the M-theory to F-theory approach has been suggested already in [Vafa '96]. However, its importance and power might have been underappreciated. Currently this limit is the only reliable way to infer information about F-theory effective actions.
- (2) Show that F-theory geometries can teach us valuable lessons about circle-reduced theories.
    - Example: How are anomalies of the higher-dimensional theories visible in the lower-dimensional effective theory?
- (3) Comment on possibility of classifying Calabi-Yau threefolds (with elliptic fibration) by using the insights from gauge theory.

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6D gauge theories / Sugra theories on circle $\Longleftrightarrow$
Geometry of resolved elliptically fibered Calabi-Yau threefolds

Our systematics is somewhat complementary to the approach and classifications of: [Morrison,Taylor etal.] [Heckman,Morrison,Rudelius,Vafa]

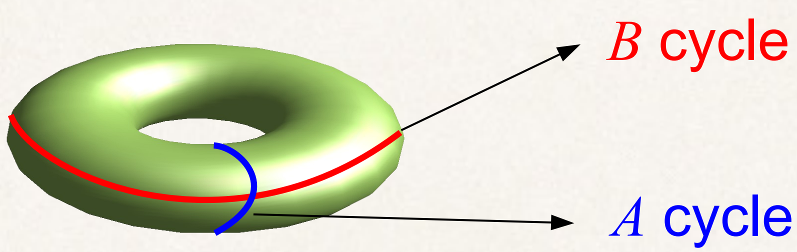
Some comments on the M-theory to F-theory duality

Approaching the problem via M-theory

- Viewing F-theory as an actually 12 dim. theory is problematic (e.g. torus volume unphysical, meaning of non-perturbative states…)

- F-theory effective actions via M-theory (as of now: definition of F-theory)
  Consider M-theory on space $T^2 \times M_9$ [Vafa]

  τ is the complex structure modulus of the T^2, v volume of T^2



<u>F-theory limit:</u>

(1) A-cycle: if small than M-theory becomes Type IIA
(2) B-cycle: T-duality $\Rightarrow$ Type IIA becomes Type IIB, τ is indeed dilaton-axion
(3) grow extra dimension: send $v \rightarrow 0$ than T-dual B-cycle becomes large

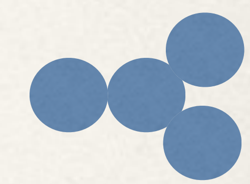
Consequences of M-theory to F-theory limit

- First step: approach M-theory via 11D supergravity on a smooth geometry
  - resolution of singular Calabi-Yau geometry
    ⇒ classification of resolutions at each co-dimensions in base

    [almost everyone who has worked on F-theory,…]

  - co-dimension 1 in base
    → non-Abelian gauge group
    → simple roots

  - co-dimension 2 in base
    → matter in representation ***R***
    → weights w of ***R***



  - Abelian gauge group factors: $n_{U(1)} + 1$ rational sections of the fibration

    zero section (assumed to exist throughout this talk)

- Second step: M-theory to F-theory limit shrinks fiber torus and resolutions and grows extra dimension (5D → 6D) - keep track of M2-brane states
  - e.g. M2-branes on two-torus fiber correspond to circle Kaluza-Klein states

Massive states in five dimensions

- 11D Sugra on smooth geometry
  $\Rightarrow$ 5D effective theory

  ⟷

  6D supergravity theory
  (gauge theory) on a circle
  $\Rightarrow$ push to Coulomb branch
  $\Rightarrow$ integrate out all massive modes

  Both types of theories need to be computed and then compared

- 5D gauge group in Coulomb branch: $U(1)^{\text{rank}G} \times U(1)^{n_{U(1)}}$

- mass of 5D state at Kaluza-Klein level n descending from 6D state in representation ***R*** of G with weights w and U(1)-charges q_m

$$m = m_{\text{CB}} + m_{\text{KK}} = w_I \zeta^I + q_m \zeta^m + \frac{n}{r}$$

Coulomb branch vevs (blow-up vevs) — circle radius (torus fiber volume)

