On the Classification of 6D RG Flows

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Based On

Work with:

M. Del Zotto, D.R. Morrison, D.S. Park,

T. Rudelius, A. Tomasiello, C. Vafa

In Particular:

hep-th/1502.05405 w/ Rudelius, Morrison, Vafa

hep-th/1601.04078 w/ Rudelius and Tomasiello

6D Theories are Interesting

Basic Reason:

$\exists$ Interacting 6D CFTs?

Answer from Strings:

6D SCFTs Exist!

More Reasons to Study

- What do 6D QFTs "Look Like"?
- Long Distance Limit of M5-branes
- Compactify to 5D/4D/3D/2D/1D

(see C. Vafa's talk)

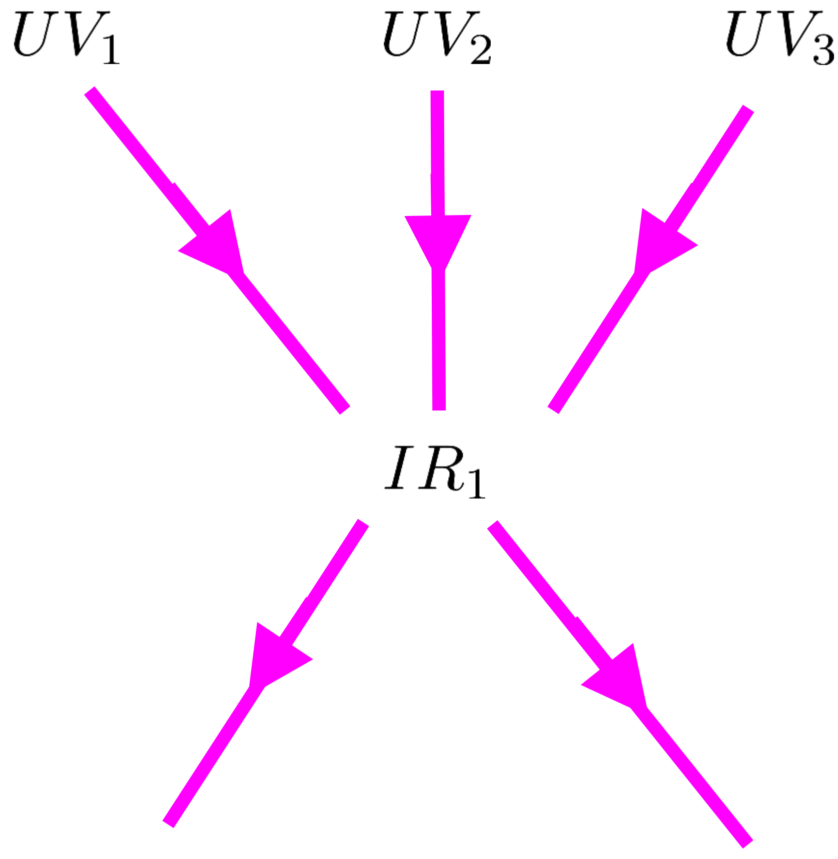
More Reasons to Study

- What do 6D QFTs "Look Like"?

Study via Flows into and out of CFTs

(see also K. Intriligator's talk)

¿RG Flows?



¿RG Flows?

Step 1: Find the Fixed points (SCFTs)

Step 2: Classify Flows Between Them

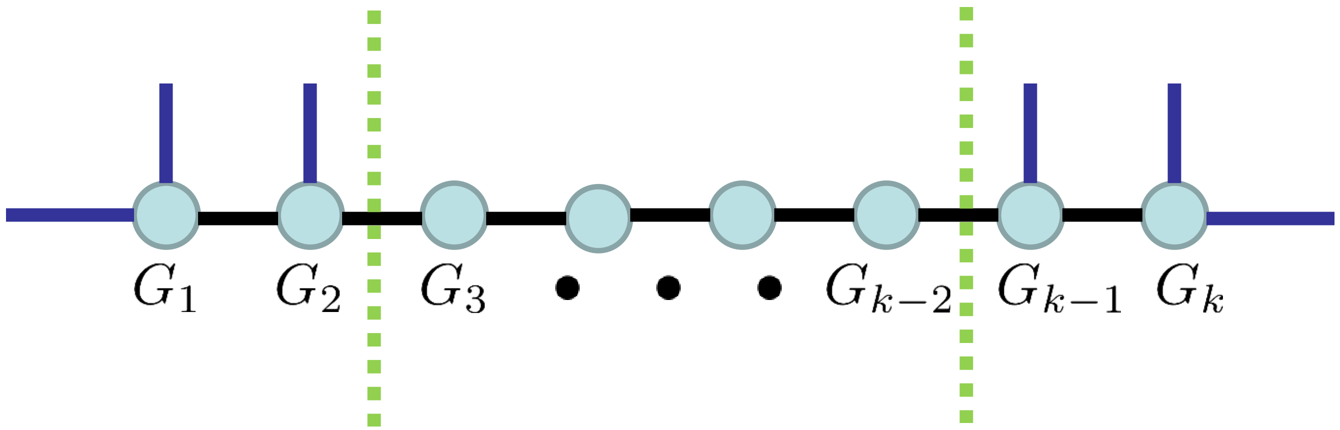
Step 1:

Classifying 6D SCFTs

(see T. Rudelius' talk for LSTs)

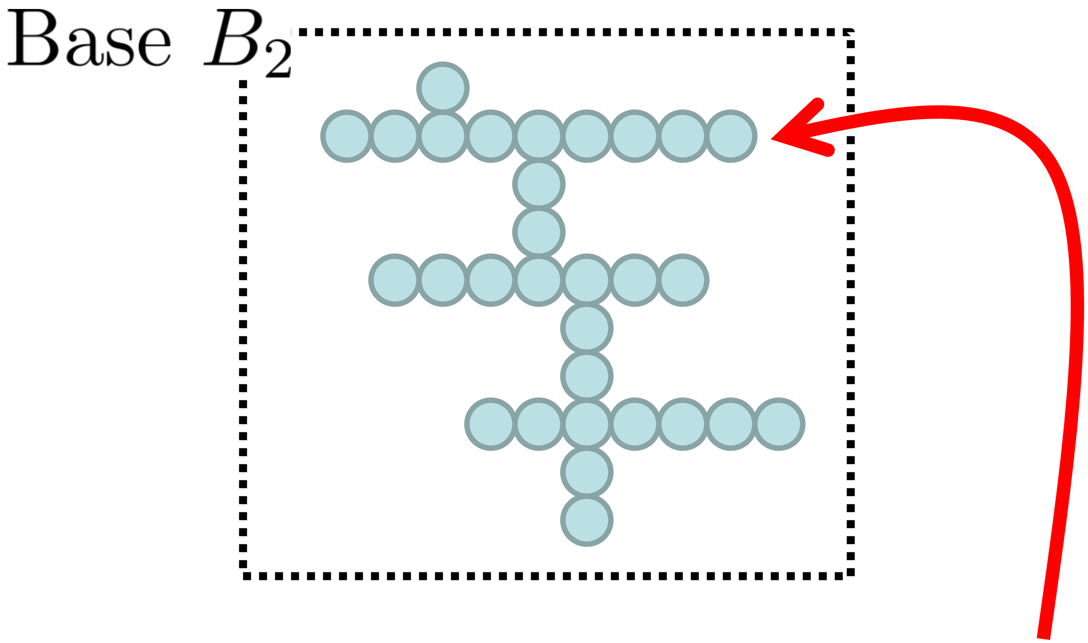
(see also K. Intriligator, Y. Tachikawa and C. Vafa's talks)

F-theory and 6D SCFTs



$$\text{6D SCFTs} = \text{Generalized Quivers}$$

Geometric Picture: $CY_3 \to B_2$

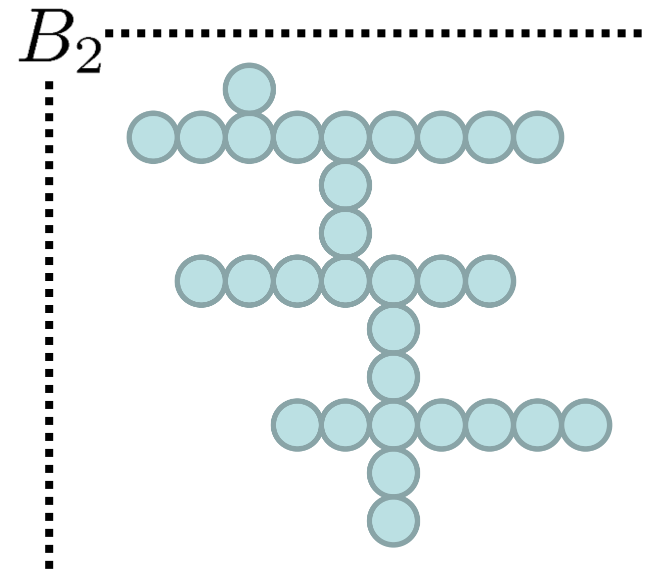


Singularities in base $\Rightarrow$ strings (D3 / $\mathbb{P}^1$)

