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5-brane webs and 5d, 6d SCFTs

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Based on the collaboration with

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[\[arXiv:1505.04439\]](#), [\[arXiv:1509.03300\]](#) and [\[arXiv:1512.08239\]](#)

24th of February 2016, F-theory at 20 at Caltech

1. Introduction

- Understanding 6d $\mathbf{N} = (1, 0)$ SCFTs that have various interesting properties.
- An indirect way to study 6d SCFTs
→ Use a 5d gauge theory which has a 6d UV completion.
- In general, gauge theories in higher dimensions are not renormalizable. The gauge coupling becomes infinitely strong at high energies.

- Those gauge theories in higher dimensions may make sense at UV if they admit a UV fixed point.
- In the case of 5d gauge theories, the UV completion is given by either a 5d SCFT or a 6d SCFT.
- When the UV completion is a 6d SCFT, KK modes of the circle compactification may be identified with an infinite tower of instantons in the 5d theory.

What is a 5d gauge theory that has a 6d UV completion?

- Example: 5d SU(2) gauge theories with N_f flavors

Seiberg 96,

Morrison, Seiberg 96

Douglas, Katz, Vafa 96

Ganor, Morrison, Seiberg 96

- (i) $N_f = 8 \rightarrow$ 6d SCFT called “E-string”

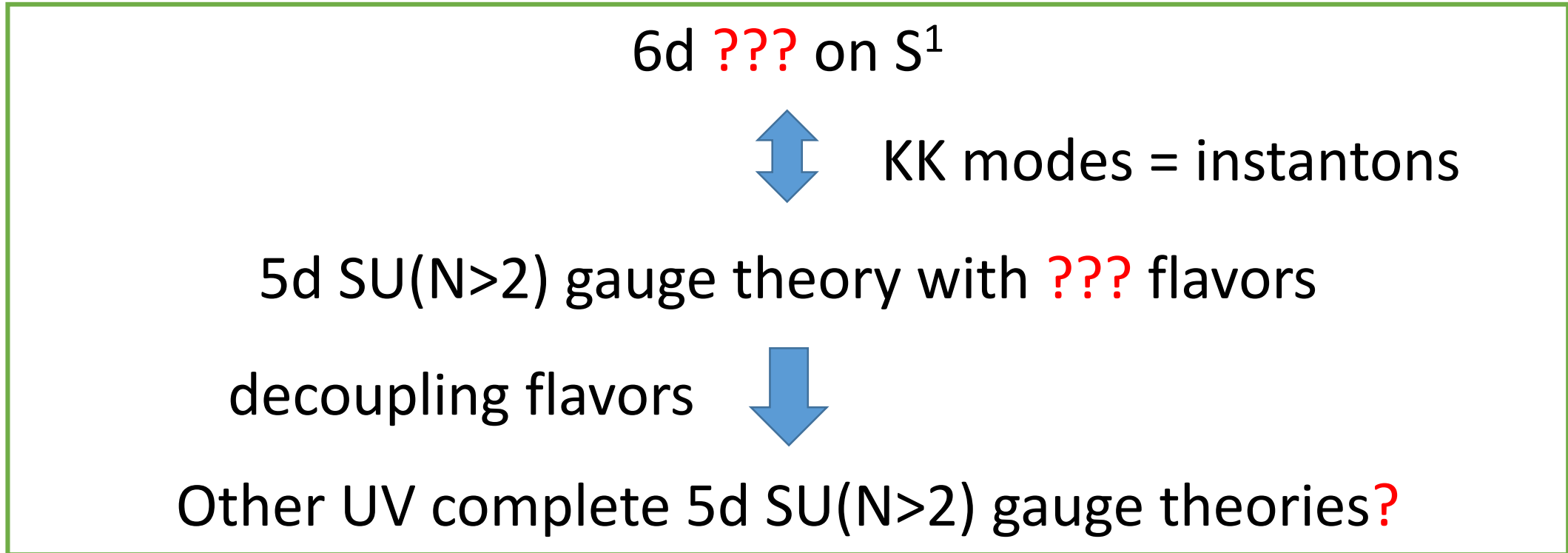
5d Nekrasov partition function = 6d elliptic genus

Kim, Kim, Lee, Park, Vafa 14

- (ii) $N_f \leq 7 \rightarrow$ 5d SCFT with E_{N_f+1} flavor symmetry

- Can we generalize this, for example to higher rank cases?

- This question turns out to be quite non-trivial!



- What is the landscape of UV complete 5d theories?

- I will address the questions from string theory.

I will argue that:

- String theory predicts **new UV complete 5d gauge theories** that admit a 6d UV completion and gives a direct relation between 5d and 6d.
- String theory also implies **new 5d “UV dualities”**.
- The landscape of UV complete 5d gauge theories may be largely expanded!

1. Introduction
2. 5d gauge theories from string theory
3. 5-brane webs and 6d SCFTs
4. New 5d UV dualities
5. Conclusion

2. 5d gauge theories from string theory

- We construct a 5d supersymmetric field theory with eight supercharges as the worldvolume theory on a 5-brane web.
- The 5-brane configuration in Type IIB string theory.

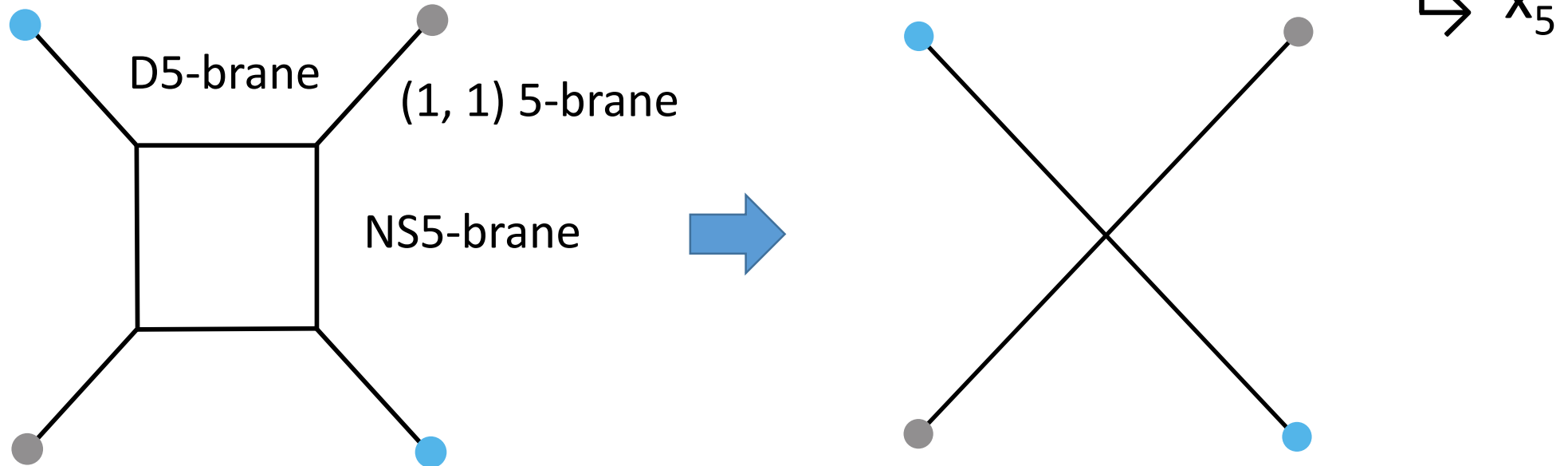
	0	1	2	3	4	5	6	7	8	9
D5-brane	×	×	×	×	×	×				
NS5-brane	×	×	×	×	×		×			
(p, q) 5-brane	×	×	×	×	×	angle				
7-brane	×	×	×	×	×			×	×	×