Anomaly cancellation and circle compactifications

M-theory on Calabi-Yau threefolds

- effective action has been studied long ago: 5D N=2 theory [Ferrara,…] [Minasian,…]
  - important to us are the Chern-Simons terms:

$$-\frac{1}{48\pi^2}\mathcal{K}_{ABC}\int_{\mathcal{M}_5} A^A\wedge F^B\wedge F^C-\frac{1}{384\pi^2}c_B\int_{\mathcal{M}_5} A^B\wedge \mathrm{Tr}(R\wedge R)$$

triple intersection numbers:

$$\mathcal{K}_{ABC}=\int_{Y_3}\omega_A\wedge\omega_B\wedge\omega_C$$

second Chern class: $A=1,\ldots,h^{1,1}(Y_3)$

$$c_A=\int_{Y_3}\omega_A\wedge c_2(Y_3)$$

  - arise from 11D sugra action including terms up to 8 derivatives

- comparison to Chern-Simons terms obtained after circle reduction
  → classical and one-loop corrections required

early works: [Morrison,Seiberg][Witten][Intriligator, Morrison,Seiberg]

Chern-Simons terms on circle side

- Classical and one-loop Chern-Simons terms
  - classical terms depend on $\Omega_{\alpha\beta},\ b^\alpha,\ a^\alpha$ (6D tensor coupling, anomaly coefficients)

    $\rightarrow$ $\Omega_{\alpha\beta},\ b^\alpha,\ a^\alpha$ fixed by geometry (intersection numbers base, location of branes, canonical class of base)

    $\rightarrow$ there are many more Chern-Simons terms in M-theory [Bonetti,TG '11]

  - one-loop CS-terms in the effective theory induced by integrating out:

$$\mathcal{K}_{ABC} = \sum_{\text{mass. states}} k_{\mathbf{r}} \cdot q_A\, q_B\, q_C\, \text{sign}(m) \qquad c_A = \sum_{\text{mass. states}} \kappa_{\mathbf{r}} \cdot q_A\, \text{sign}(m)$$

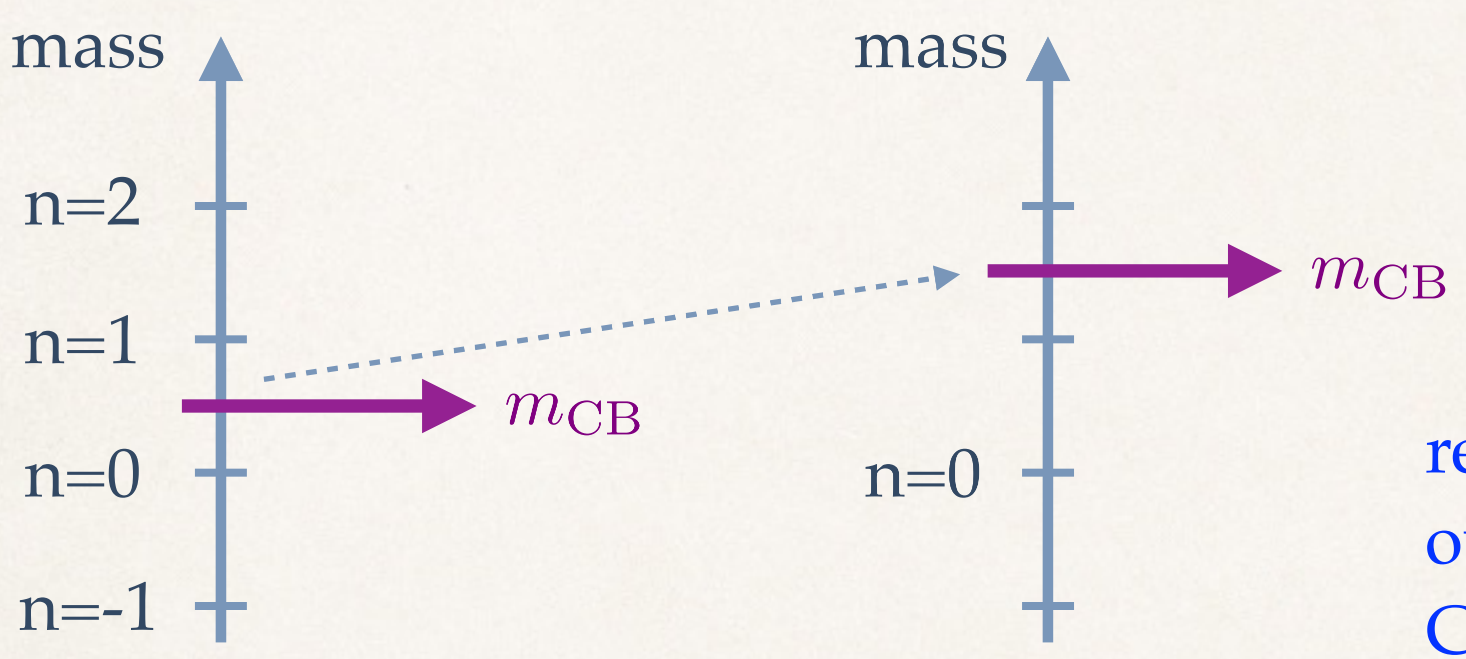
- massive spin 1/2 fermions
- massive spin 3/2 fermions
- massive 'self-dual' tensors