Singularities in fiber $\Rightarrow$ particles ($7 - 7'$ strings)

SCFT Limit

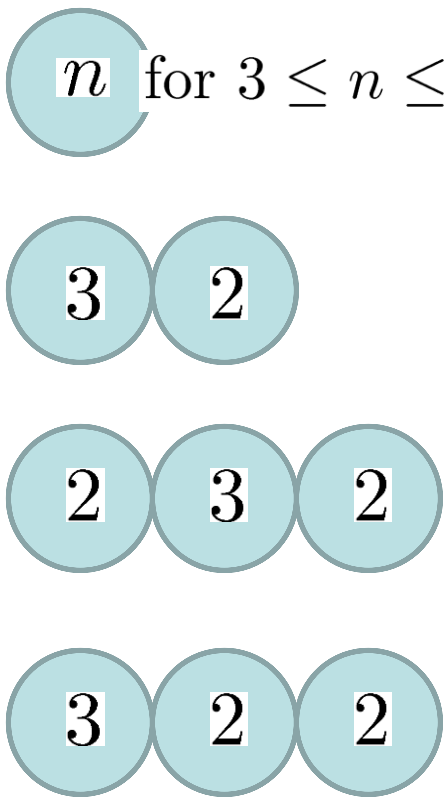
Start: A smooth base B_2



End: To get a CFT, sim. contract curves of B_2

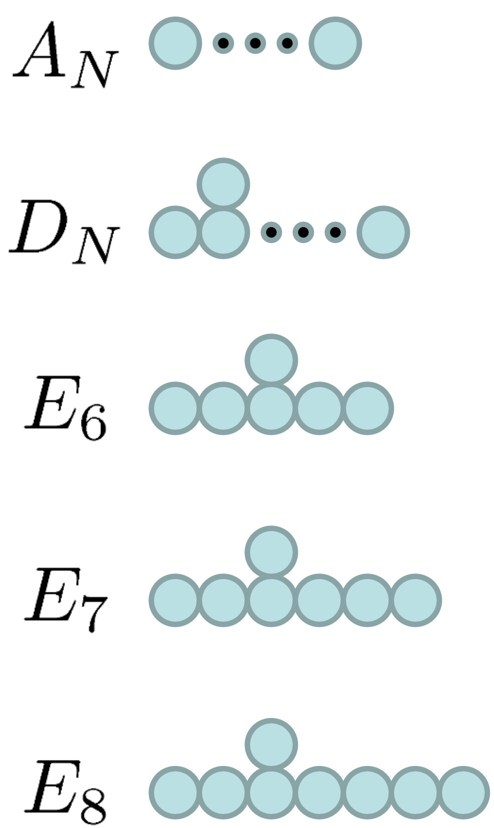
Building Blocks

"Non-Higgsable Clusters"

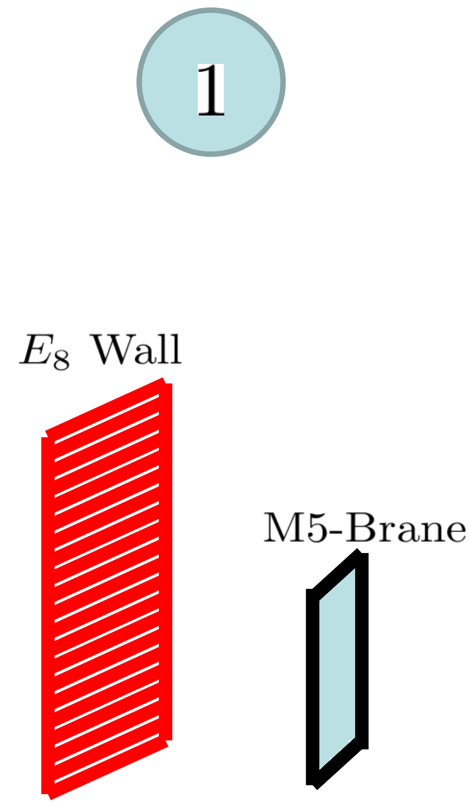


for $3 \leq n \leq 12$

$(2,0)$ Theories



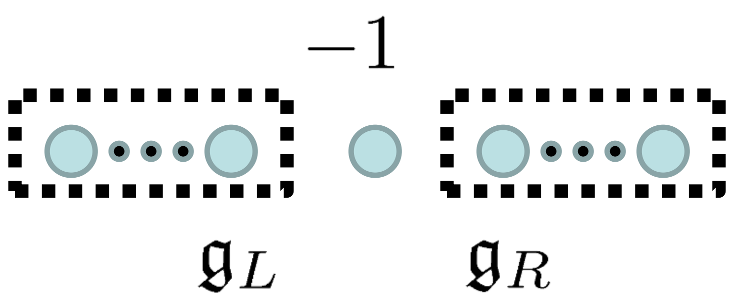
E-String Theory



Building a Base

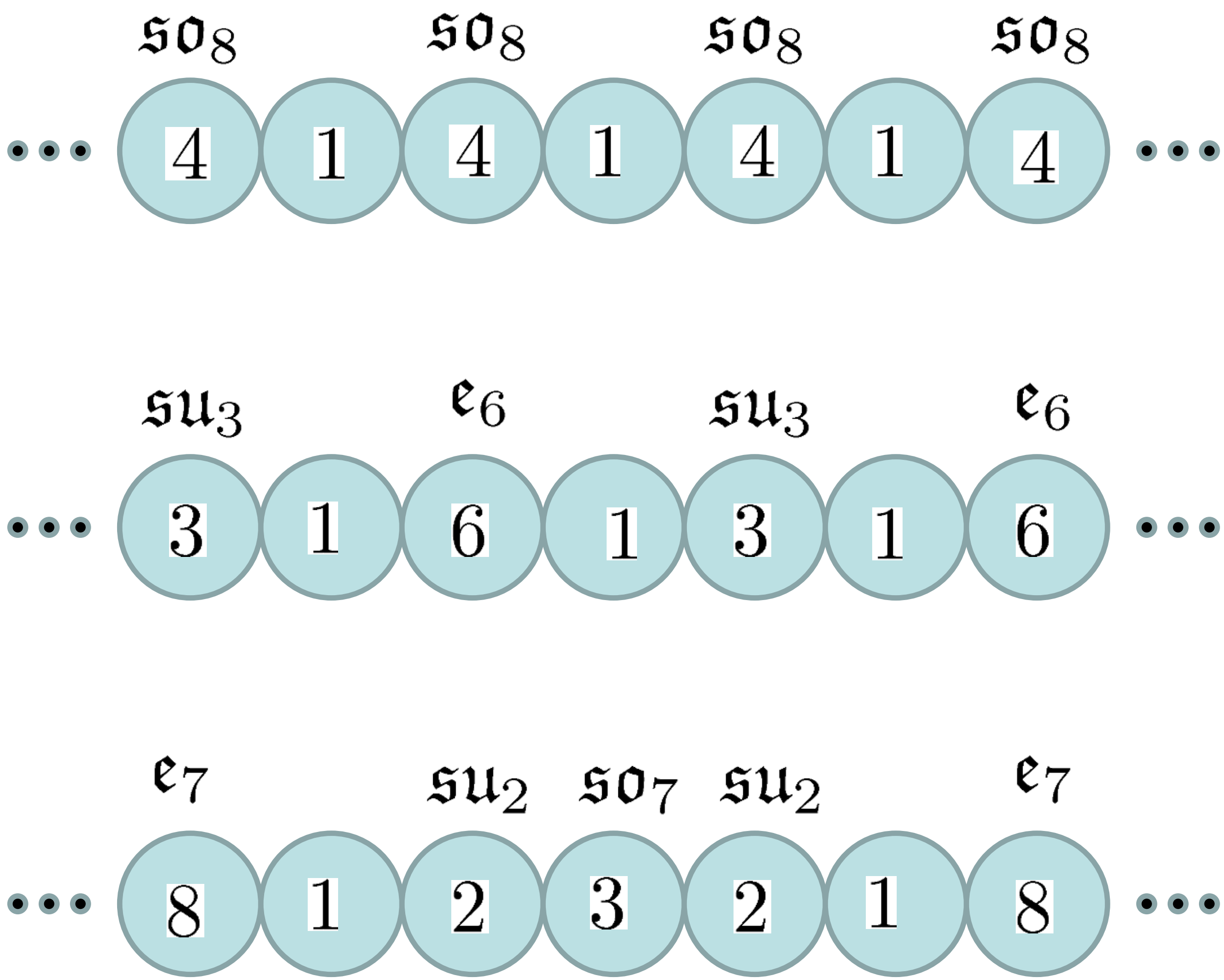
In base B_2, "gluing" of building blocks:

classified by Morrison and Taylor '12 (see also JJH Morrison Vafa '13)