5-brane web

Aharony, Hanany 97,
 Aharony, Hanany, Kol 97
 DeWolfe, Iqbal, Hanany, Katz 99

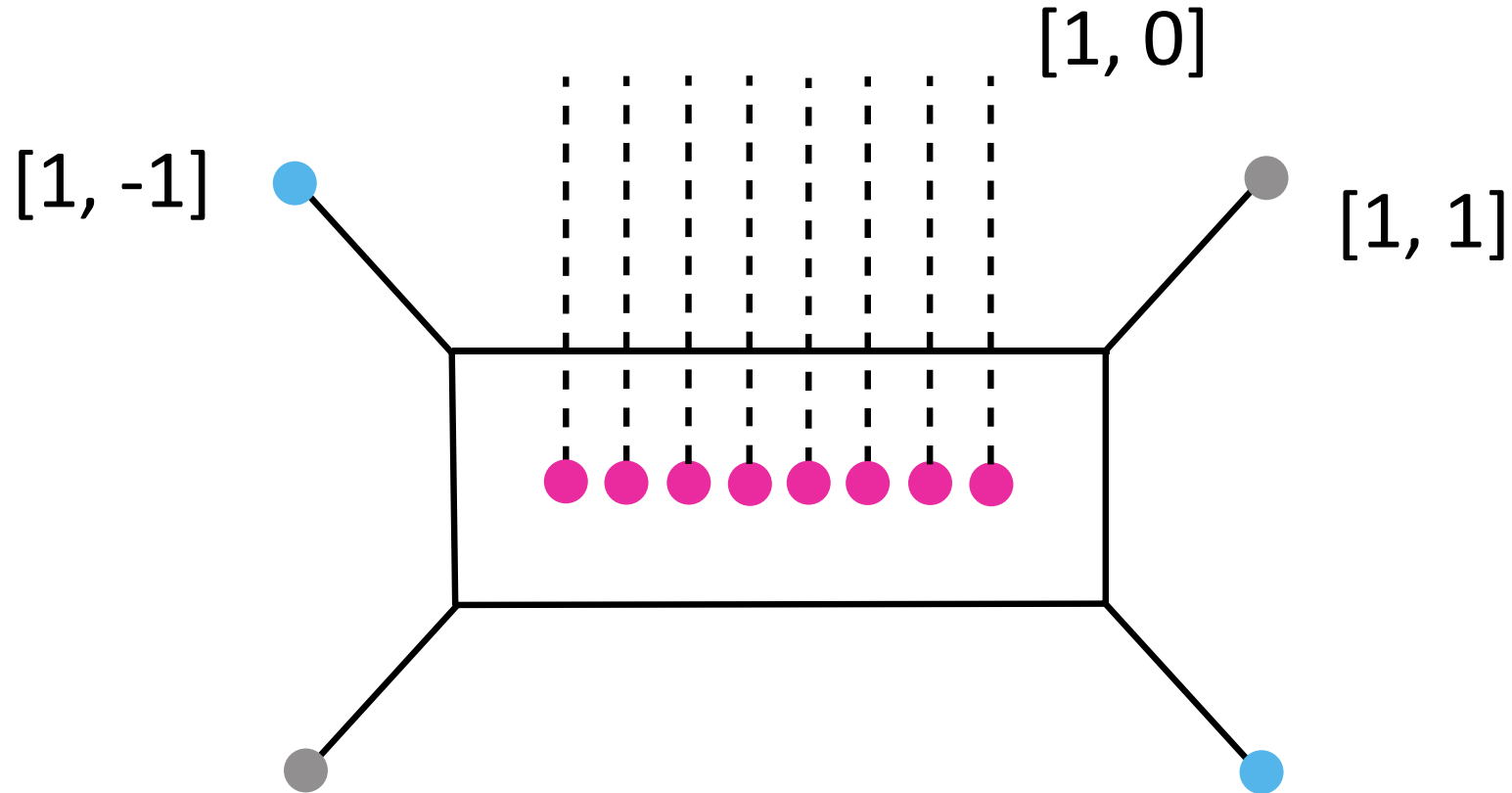
- Ex. Pure $SU(2)$ gauge theory with $\theta = 0$.

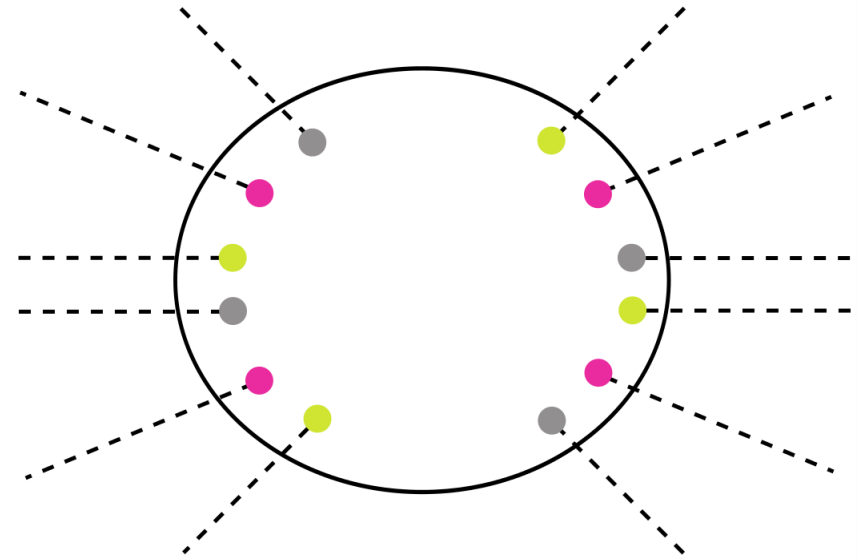
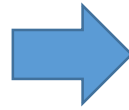
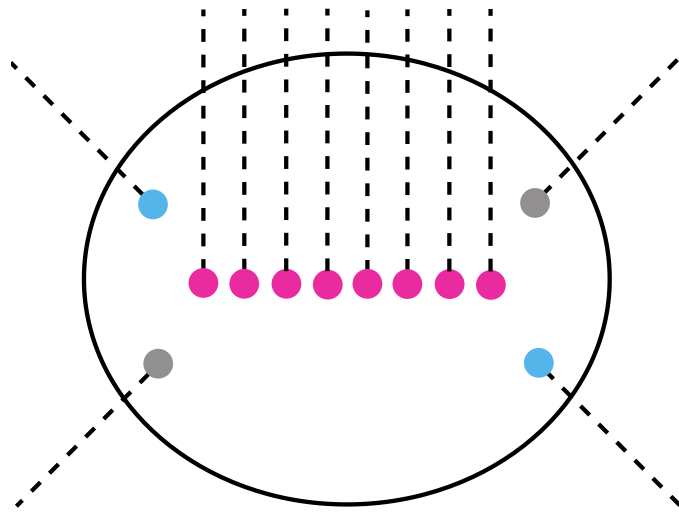


- This theory has a fixed point where the gauge coupling is infinitely strong.
- Enhancement of flavor symmetry: $U(1) \rightarrow SU(2) = E_1$.