[Bonetti,TG,Hohenegger '13]

$\Rightarrow$ massive Kaluza-Klein states of all 6D fields that carry chirality and contribute to the 6D anomaly

Jumping Chern-Simons terms

- match was still not possible for certain geometries



[TG,Kapfer,Keitel '13]

regularization of infinite sum over KK modes in one-loop CS terms gets modified

- While one-loop CS terms are independent of the precise numerical value of the mass of a state, they do depend on

(1) sign of CB mass sign$(m_{\rm CB})$
(2) hierarchy of $m^n_{\rm KK} = n/r$ and $m_{\rm CB}$

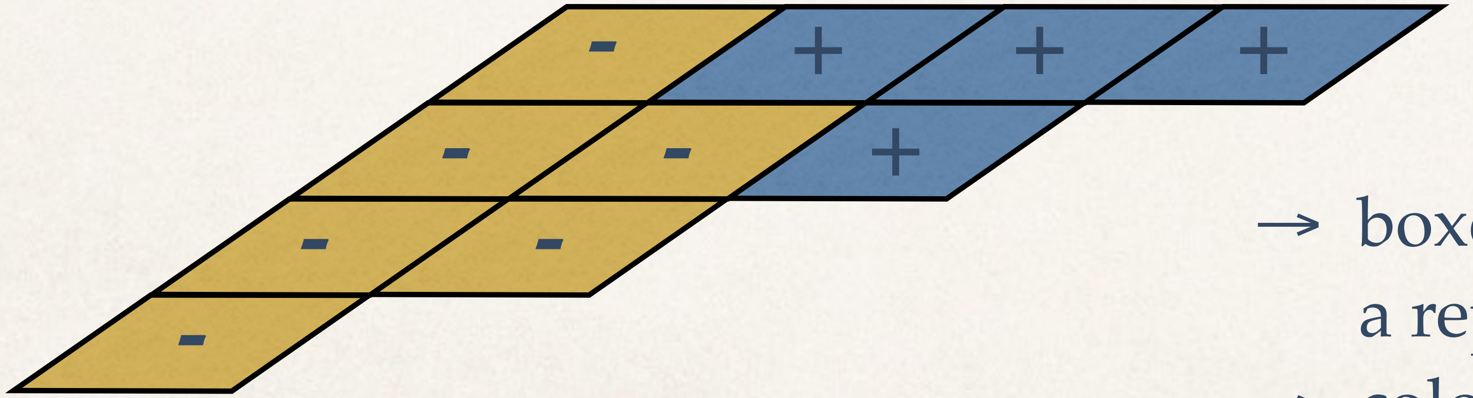
→ associate an integer label $\ell_{w,q}$ to each massive state:
$m_{\rm CB}$ is between mass of $\ell_{w,q}$ and $\ell_{w,q}+1$ Kaluza-Klein state

Extending box graphs

- Box graphs have been introduced to systematically classify the sign-information and realized Coulomb branch phases

[Hayashi, Lawrie, Morrison, Schäfer-Nameki][Braun,Schäfer-Nameki]…

- Example: anti-symmetric representation **10** of SU(5)



→ boxes are for the weights w of a representation
→ colors indicate sign-information
→ clear rules and allowed connections studied