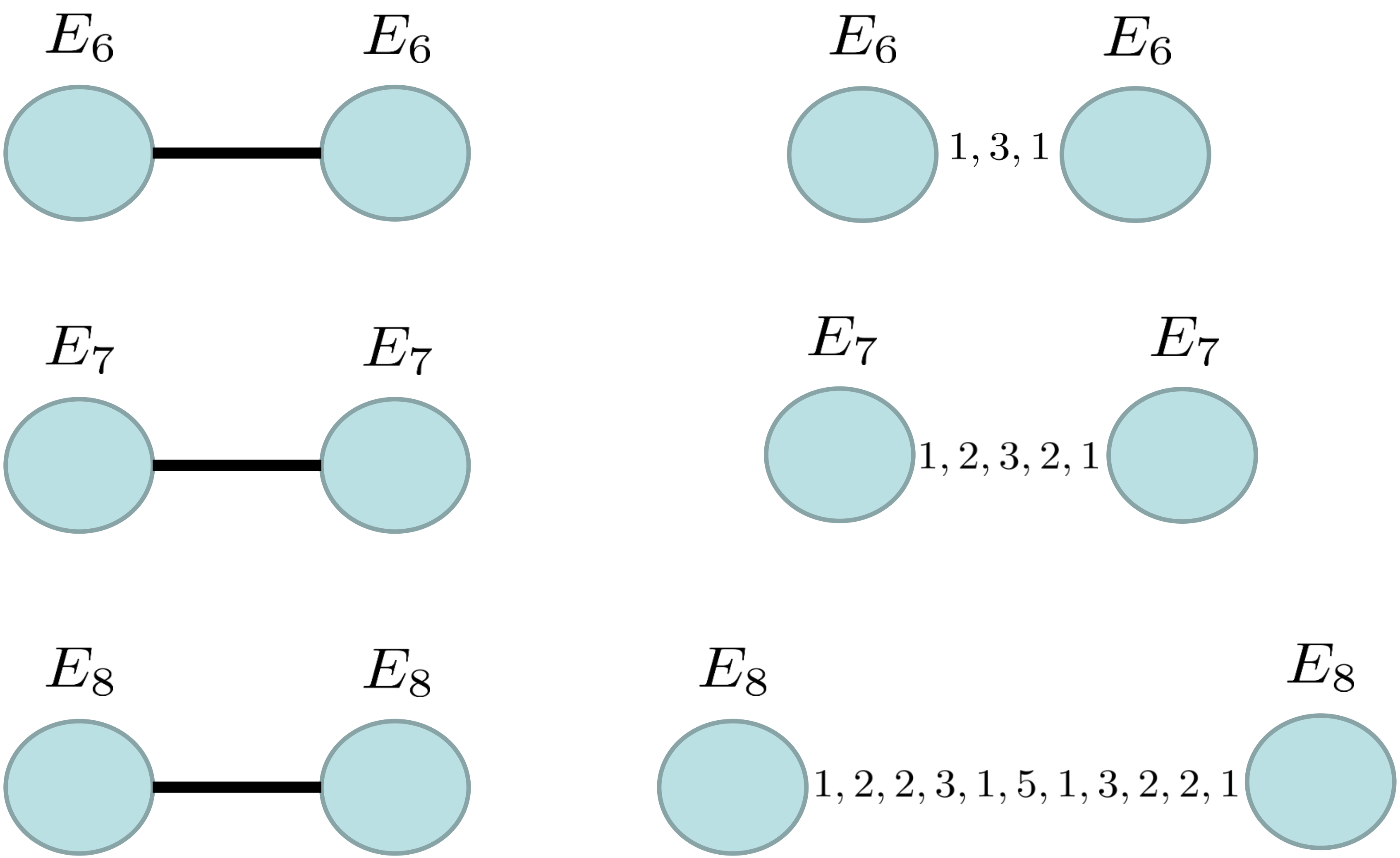
$\mathfrak{g}_L \times \mathfrak{g}_R \subset \mathfrak{e}_8$

Examples



$\mathfrak{so}_8$ $\mathfrak{so}_8$ $\mathfrak{so}_8$ $\mathfrak{so}_8$
4 1 4 1 4 1 4
$\mathfrak{su}_3$ $\mathfrak{e}_6$ $\mathfrak{su}_3$ $\mathfrak{e}_6$
3 1 6 1 3 1 6
$\mathfrak{e}_7$ $\mathfrak{su}_2$ $\mathfrak{so}_7$ $\mathfrak{su}_2$ $\mathfrak{e}_7$
8 1 2 3 2 1 8

E-Type Quivers



Useful Terminology: I / II

Split Up NHCs into two groups:

$\mathbf{I}^{l} = 1, 2, ..., 2$

"instantons"

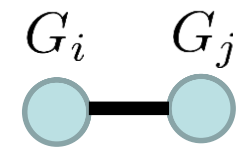
			$\frac{1}{2}\mathbf{56}$		$\mathbf{I}^3$	$\mathbf{I}^2$	$\mathbf{I}^1$	
	$\mathfrak{so}_8$	$\mathfrak{e}_6$	$\mathfrak{e}_7$	$\mathfrak{e}_7$	$\mathfrak{e}_8$	$\mathfrak{e}_8$	$\mathfrak{e}_8$	$\mathfrak{e}_8$
DE-type:	4,	6,	7,	8,	9,	10,	11,	12

non-DE-type: 1, 2, 3, 23, 232, 223, 5

Useful Terminology: II / II

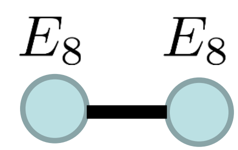
Define a Base Quiver by minimal fiber types:

Nodes: DE-type curves 

Links: Connecting DE-type curves 

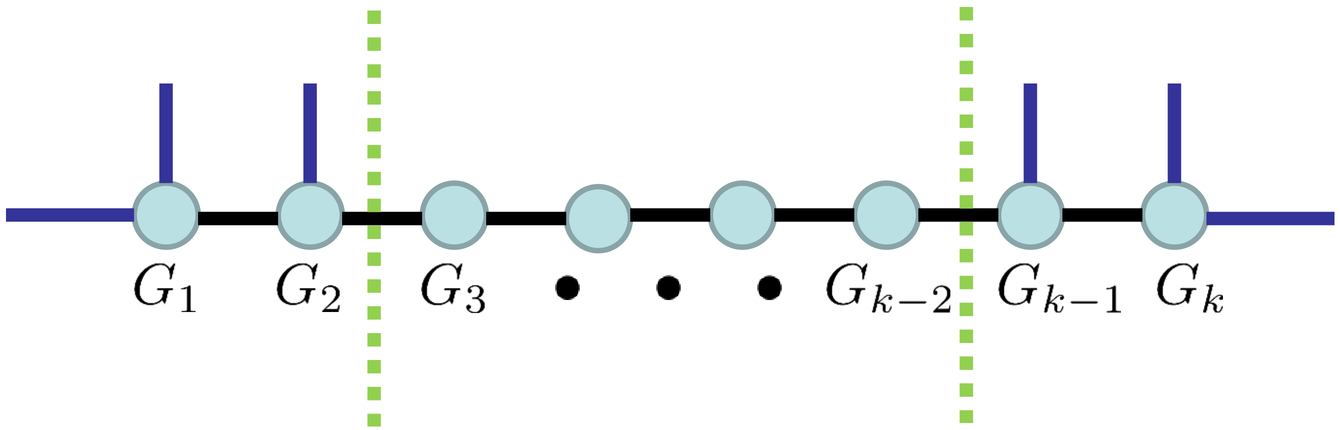
Example:

$$(12), 1, 223, 1, 5, 1, 322, 1, (12)$$



Base Quivers

The Base Quivers have a *very* simple structure!



$$G_1 \subseteq G_2 \subseteq \cdots \subseteq G_m \supseteq \cdots \supseteq G_{k-1} \supseteq G_k$$

Completeness?

There are a few outlying IIA constructions

(Hanany Zaffaroni ’96)

Can view as quotients of an F-theory geometry

(Bhardwaj, Del Zotto, JJH, Morrison, Rudelius, Vafa ’15)