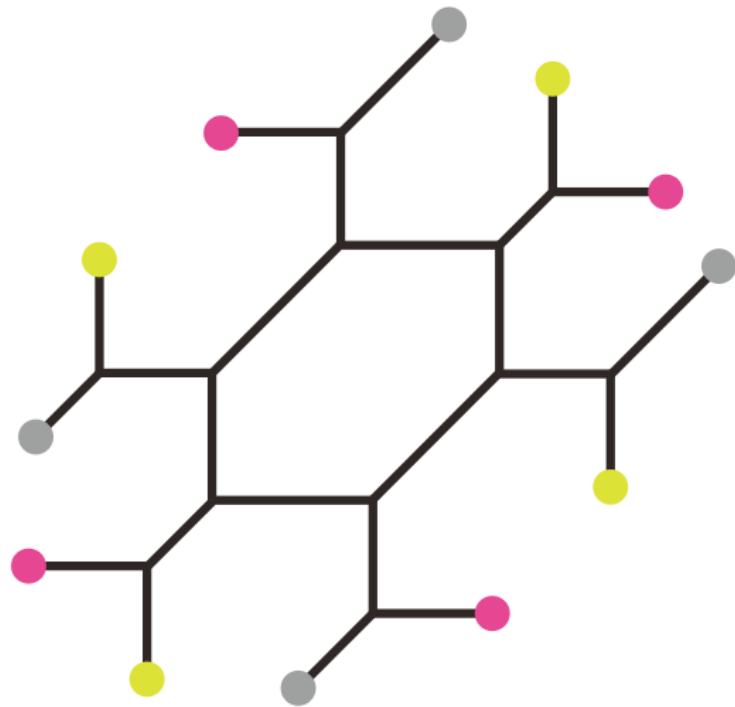
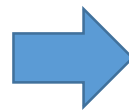
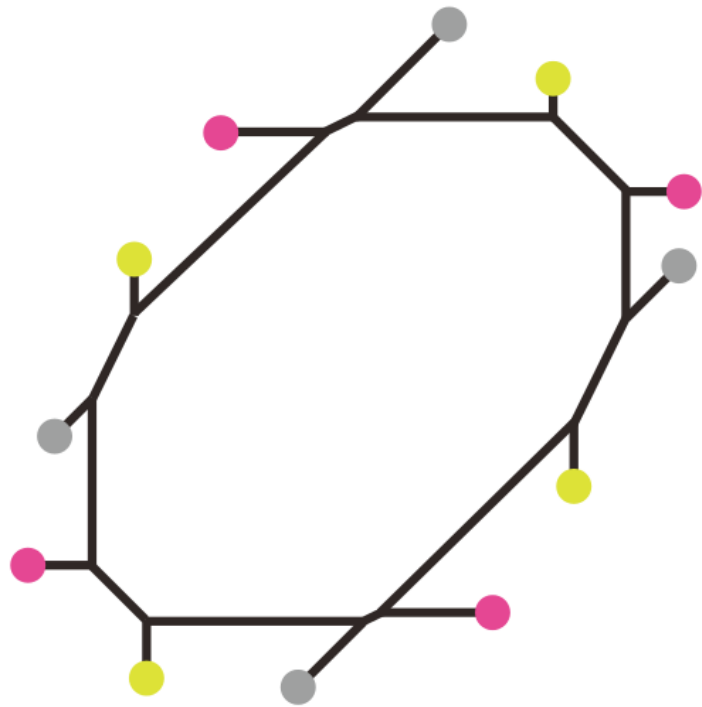
Seiberg 96

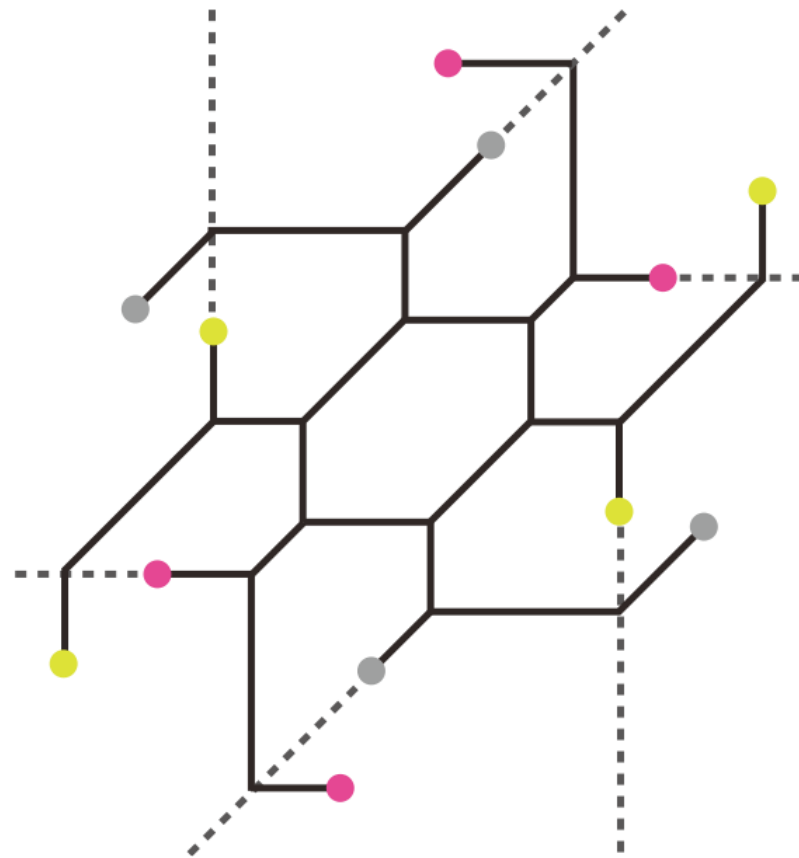
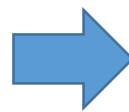
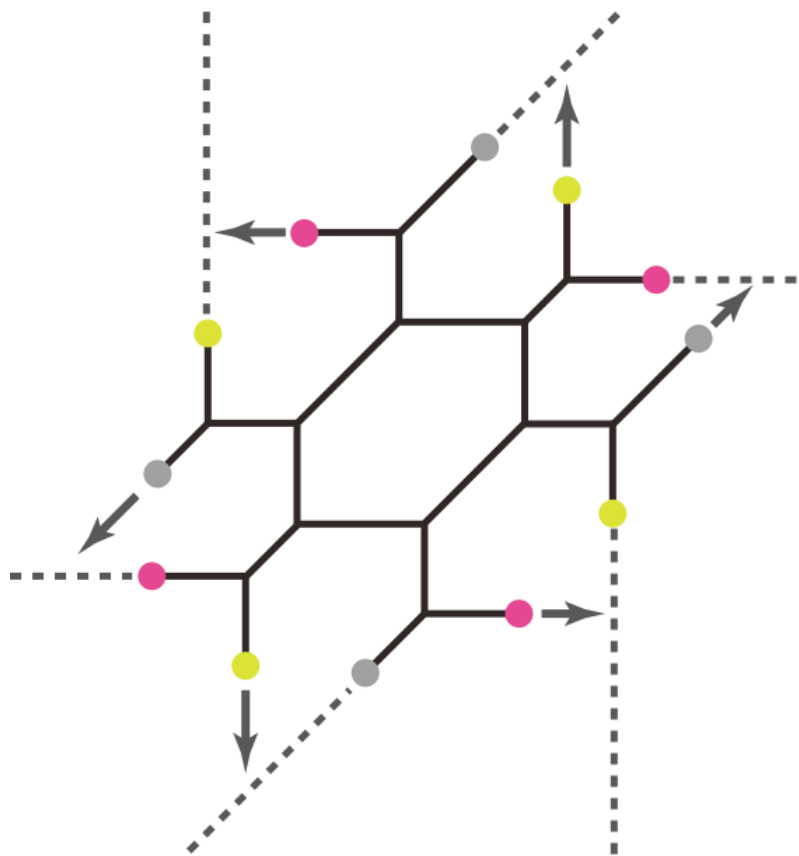
- What happens if we consider the $SU(2)$ gauge theory with 8 flavors? We know that its UV fixed point is the 6d E-string theory.