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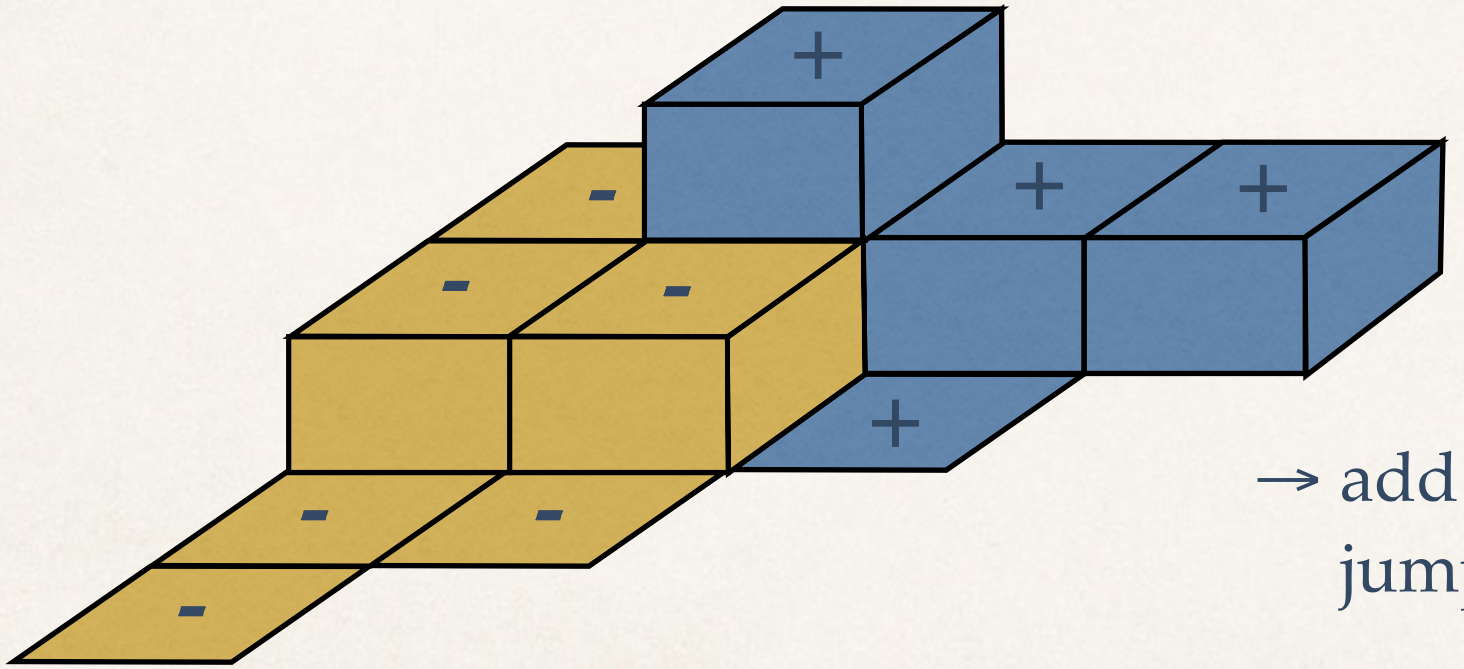