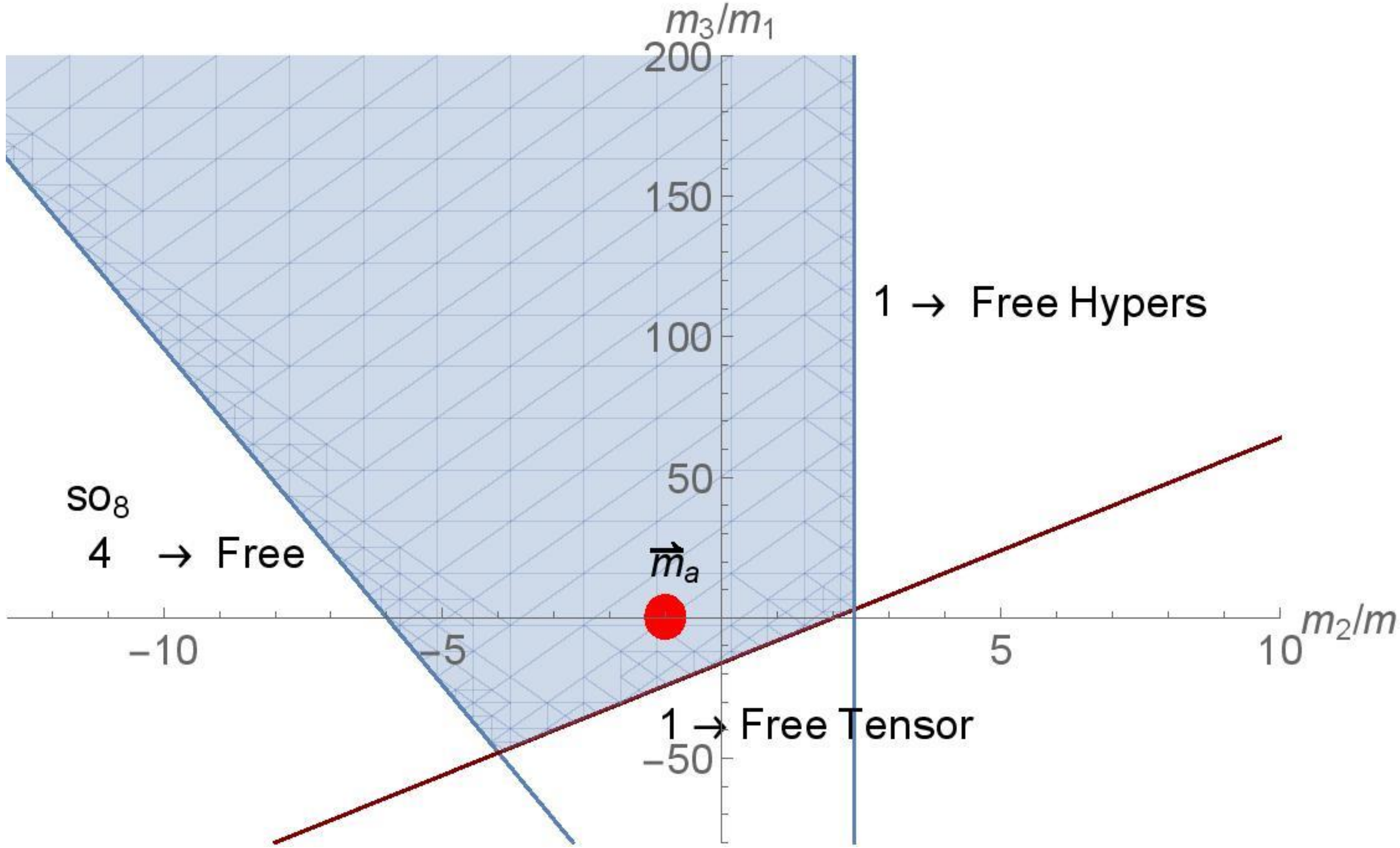
So, in this sense it really is complete

Step 2:
Classifying RG Flows

(JJH Rudelius '15)

Crude Version

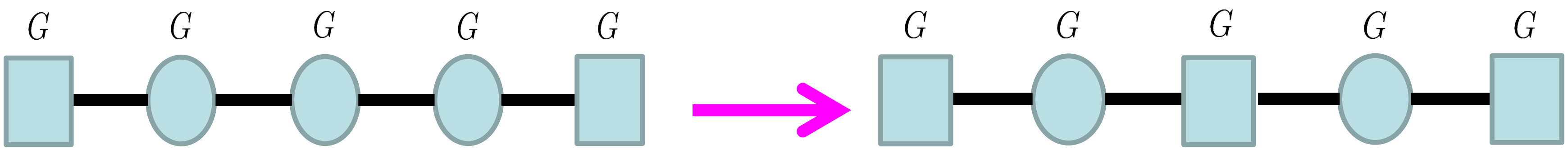
Brute Force C-theorems:



(see also Cordova, Dumitrescu, Intriligator '15)

Geometric Flows

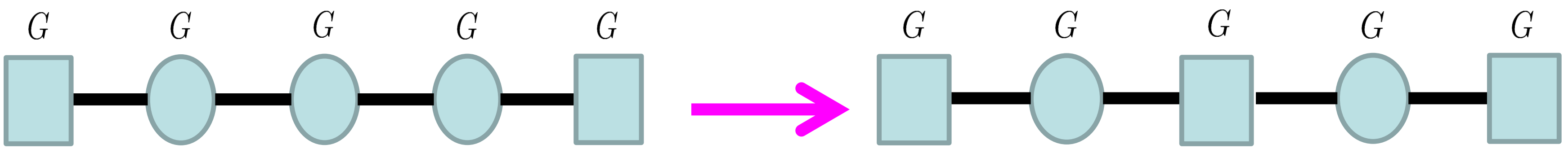
Tensor / Kähler deformations:



Higgsing / cplx deformations:

Tensor Branch Flows

Tensor / Kähler deformations:



Decoupled SCFTs + Free Vectors

(Can also cut on the links)

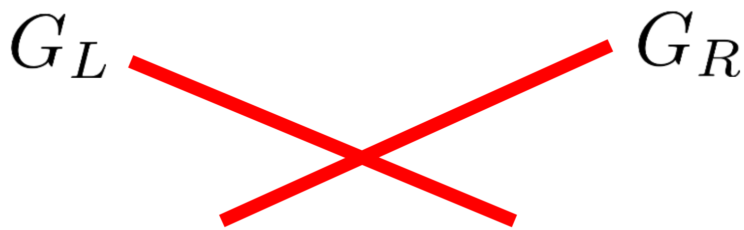
Higgs Branch Flows

Higgsing / cplx deformations:

But in what sense is this a vev for "matter"?

7-Brane Perspective

$G_L \times G_R$ Hitchin System + Sources



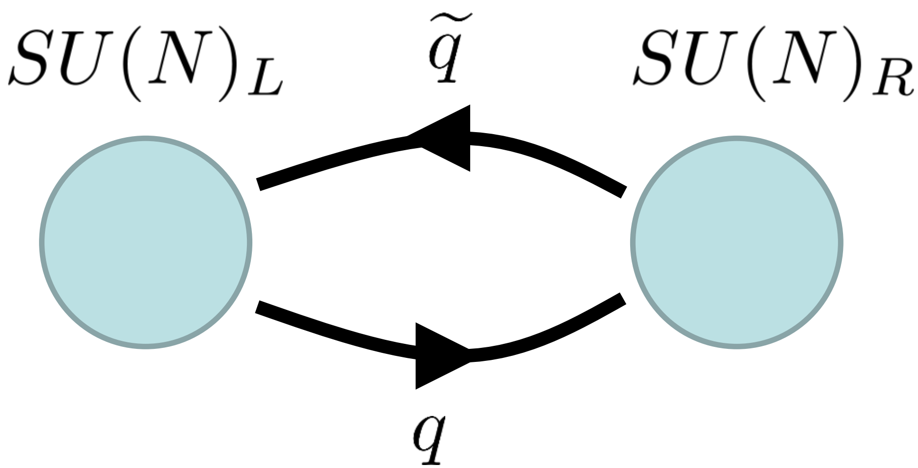
$$\overline{\partial}_A \Phi = \delta_p \, \mu_{\mathbb{C}} \qquad \overline{\partial}_{A'} \Phi' = \delta_p \, \mu'_{\mathbb{C}}$$

$$F + [\Phi, \Phi^\dagger] = \delta_p \, \mu_{\mathbb{R}} \qquad F' + [\Phi', \Phi'^\dagger] = \delta_p \, \mu'_{\mathbb{R}}$$

Higher Order Singularities: Anderson, JJH, Katz, Schaposnik (to appear)

μ's and Breaking Patterns

Example at weak coupling:



$$\mu^a_{\mathbb{R}} = q^\dagger T^a q - \widetilde{q} T^a \widetilde{q}^\dagger$$

$$\mu^a_{\mathbb{C}} = q^c T^a q$$

μ's and Breaking Patterns

Example at strong coupling:

$\mu_{\mathbb{R}} \in \mathfrak{e}_8$

$\mu_{\mathbb{C}} \in \mathfrak{e}_8 \otimes \mathbb{C}$

Main Idea: Track what is *unbroken*

Digression:

Nilpotent Orbits

Nilpotent μ

$\mu \in \mathfrak{g}_{\mathbb{C}}$ can be written as: $\mu = \mu_{\text{simple}} + \mu_{\text{nilpotent}}$

"T-Branes"

Orbit: Conjugate μ by $\text{adj}_{\mathfrak{g}}$

Focus today: $\mu_{\text{nilpotent}}$ (actually covers most flows)

Nilpotent μ

Classical Algebras: $\mu = \oplus J_{\mu_i \times \mu_i}$

$\mu \in \mathfrak{sl}_N$, Partition of $N = \sum \mu_i$

$\mathfrak{so}$ and $\mathfrak{sp}$ similar

Exceptional Algebras: "Bala-Carter Label"

$[\mu, T] = 0? \quad [\mu, T] \neq 0?$

Examples

$\mathfrak{su}(N)$: $\mu = (\mu_1^{d_1}, ..., \mu_k^{d_k})$ $\qquad$ $\mathfrak{g}_{\text{unbroken}} = \mathfrak{s}(\bigoplus \mathfrak{u}(d_i))$