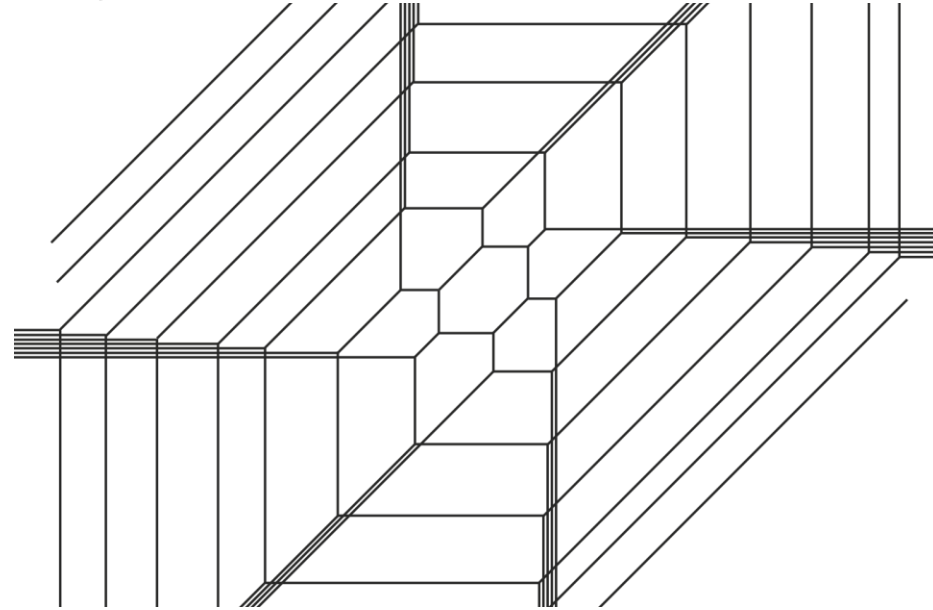


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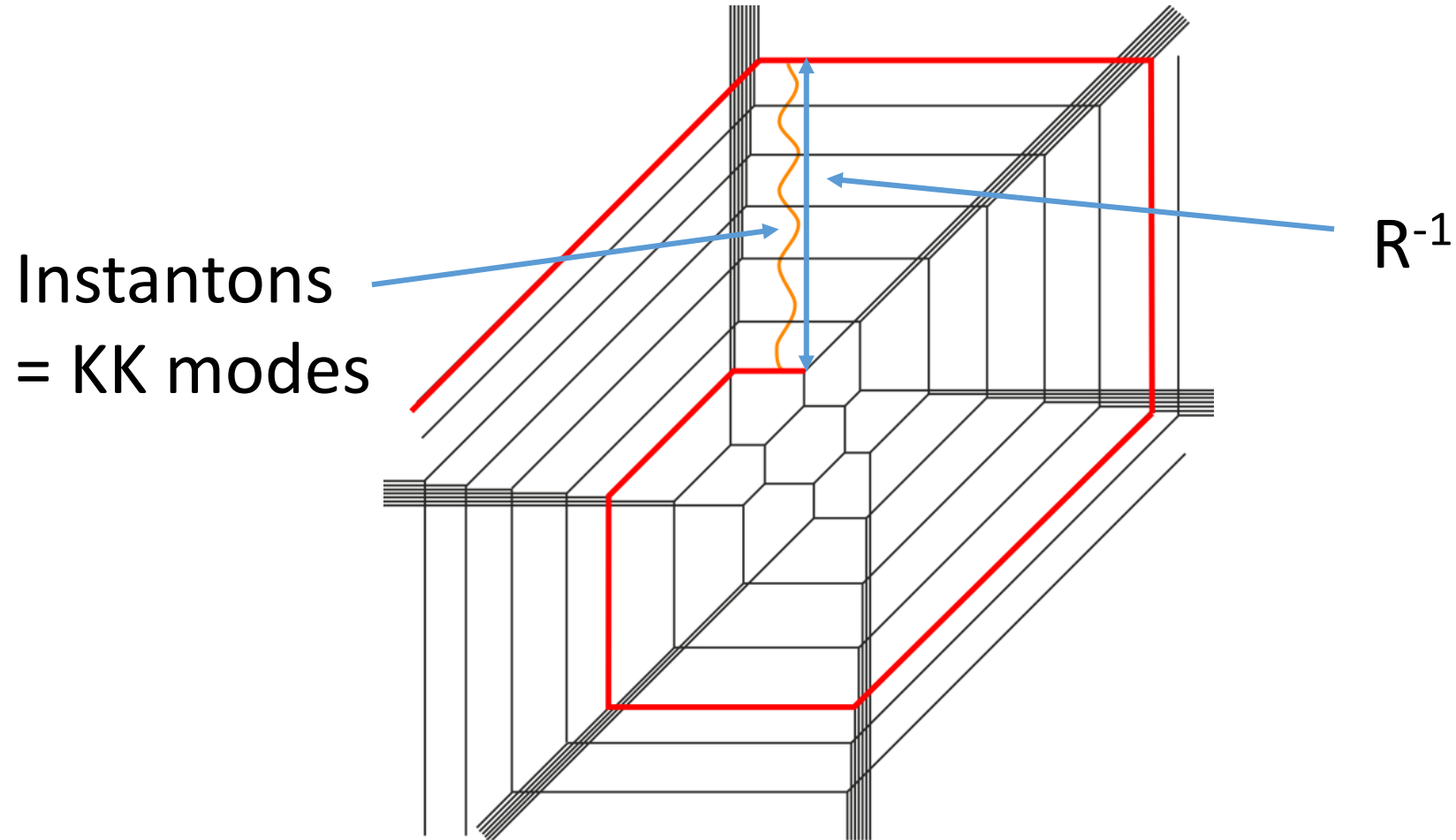


- Then the 5-brane web starts rotating when we pull 7-branes to infinity.



- The topological vertex computation reproduces the Nekrasov partition function of 5d $SU(2)$ gauge theory with 8 flavors

- The infinitely rotating structure is essential to reproduce the infinite tower of KK modes.



- Conjectures:

- The existence of a standard 5-brane web implies the corresponding 5d theory has a **5d** UV fixed point.
- The existence of an infinitely rotating 5-brane web implies the corresponding 5d theory has a **6d** UV fixed point.