→ add information about ℓ_w jump levels

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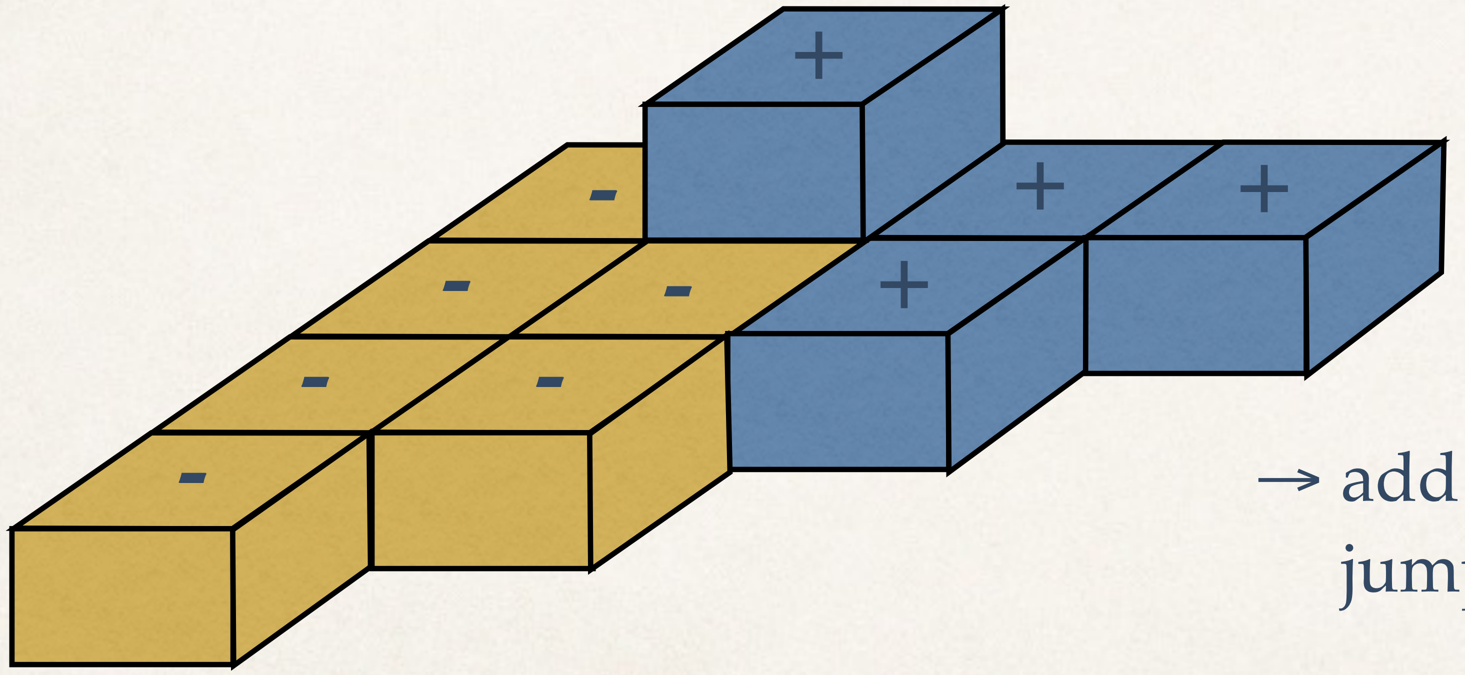


→ add information about ℓ_w jump levels

What are the rules for such diagrams? What are the inherent symmetries?

Large gauge transformations

- large gauge transformations precisely induce certain types of jumps
  - 6D gauge transformation: $\hat{\Lambda}^{\mathcal{I}}(x,y) = \begin{cases} -k^I y \\ 0 \end{cases}$ $\qquad \hat{\Lambda}^m(x,y) = -k^m y$

    circle coordinate
  - shift of Coulomb branch vevs: $\tilde{\zeta}^I = \zeta^I + \frac{k^I}{r}\,, \qquad \tilde{\zeta}^m = \zeta^m + \frac{k^m}{r}\,,$

    mix of 5D vectors:
    $$\tilde{A}^I = A^I - k^I A^0$$
    $$\tilde{A}^m = A^m - k^m A^0$$
    mixing-in the Kaluza-Klein vector A^0 rearrangement of whole Kaluza-Klein tower

- large gauge transformations act on non-trivially on $\ell_{w,q}$ $\Rightarrow$ Symmetry?
- Yes! But only if one takes into account 6D anomalies!

  One-loop corrected effective 5D theories differing by the above transformation are identified if and only if 6D anomalies are cancelled.

Discovering the 6D anomaly conditions

- Recall the form of the 6D anomaly cancellation conditions: e.g.

pure Abelian: $$-(b^\alpha_{mn} b^\beta_{pq} + b^\alpha_{mp} b^\beta_{nq} + b^\alpha_{mq} b^\beta_{np})\Omega_{\alpha\beta} = \sum_q F_{1/2}(q)\; q_m q_n q_p q_q$$

pure non-Abelian: $$-3\frac{b^\alpha}{\lambda(G)}\frac{b^\beta}{\lambda(G)}\Omega_{\alpha\beta} = \sum_{\boldsymbol{R}} F_{1/2}(\boldsymbol{R})\; C_{\boldsymbol{R}}$$

$$\mathrm{tr}_R \hat{F}^4 = B_R\, \mathrm{tr}_f \hat{F}^4 + C_R\, (\mathrm{tr}_f \hat{F}^2)^2$$