$\mathfrak{e}_6 : \mu_{BC} = A_1$ $\qquad$ $\mathfrak{g}_{\text{unbroken}} = \mathfrak{su}(6)$

$\mathfrak{e}_7 : \mu_{BC} = A_1$ $\qquad$ $\mathfrak{g}_{\text{unbroken}} = \mathfrak{so}(12)$

$\mathfrak{e}_8 : \mu_{BC} = A_1$ $\qquad$ $\mathfrak{g}_{\text{unbroken}} = \mathfrak{e}_7$

Partial Ordering

"How Much is Broken?" $\Longrightarrow \mu < \nu$

Example for $\mathfrak{su}(N)$:

$$\mu = (\mu_1, ..., \mu_N),\ \nu = (\nu_1, ..., \nu_N)$$

$$\mu < \nu \Longleftrightarrow \sum_{i=1}^{k} \mu_i < \sum_{i=1}^{k} \nu_i$$

End of Digression

Nilpotent Hierarchies

Expect $\mu_{\text{nilpotent}}$ to trigger Higgs Branch Flows

$$\mu < \nu \Longrightarrow T^{UV}[\mu] \to T^{IR}[\nu]$$

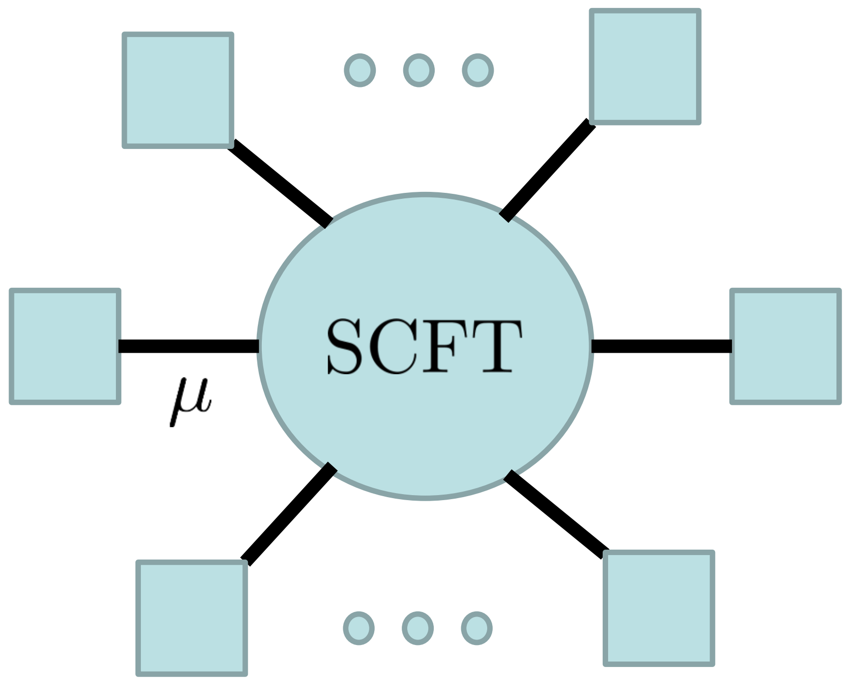
But Nilpotent Cone is fully known!

Natural Questions

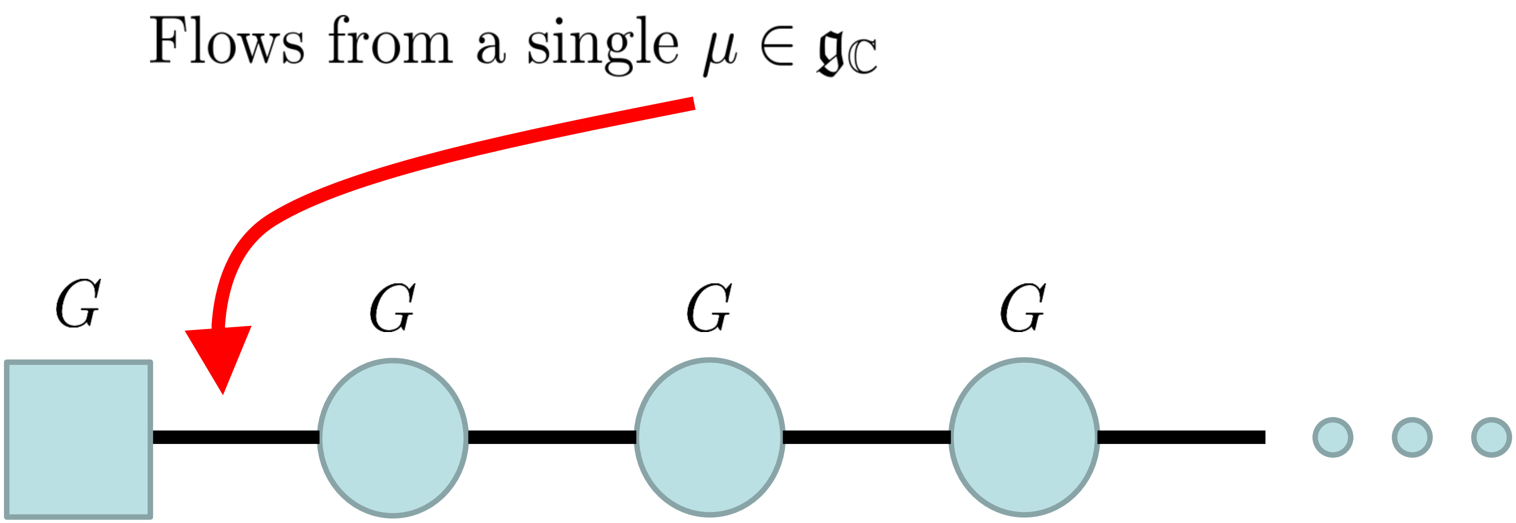
- Can we match RG Flows with partial ordering?

- Which set of μ's goes with a given CY_3???