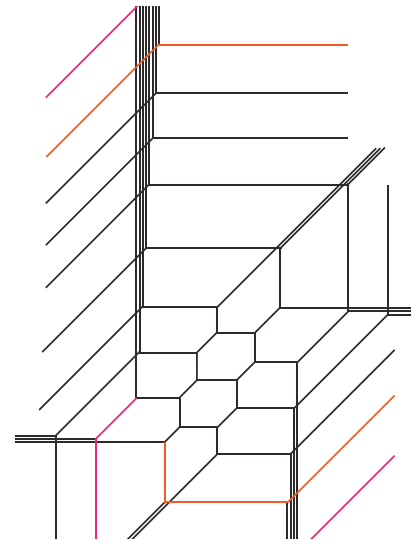
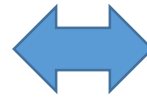
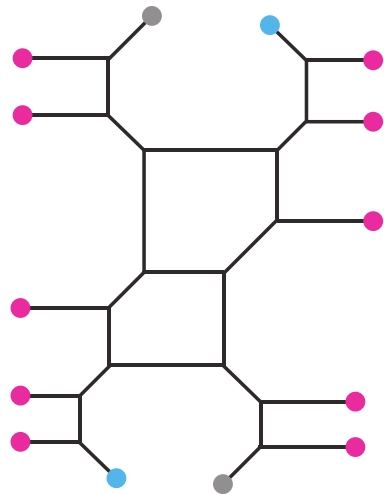
- Then let us think about the generalization by utilizing the infinitely rotating structure of 5-brane webs!

3. 6d SCFTs and 5-brane webs

- The generalization to higher rank cases is straightforward. The infinitely rotating structure of a 5-brane web appears when it yields a 5d $SU(n)$ gauge theory with $N_f = 2n+4$ flavors.

Kim, Taki, Yagi 15

Ex. 5-brane web for 5d $SU(3)$ with 10 flavors



- Does it corresponds to a 6d SCFT at UV? → Yes!

Hayashi, Kim, Lee, Taki, Yagi 15
Yonekura 15

- Our claim is

6d (D_{n+2}, D_{n+2}) minimal conformal matter theory on S^1



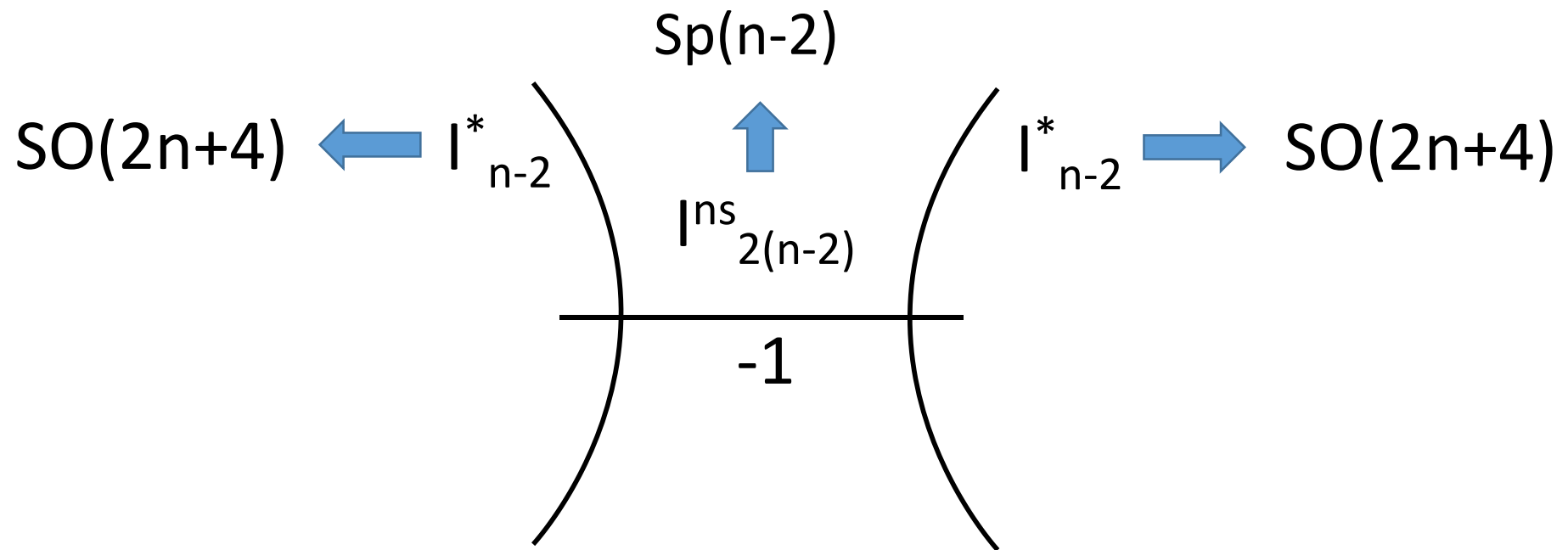
5d $SU(n)$ gauge theory with $N_f = 2n+4$ & CS-level 0

- In fact, we can “derive” this claim by using a brane construction in string theory.

Hayashi, Kim, Lee, Taki, Yagi 15

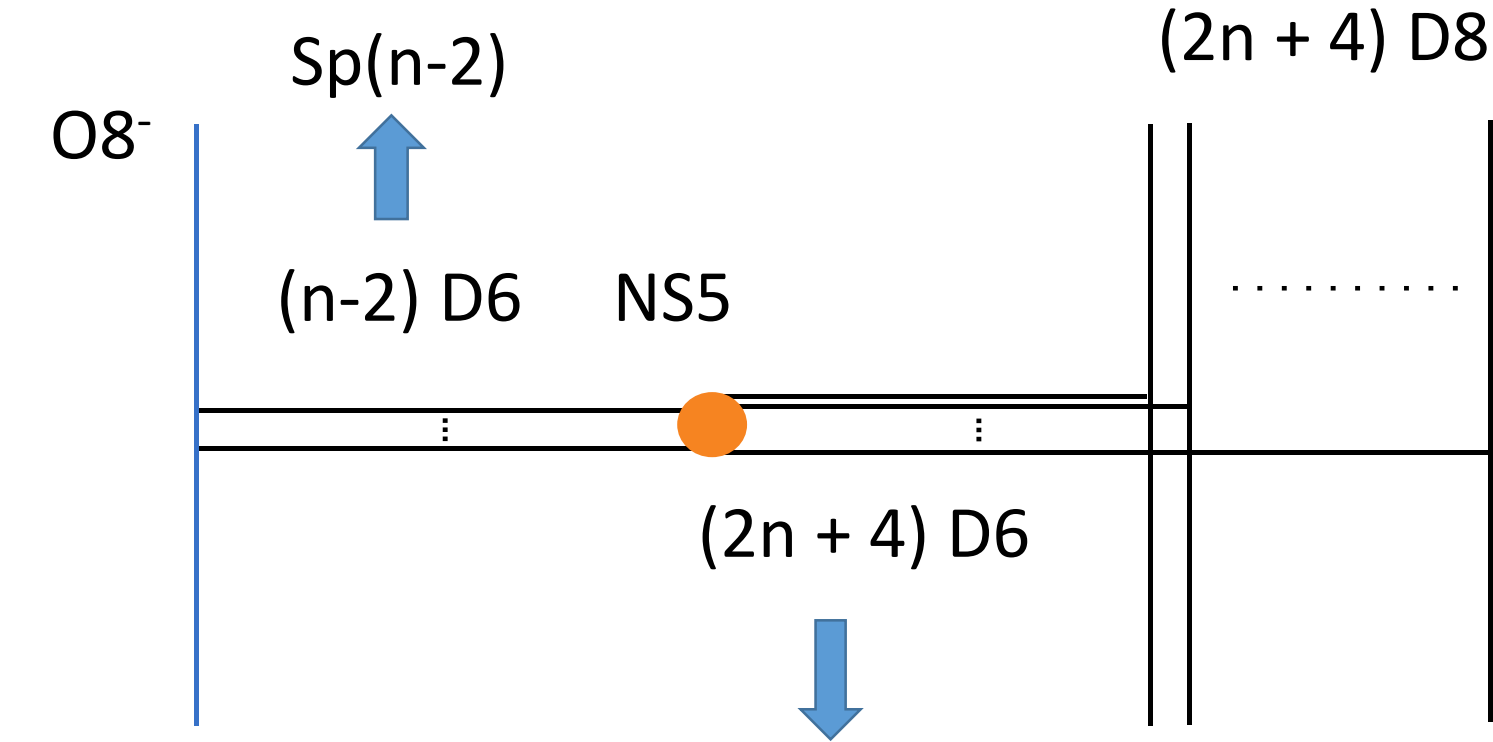
- On the tensor branch, the 6d (D_{n+2}, D_{n+2}) minimal conformal matter theory becomes a 6d $Sp(n-2)$ gauge theory with $2n+4$ fundamental hypermultiplets, coupled to a tensor multiplet.