- differ from one-loop Chern-Simons terms: e.g.

pure Abelian: $$\mathcal{K}_{mnp} = \frac{1}{2}\sum_q F_{1/2}(q)\; q_m q_n q_p\, (2\ell_q + 1)\mathrm{sign}(m^q_{\mathrm{CB}})$$

pure non-Abelian: $$\mathcal{K}_{IJK} = \frac{1}{2}\sum_{\boldsymbol{R}} F_{1/2}(\boldsymbol{R}) \sum_{w\in \boldsymbol{R}} w_I w_J w_K\, (2\ell_w + 1)\mathrm{sign}(m^w_{\mathrm{CB}})$$

Discovering the 6D anomaly conditions

- We were able to show:

6D anomalies:

$$\sum_q F_{1/2}(q)\; q_m q_n q_p q_q$$

$$\sum_{\boldsymbol{R}} F_{1/2}(\boldsymbol{R})\; C_{\boldsymbol{R}}$$

Properties of 1-loop CS-terms:

(1) additional $q_r\,, w_I$ from large gauge transformation acting on

$$\delta_r\,(2\ell_q+1)\mathrm{sign}(m^q_{\rm CB}) \longrightarrow q_r$$

$$\delta_L\,(2\ell_w+1)\mathrm{sign}(m^w_{\rm CB}) \longrightarrow w_L$$

(2) identities involving four weights to obtain $C_{\boldsymbol{R}}$

[TG,Kapfer '15]

Change in classical and 1-loop Chern-Simons terms cancel if and only if 6D anomalies are cancelled

Arithmetic structures and anomaly cancellation

- A proof anomaly cancellation for F-theory geometries requires to show that the large gauge transformations are actually a geometric symmetry.

  - for models with only Abelian gauge symmetries this is possible [TG,Kapfer '15]

    large gauge transformations ≅ Mordell-Weil group of rational sections

    $$MW(Y_3) \cong \mathbb{Z}^{n_{U(1)}} \oplus \mathbb{Z}_{k_1} \oplus \ldots \oplus \mathbb{Z}_{k_{n_{\mathrm{tor}}}}$$

    key: in the M-theory to F-theory limit we are free to pick the 'zero-section' that specifies the Kaluza-Klein vector (any choice works fine!)

  - define new arithmetic structures for non-Abelian large gauge transformations
    define arithmetic structure on geometries with multi-sections
    → mathematical meanings?

    [TG,Kapfer,Klevers '15]

  - Higgsing might connect non-Abelian models to Abelian models
    → arithmetic structures need to be compatible

Comments on the classification of threefolds

Characteristic topological data

- Wall's classification theorem:
  - The homotopy types of Calabi-Yau threefolds are classified by the following numerical characteristics:

    Hodge numbers: $h^{1,1}(Y_3),\ h^{2,1}(Y_3)$

    Triple intersection numbers: $\mathcal{K}_{ABC}$

    Second Chern class: c_A

  - Spaces differing in these numerical data cannot be continuously deformed into each other without encountering singularities.
- Are these data systematically specified by the F-theory effective actions on an additional circle?

Information from gauge theories on circles

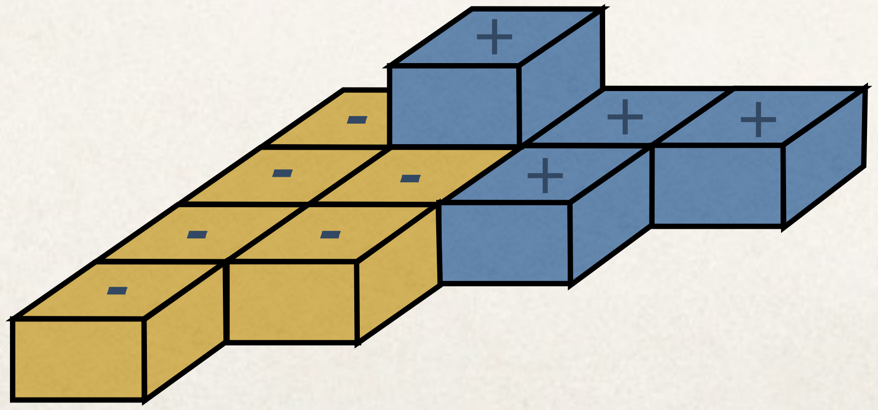
- (1) Hodge numbers are determined by spectrum (in the following: only non-Abelian)