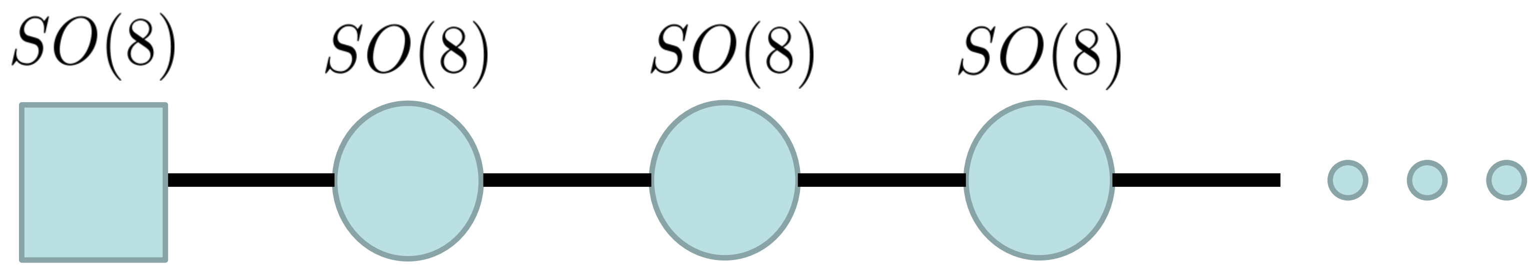
General Picture



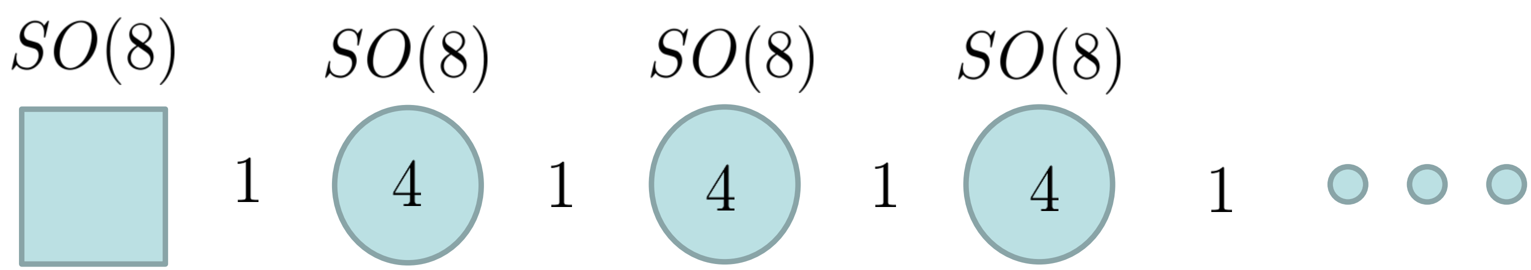
Focus for Today

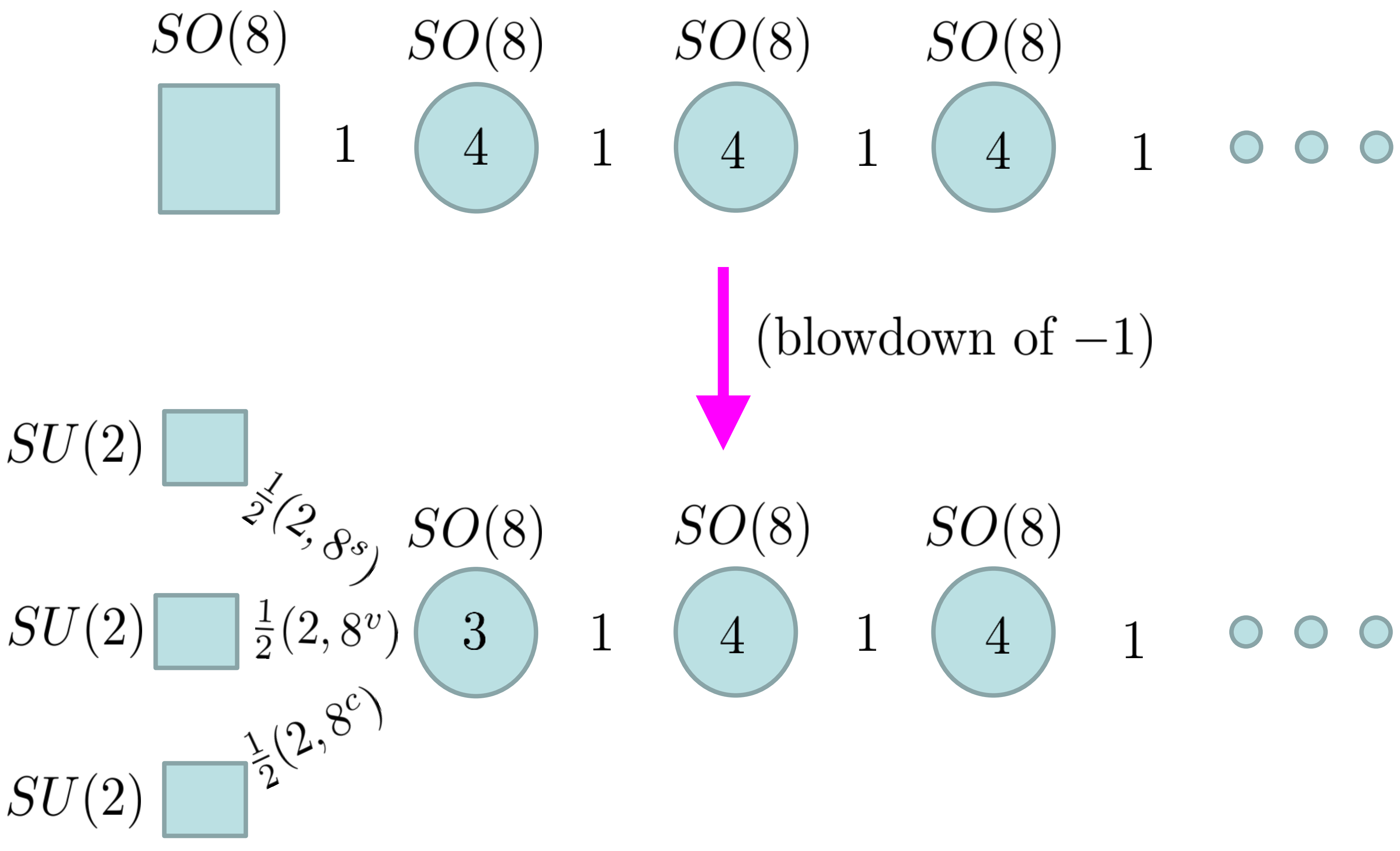


Example 1: $SO(8)$ Flows

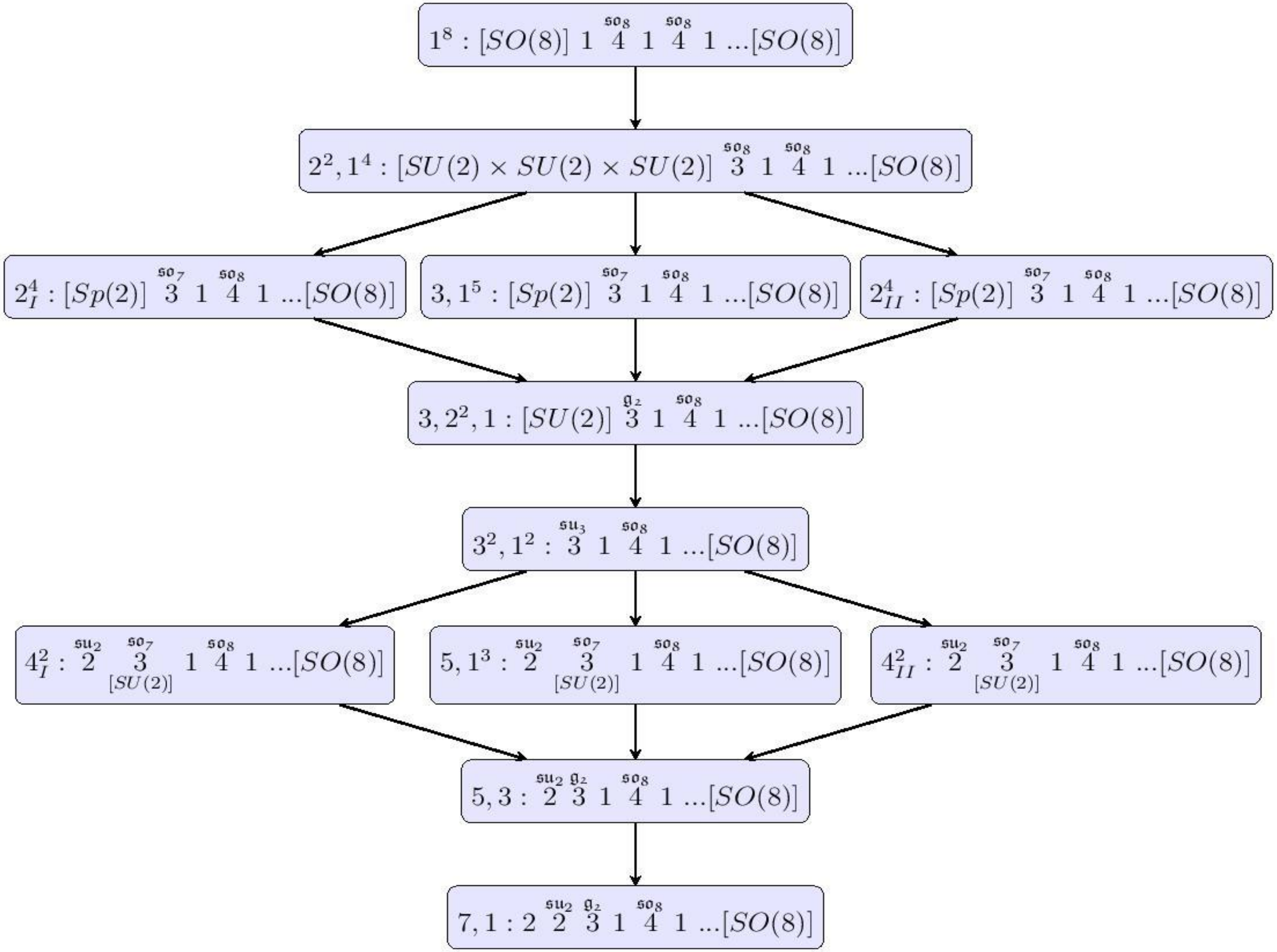


Example 1: $SO(8)$ Flows



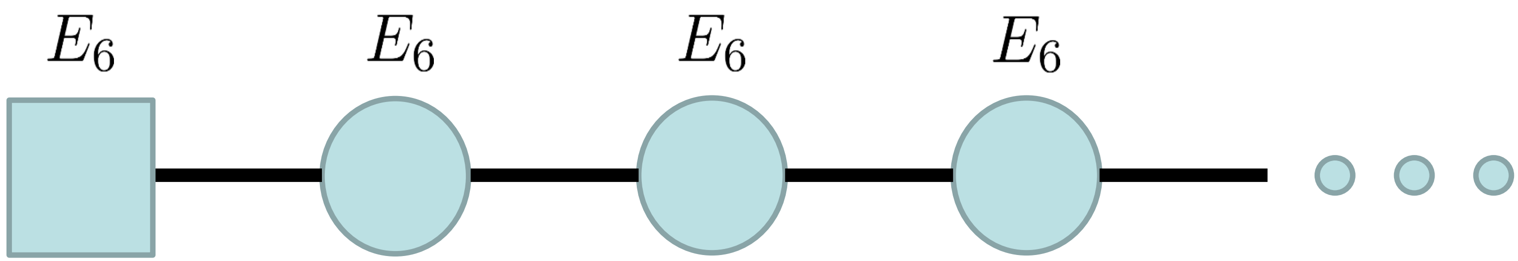


SO(8)
SO(8)
SO(8)
SO(8)
1
4
1
4
1
4
1
(blowdown of −1)
SU(2)
1/2(2, 8^s)
SO(8)
SO(8)
SO(8)
SU(2)
1/2(2, 8^v)
3
1
4
1
4
1
1/2(2, 8^c)
SU(2)