Del Zotto, Heckman, Tomasiello, Vafa 14



- The type IIA brane configuration

Brunner, Karch 97
Hanany, Zaffaroni 97



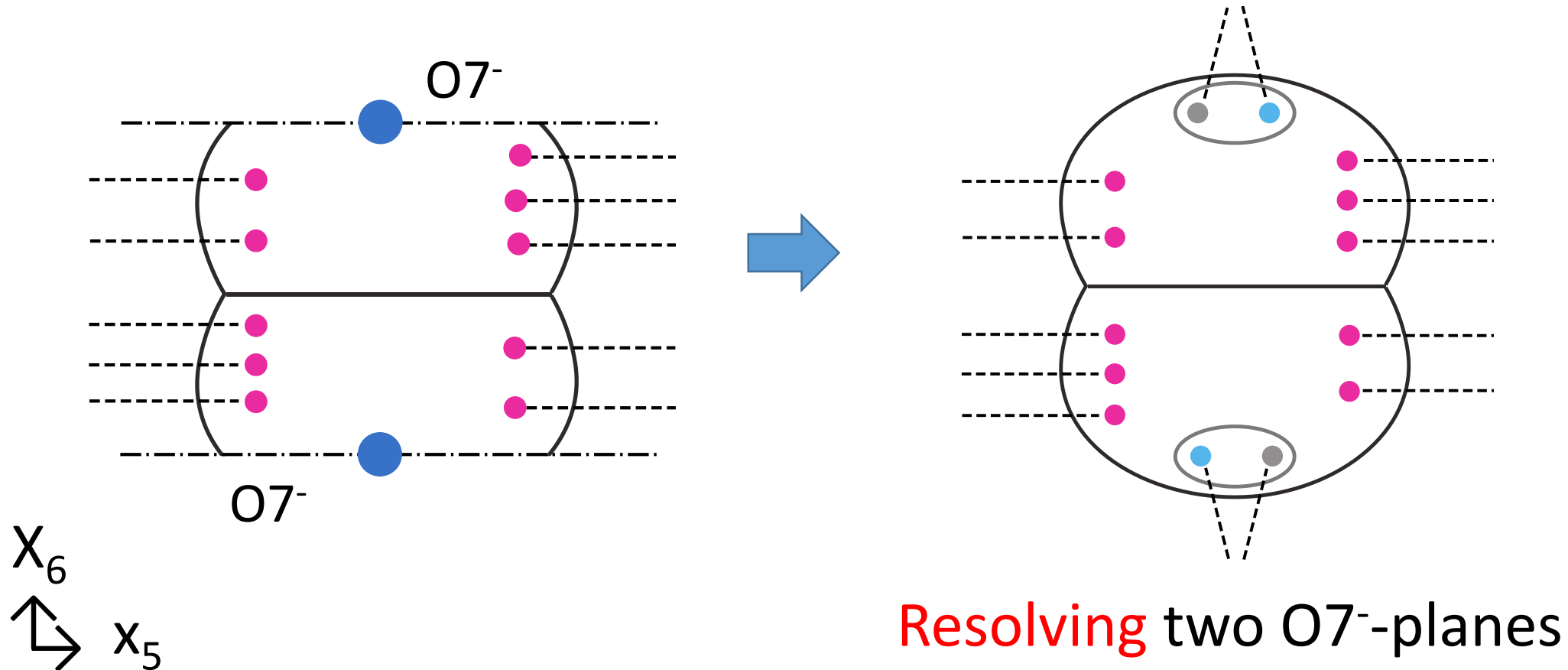
$x_{7,8,9}$
↗ ↘
 x_5

$$U(2n+4) \subset SO(4n+8)$$

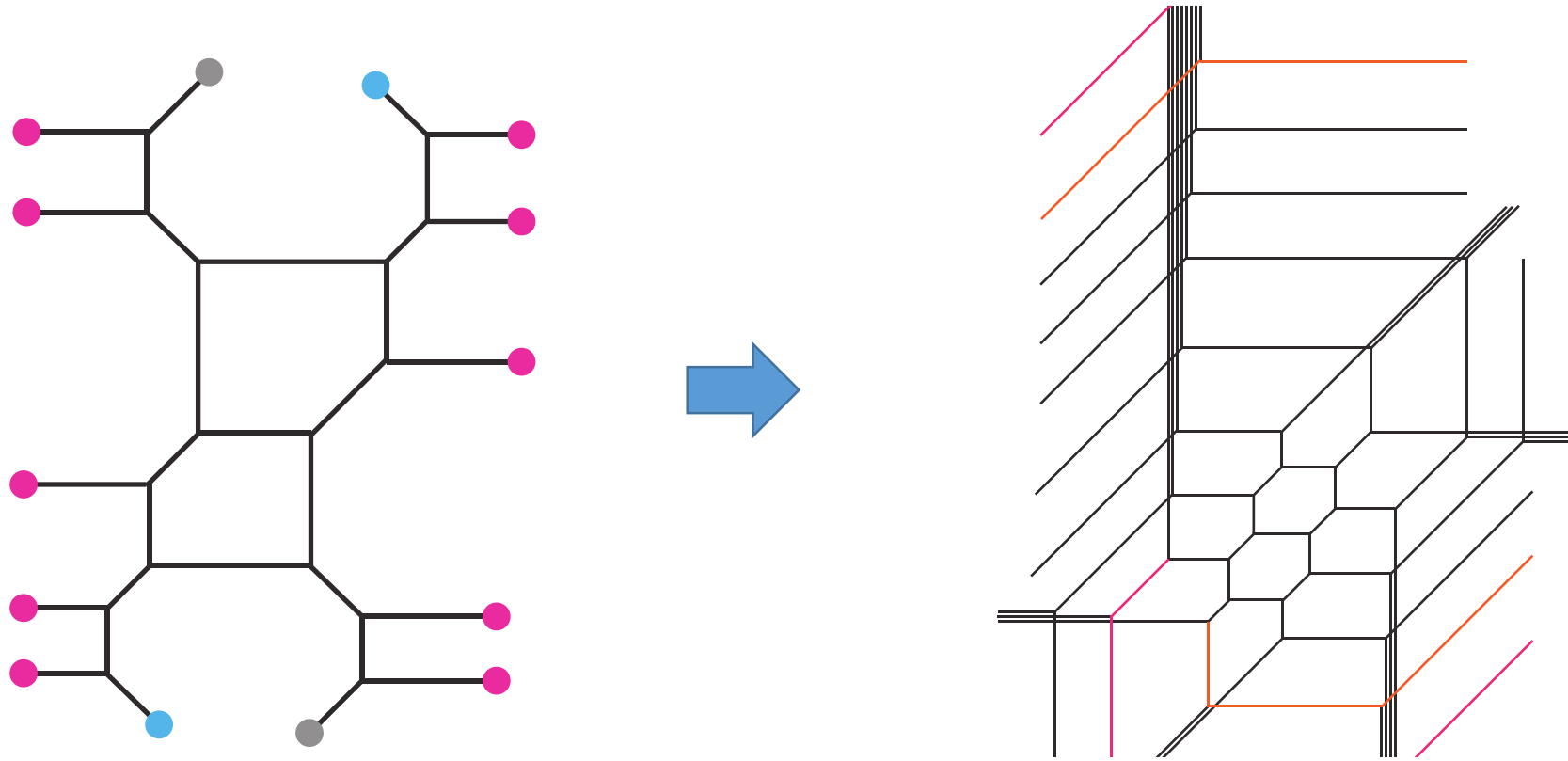
	0	1	2	3	4	5	6	7	8	9
D6-brane	×	×	×	×	×	×	×			
NS5-brane	×	×	×	×	×		×			
D8-brane	×	×	×	×	×		×	×	×	×
O8-plane	×	×	×	×	×		×	×	×	×