$$h^{1,1}(Y_3) = T + 2 + \mathrm{rank}(G)$$
$$h^{2,1}(Y_3) = H_{neut} - 1$$

- (2) Intersection numbers and second Chern class determined by:
  - <u>classical data:</u> $\Omega_{\alpha\beta},\ b^\alpha,\ a^\alpha$

$$\mathcal{K}_{0\alpha\beta} = \Omega_{\alpha\beta} \qquad \mathcal{K}_{IJ\alpha} = -\mathcal{C}_{IJ}\Omega_{\alpha\beta}b^\beta \qquad c_\alpha = -12\Omega_{\alpha\beta}a^\beta$$

  - <u>one-loop data:</u> spectrum plus circle info (sign table, jump levels)



→ extended box graphs as convenient object for classification?

Intersection and Chern class data

- Intersection numbers and Chern classes (non-Abelian groups only)

$$\mathcal{K}_{000} = \frac{1}{120}\Big(2(T_{sd} - T_{asd}) - F_{1/2} - 5F_{3/2}\Big) + \frac{1}{4}\sum_R F_{1/2}(R) \sum_{w\in R} l_w^2\ (l_w+1)^2\,,$$

$$\mathcal{K}_{00I} = \frac{1}{6}\sum_R F_{1/2}(R) \sum_{w\in R} l_w\ (l_w+1)\ (2l_w+1)\ w_I\ \ \mathrm{sign}(m^w_{\mathrm{CB}})\,,$$

$$\mathcal{K}_{0IJ} = \frac{1}{12}\sum_R F_{1/2}(R) \sum_{w\in R} (1+6\ l_w\ (l_w+1))\ w_I w_J\,,$$

$$\mathcal{K}_{IJK} = \frac{1}{2}\sum_R F_{1/2}(R) \sum_{w\in R} (2l_w+1)\ w_I w_J w_K\ \ \mathrm{sign}(m^w_{\mathrm{CB}})\,,$$

$$c_0 = \frac{1}{6}\Big(19F_{3/2} - F_{1/2} - 4(T_{sd} - T_{asd})\Big) - \sum_R F_{1/2}(R) \sum_{w\in R} l_w\ (l_w+1)\,,$$

$$c_I = -\sum_R F_{1/2}(R) \sum_{w\in R} (2l_w+1)\ w_I\ \ \mathrm{sign}(m^w_{\mathrm{CB}})\,,$$

Two classification problems

- (1) given a 6D N=(1,0) anomaly-free theory in tensor Coulomb branch
  $\rightarrow$ construct classifying topological data of Calabi-Yau threefolds

  - Systematics: - start with gauge theories of non-Higgsable clusters [Morrison, Taylor]
    - successive un-Higgsing of gauge groups in field theory
    - generate tree of topological data

  - Challenges: - moding out symmetries
    - detecting additional geometric constraints
    - extending analysis to non-resolvable geometries

  $\Rightarrow$ String Universality: Is F-theory a theory of F-rything in 6D?

- (2) given an set of Calabi-Yau geometries, as provided e.g. by the Kreuzer-Skarke list
  $\rightarrow$ classify 6D N=(1,0) theories associated to elliptic fibrations

Conclusions

- The map between the Calabi-Yau threefold geometry and a 6D sugra theory on a circle has been established at **classical and one-loop** level.
  - Anomalies can be inferred from Chern-Simons terms, if the latter are probed by large gauge transformations.
  - Large gauge transformations map to arithmetic structures on ell. fibrations.
    - for Abelian gauge groups one encounters the Mordell-Weil group
      → general proof of anomaly cancellation for viable F-theory geometries
    - new arithmetic structures for non-Abelian groups and geometries with multi-sections.
  - Extension to 3D/4D has been worked out for chiral spectrum induced by fluxes

- F-theory provides a novel way to approach **classification problems**.
  - topological data of resolved Calabi-Yau threefolds are robust and corse information directly related to corse information about gauge theories
    - Are there mathematical restrictions on viable Hodge numbers, intersection numbers Chern classes? → ruling out 6D gauge theories coupled to gravity?

Thank you for your attention!