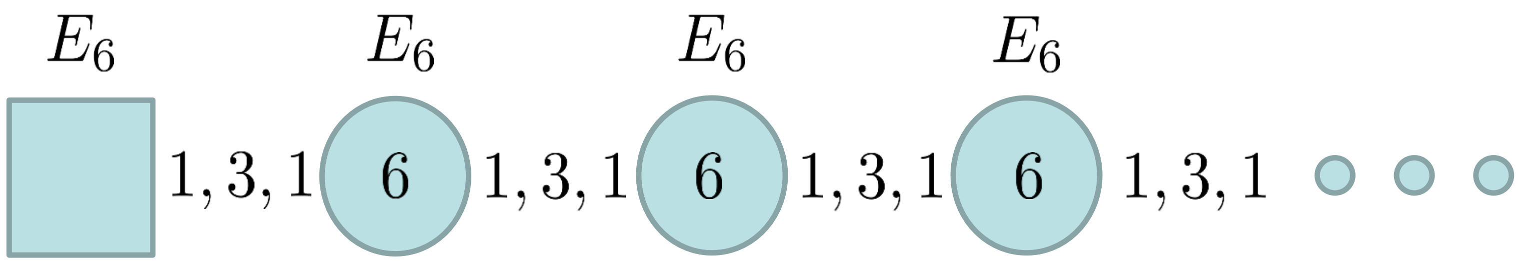


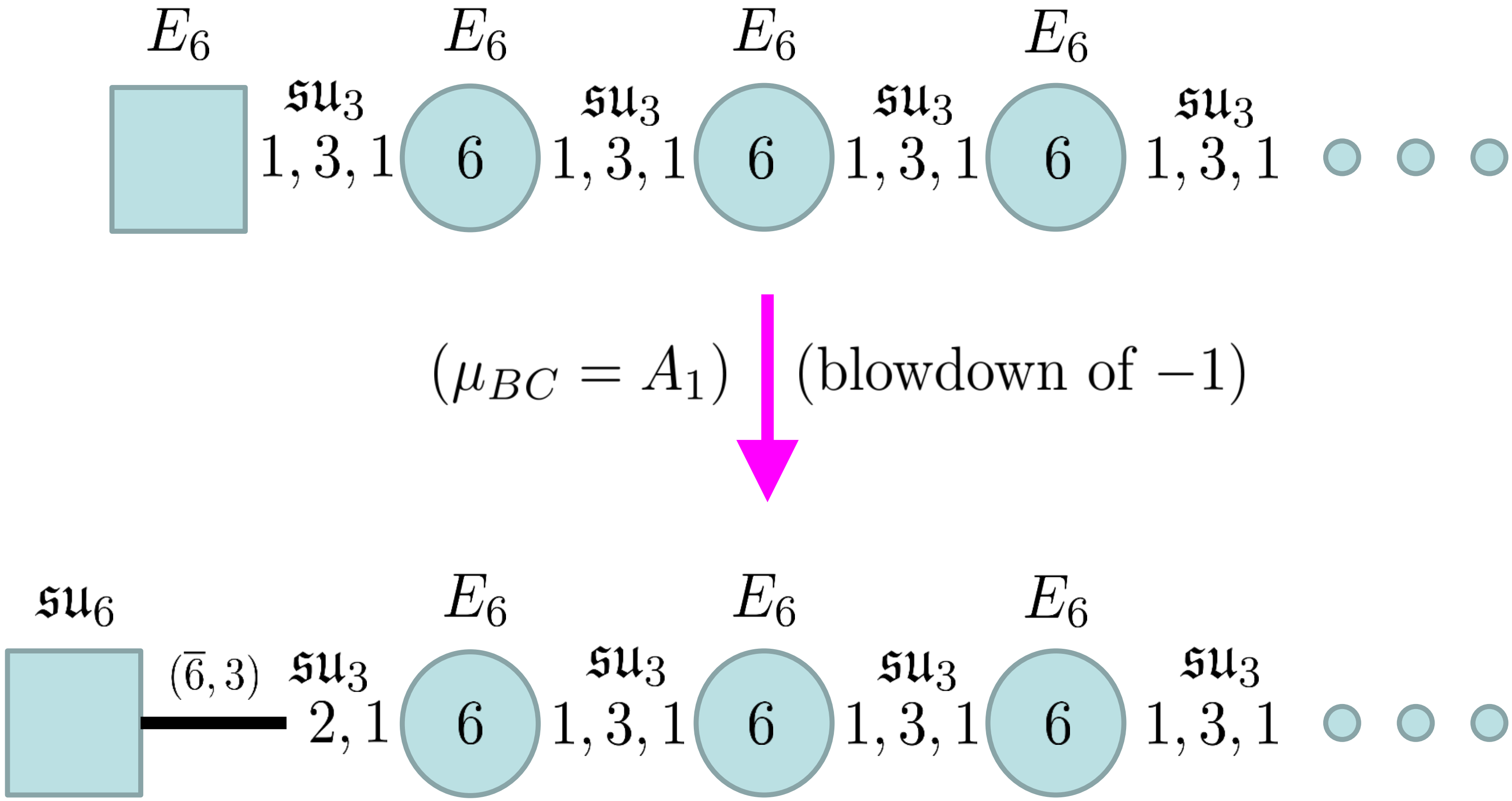
$1^8 : [SO(8)]\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$2^2, 1^4 : [SU(2) \times SU(2) \times SU(2)]\ \overset{\mathfrak{so}_8}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$2^4_I : [Sp(2)]\ \overset{\mathfrak{so}_7}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$3, 1^5 : [Sp(2)]\ \overset{\mathfrak{so}_7}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$2^4_{II} : [Sp(2)]\ \overset{\mathfrak{so}_7}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$3, 2^2, 1 : [SU(2)]\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$3^2, 1^2 : \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$4^2_I : \overset{\mathfrak{su}_2}{2}\ \underset{[SU(2)]}{\overset{\mathfrak{so}_7}{3}}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$5, 1^3 : \overset{\mathfrak{su}_2}{2}\ \underset{[SU(2)]}{\overset{\mathfrak{so}_7}{3}}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$4^2_{II} : \overset{\mathfrak{su}_2}{2}\ \underset{[SU(2)]}{\overset{\mathfrak{so}_7}{3}}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$5, 3 : \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$
$7, 1 : 2\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{so}_8}{4}\ 1\ ...[SO(8)]$