- We then compactify the x_6 -direction on S^1 and perform the **T-duality** along the direction.

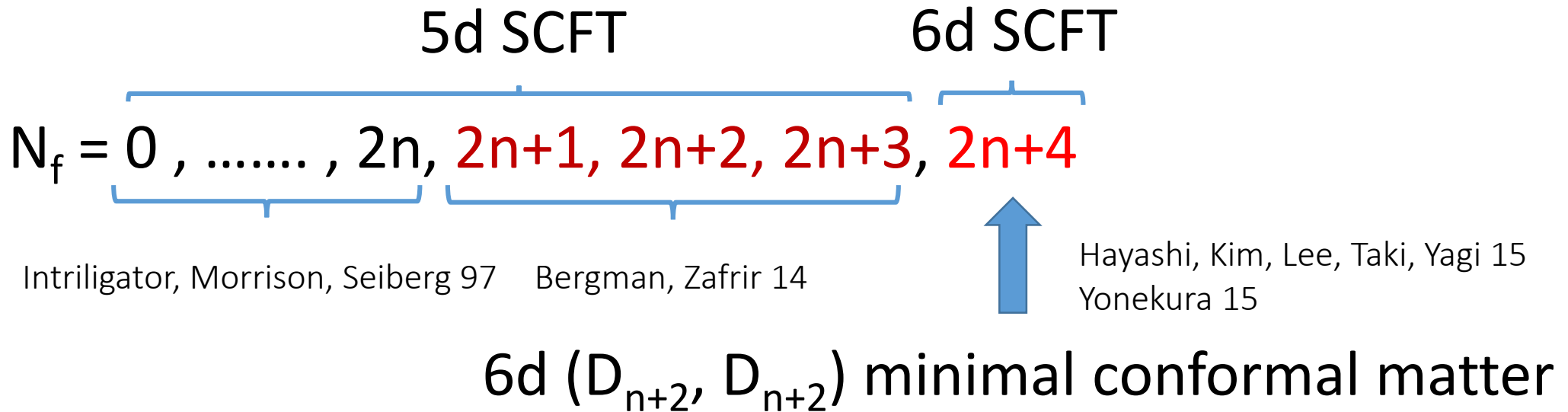
Ex. $n = 3$



- Pulling out the 7-branes outside the 5-brane loop yields the 5-brane web for the $SU(3)$ gauge theory with $N_f = 10$ flavors.



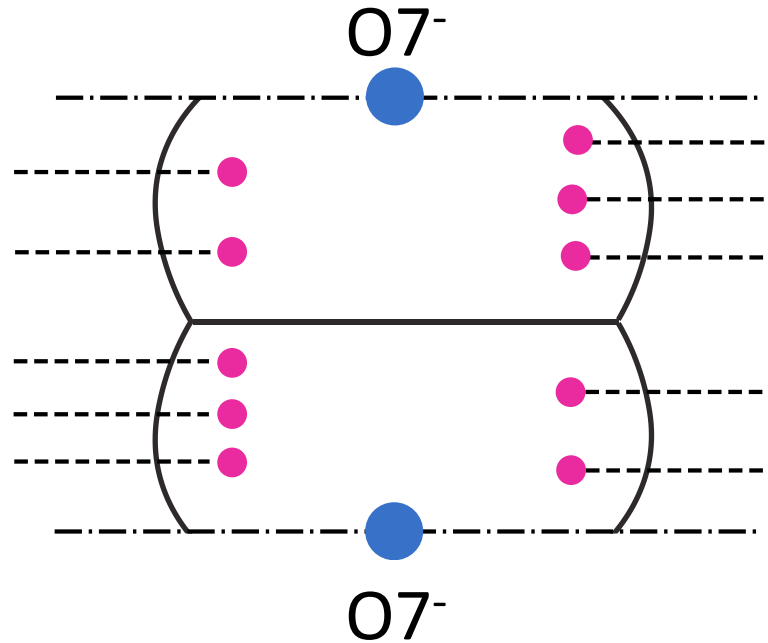
- By decoupling flavors, we obtain 5d SU(n) gauge theories whose UV completion is 5d SCFTs.



- The condition for the UV completion

$$N_f + 2|k| \leq 2n + 4$$

- The infinitely rotating 5-brane web hides the structure of a “circle” given by the two $O7^-$ -planes.



T-duality



6d gauge theory on S^1



Resolving $O7^-$ -planes

5d gauge theory

- It is possible to generalize the analysis to other 6d SCFTs constructed by type IIA brane configurations.

(i) Including more NS5-branes gives **new** 5d A-type quiver gauge theories.

Yonekura 15
Zafrir 15,
Hayashi, Kim, Lee, Yagi 15
Ohmori, Shimizu 15

(ii) Including an ON^0 -plane gives **new** 5d D-type quiver gauge theories.

Hayashi, Kim, Lee, Taki, Yagi 15

- **Largely expands the landscape of 5d gauge theories!**

Examples:

(i) 6d $SU(2n)$ with $N_f = 2n+8$ and $N_a = 1$ on S^1



5d $[n+3] - SU(n+1) - SU(n+1) - [n+3]$

(ii) 6d $[8] - SU(2n) - Sp(2n-4) - SO(4n) - Sp(2n-4) - [2n]$ on S^1



5d $[4] - SU(n+3) \left\{ \begin{array}{l} \backslash \\ / \end{array} \right. SU(2n+2) - SU(2n-2) - SU(2n-6) - [2n-8]$
 $[4] - SU(n+3)$

4. New 5d UV dualities

- The derivation using the brane configuration also implies another interesting 5d dualities.
- 5d $SU(n) \leftrightarrow 5d Sp(n-1)$ duality