Example 2: E_6 Flows

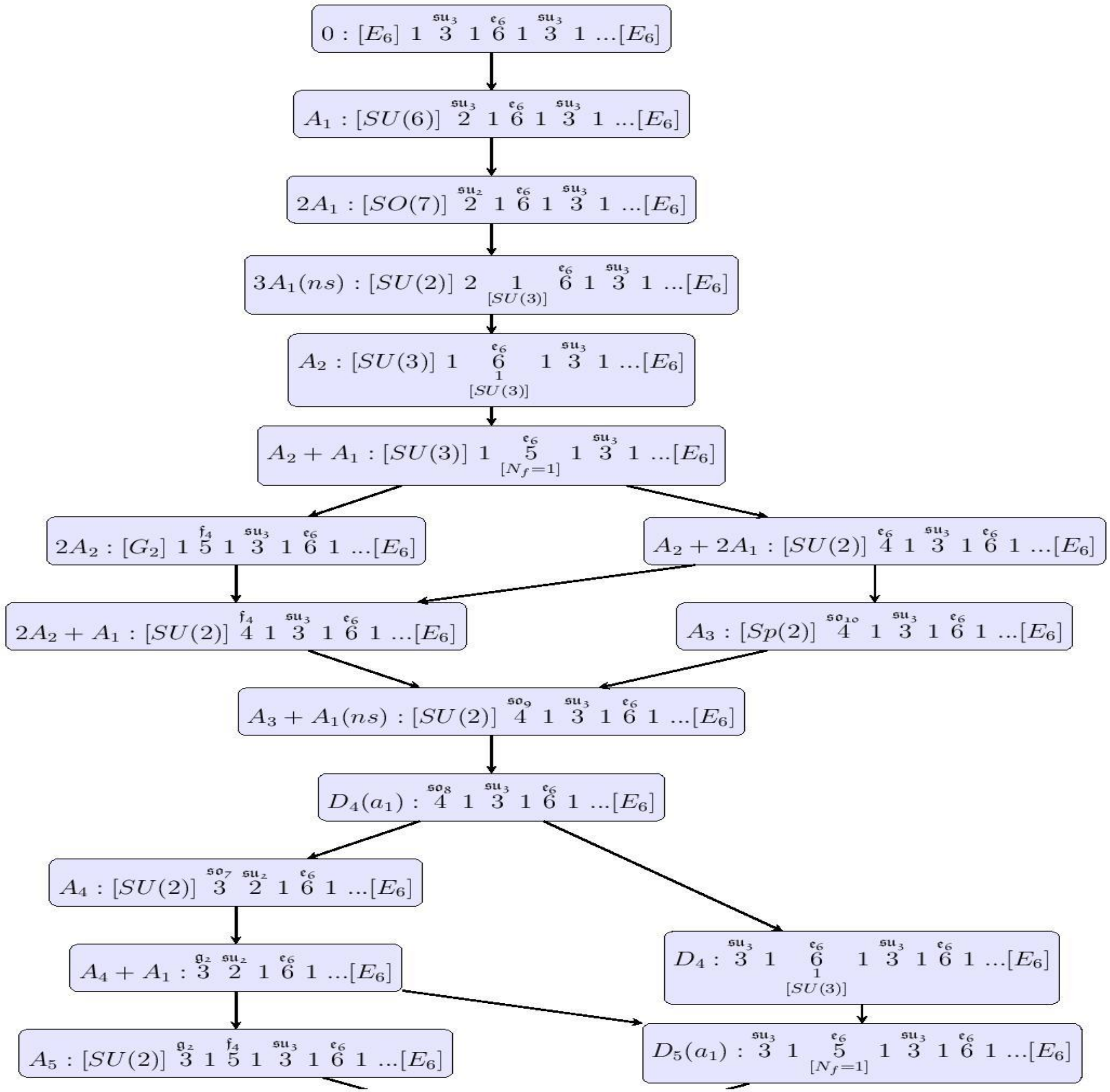


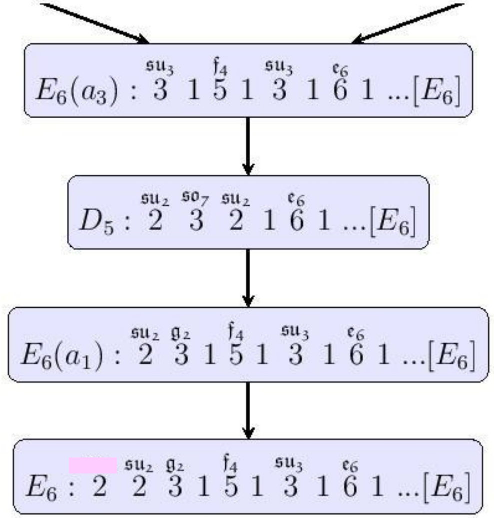
Example 2: E_6 Flows





E_6
$\mathfrak{su}_3$
1, 3, 1
E_6
6
$\mathfrak{su}_3$
1, 3, 1
E_6
6
$\mathfrak{su}_3$
1, 3, 1
E_6
6
$\mathfrak{su}_3$
1, 3, 1
$(\mu_{BC} = A_1)$
(blowdown of -1)
$\mathfrak{su}_6$
$(\overline{6}, 3)$
$\mathfrak{su}_3$
2, 1
E_6
6
$\mathfrak{su}_3$
1, 3, 1
E_6
6
$\mathfrak{su}_3$
1, 3, 1
E_6
6
$\mathfrak{su}_3$
1, 3, 1


$0 : [E_6]\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ ...[E_6]$
$A_1 : [SU(6)]\ \overset{\mathfrak{su}_3}{2}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ ...[E_6]$
$2A_1 : [SO(7)]\ \overset{\mathfrak{su}_2}{2}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ ...[E_6]$
$3A_1(ns) : [SU(2)]\ 2\ \underset{[SU(3)]}{1}\ \overset{\mathfrak{e}_6}{6}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ ...[E_6]$
$A_2 : [SU(3)]\ 1\ \underset{[SU(3)]}{\underset{1}{\overset{\mathfrak{e}_6}{6}}}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ ...[E_6]$
$A_2 + A_1 : [SU(3)]\ 1\ \underset{[N_f=1]}{\overset{\mathfrak{e}_6}{5}}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ ...[E_6]$
$2A_2 : [G_2]\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$
$A_2 + 2A_1 : [SU(2)]\ \overset{\mathfrak{e}_6}{4}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$
$2A_2 + A_1 : [SU(2)]\ \overset{\mathfrak{f}_4}{4}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$
$A_3 : [Sp(2)]\ \overset{\mathfrak{so}_{10}}{4}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$
$A_3 + A_1(ns) : [SU(2)]\ \overset{\mathfrak{so}_9}{4}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$
$D_4(a_1) : \overset{\mathfrak{so}_8}{4}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$
$A_4 : [SU(2)]\ \overset{\mathfrak{so}_7}{3}\ \overset{\mathfrak{su}_2}{2}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$
$D_4 : \overset{\mathfrak{su}_3}{3}\ 1\ \underset{[SU(3)]}{\underset{1}{\overset{\mathfrak{e}_6}{6}}}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$
$A_4 + A_1 : \overset{\mathfrak{g}_2}{3}\ \overset{\mathfrak{su}_2}{2}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$
$A_5 : [SU(2)]\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$
$D_5(a_1) : \overset{\mathfrak{su}_3}{3}\ 1\ \underset{[N_f=1]}{\overset{\mathfrak{e}_6}{5}}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$


$E_6(a_3) : \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$
$D_5 : \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{so}_7}{3}\ \overset{\mathfrak{su}_2}{2}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$
$E_6(a_1) : \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$
$E_6 : 2\ \overset{\mathfrak{su}_2}{2}\ \overset{\mathfrak{g}_2}{3}\ 1\ \overset{\mathfrak{f}_4}{5}\ 1\ \overset{\mathfrak{su}_3}{3}\ 1\ \overset{\mathfrak{e}_6}{6}\ 1\ ...[E_6]$

Summary of Results

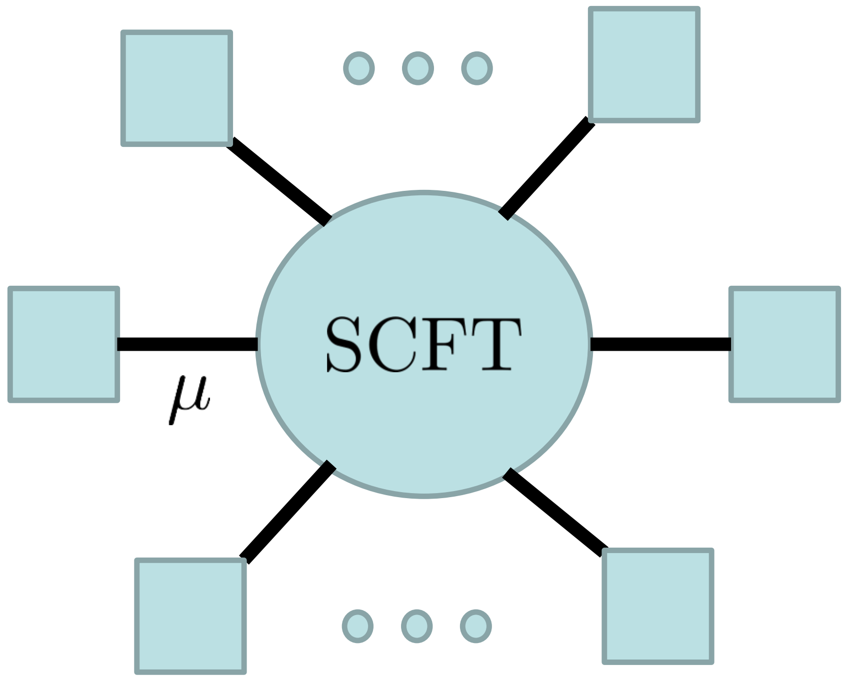
- Perfect Match of μ ordering to 6D RG Flows

- Carried out explicitly for $\mathfrak{g}_{\mathbb{C}} = ABCDEFG$

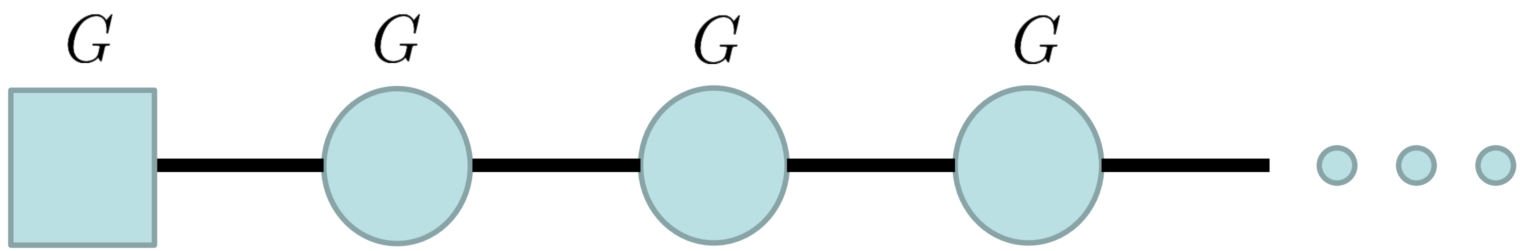
- Calculate Flavor Symmetries (including $\mathfrak{u}(1)$'s)

  Hard to do geometrically! (c.f. Bertolini, Merkx, Morrison '15)

Future Directions (I / II)



Future Directions (II / II)



Generalized Quiver Representations...

Non-Perturbative Extension of B-Branes?

Generalized Θ/Π/slope-Stability?