Gaiotto, Kim 15

5d $SU(n)$ gauge theory with N_f flavors & $|\kappa| = (n + 2 - N_f/2)$

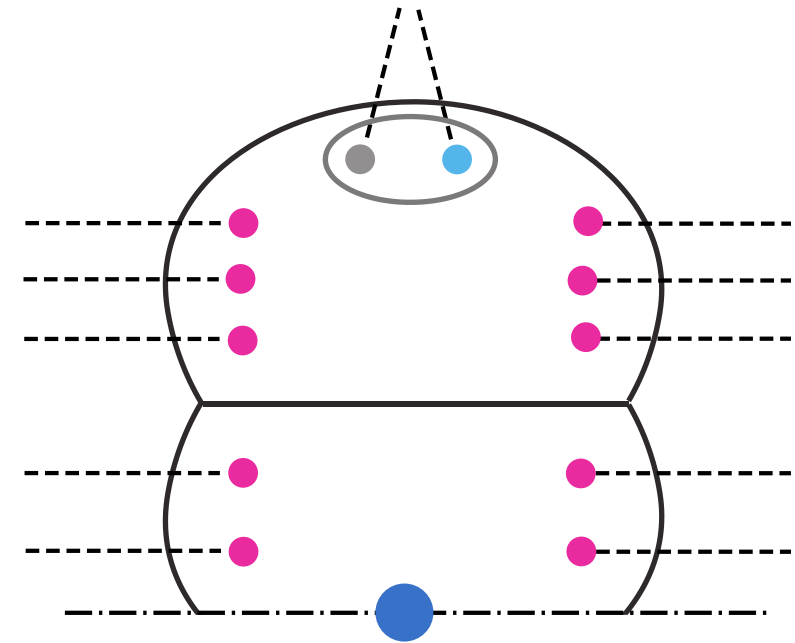
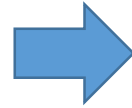
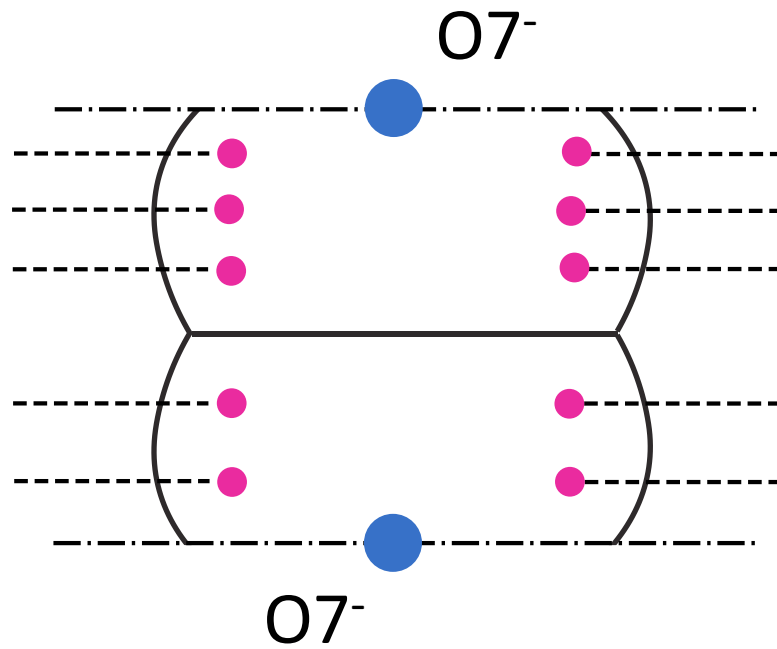


5d $Sp(n-1)$ gauge theory with N_f flavors

Why ???

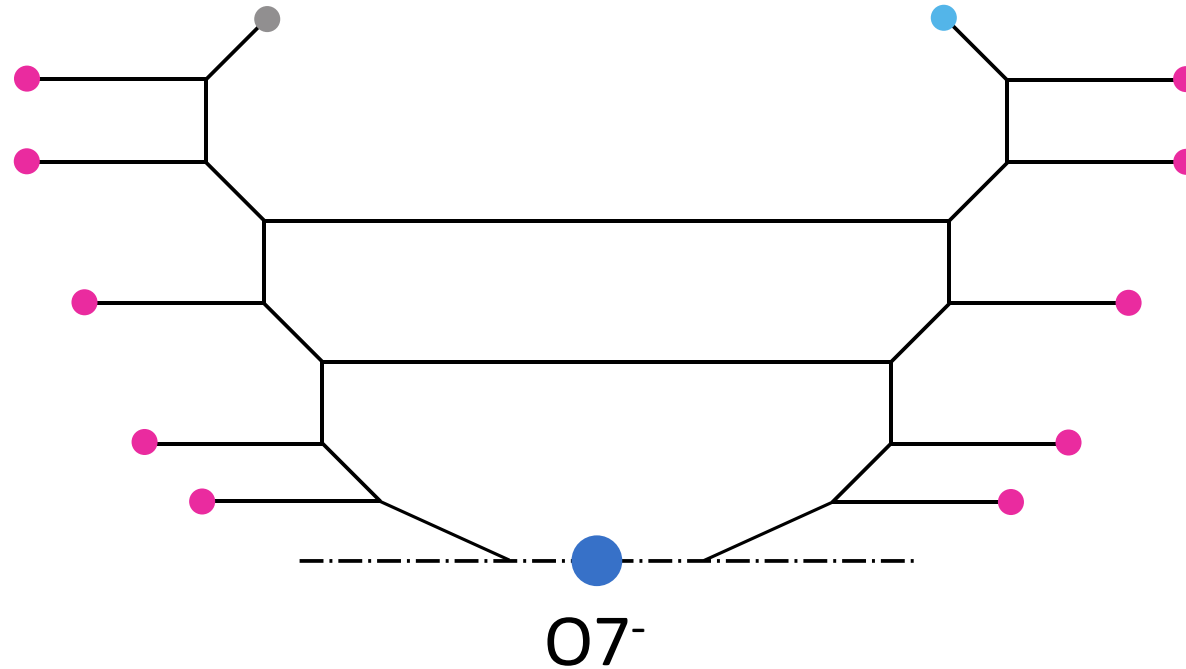
- The derivation again starts from the Type IIA brane configuration for the 6d $Sp(n-2)$ gauge theory with $N_f = 2n+4$ flavors, coupled to a tensor multiplet.

Ex. $n = 3$



Resolving **one** $O7^-$ -plane

- The resulting brane configuration is a 5d $Sp(2)$ gauge theory with $N_f = 10$.



- In general, we obtain a 5d $Sp(n-1)$ gauge theory with $N_f = 2n+4$.

(D_{n+2}, D_{n+2}) minimal conformal matter



$SU(n)$ with $N_f = 2n+4$



$Sp(n-1)$ with $N_f = 2n+4$

- Decoupling flavors on both sides in the same way reproduces the 5d SU – Sp duality.
- The generalization to other 6d SCFTs yields various **new** 5d UV dualities.

Hayashi, Kim, Lee, Yagi 15

Hayashi, Kim, Lee, Taki, Yagi 15

Examples:

(i) $[n+3] - \text{SU}(n+1) - \text{SU}(n+1) - [n+3]$



$\text{SU}(2n+1)$ with $N_f = 2n+7$ and $N_a = 1$

(ii) $[4] - \text{SU}(n+3)$
 $[4] - \text{SU}(n+3) \begin{cases} \diagdown \\ \diagup \end{cases} \text{SU}(2n+2) - \text{SU}(2n-2) - \text{SU}(2n-6) - [2n-8]$



$[8] - \text{SU}(2n+3) - \text{Sp}(2n-1) - \text{SO}(4n-2) - \text{Sp}(2n-5) - [2n-4]$

5. Conclusion

- 5-brane webs predict a large class of **new** 5d gauge theories that have a 5d or 6d UV completion.
- The combination of the T-duality and the resolution of O7-planes gives a way to directly identify the 6d UV completion of the new 5d gauge theories.
- The method also implies various **new** 5d UV dualities.