F-theory at terminal singularities

Iñaki García-Etxebarria

MAX-PLANCK-GESELLSCHAFT

Max-Planck-Institut für Physik
(Werner-Heisenberg-Institut)

in collaboration with D. Regalado

arXiv:1512.06434

Probing rigid singularities

We will be interested in understanding the four dimensional physics coming from (probe D3 branes on) F-theory compactifications in the presence of singularities that do no admit supersymmetric smoothings. I.e. they cannot be resolved or deformed into a smooth space without spending energy.

Probing rigid singularities

We will be interested in understanding the four dimensional physics coming from (probe D3 branes on) F-theory compactifications in the presence of singularities that do no admit supersymmetric smoothings. I.e. they cannot be resolved or deformed into a smooth space without spending energy.

- Complex codimension 4 Calabi-Yau singularity in a geometry with a F-theory limit. There are many such geometries, and we will only scratch the surface.

Probing rigid singularities

We will be interested in understanding the four dimensional physics coming from (probe D3 branes on) F-theory compactifications in the presence of singularities that do no admit supersymmetric smoothings. I.e. they cannot be resolved or deformed into a smooth space without spending energy.

- Complex codimension 4 Calabi-Yau singularity in a geometry with a F-theory limit. There are many such geometries, and we will only scratch the surface.
- Simplest case: $\mathbb{Z}_k$ orbifolds of $\mathbb{C}^3 \times T^2$, with non-trivial T^2 action and isolated fixed points.

(Such orbifolds have appeared for two-folds [Dasgupta, Mukhi '96] and threefolds [Witten '96], but in these cases they are deformable.)

Summary of results

Calabi-Yau fourfolds of the form $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$ can be classified completely: the orbifold actions preserving susy were classified in [Morrison, Stevens '84], [Anno '03], [Font, López '04]. We focus on the cases preserving at least 12 supercharges.

Summary of results

Calabi-Yau fourfolds of the form $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$ can be classified completely: the orbifold actions preserving susy were classified in [Morrison, Stevens '84], [Anno '03], [Font, López '04]. We focus on the cases preserving at least 12 supercharges.

In the F-theory limit, adding D3 brane probes:

- $k = 1$ gives IIB string theory → 4d $U(N)$ $\mathcal{N} = 4$ SYM.
- $k = 2$ gives IIB w/ O3 plane → 4d $\mathcal{N} = 4$ SYM w/ orthogonal or symplectic groups.

Summary of results

Calabi-Yau fourfolds of the form $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$ can be classified completely: the orbifold actions preserving susy were classified in [Morrison, Stevens '84], [Anno '03], [Font, López '04]. We focus on the cases preserving at least 12 supercharges.

In the F-theory limit, adding D3 brane probes:

- $k = 1$ gives IIB string theory → 4d $U(N)$ $\mathcal{N} = 4$ SYM.
- $k = 2$ gives IIB w/ O3 plane → 4d $\mathcal{N} = 4$ SYM w/ orthogonal or symplectic groups. (Locally $\mathbb{C}^4/\mathbb{Z}_2$, so at least in some cases such rigid singularities make perfect physical sense.)

Summary of results

Calabi-Yau fourfolds of the form $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$ can be classified completely: the orbifold actions preserving susy were classified in [Morrison, Stevens '84], [Anno '03], [Font, López '04]. We focus on the cases preserving at least 12 supercharges.

In the F-theory limit, adding D3 brane probes:

- $k = 1$ gives IIB string theory → 4d $U(N)$ $\mathcal{N} = 4$ SYM.
- $k = 2$ gives IIB w/ O3 plane → 4d $\mathcal{N} = 4$ SYM w/ orthogonal or symplectic groups. (Locally $\mathbb{C}^4/\mathbb{Z}_2$, so at least in some cases such rigid singularities make perfect physical sense.)
- $k = 3, 4, 6$ give IIB w/ exotic "OF3" plane → 4d $\mathcal{N} = 3$ SCFTs.

Summary of results

Calabi-Yau fourfolds of the form $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_k$ can be classified completely: the orbifold actions preserving susy were classified in [Morrison, Stevens '84], [Anno '03], [Font, López '04]. We focus on the cases preserving at least 12 supercharges.

In the F-theory limit, adding D3 brane probes:

- $k = 1$ gives IIB string theory $\rightarrow$ 4d $U(N)$ $\mathcal{N} = 4$ SYM.
- $k = 2$ gives IIB w/ O3 plane $\rightarrow$ 4d $\mathcal{N} = 4$ SYM w/ orthogonal or symplectic groups. (Locally $\mathbb{C}^4/\mathbb{Z}_2$, so at least in some cases such rigid singularities make perfect physical sense.)
- $k = 3, 4, 6$ give IIB w/ exotic "OF3" plane $\rightarrow$ 4d $\mathcal{N} = 3$ SCFTs.
  - [Ferrara, Porrati, Zaffaroni '98] propose a construction of exotic AdS_5 holographic backgrounds preserving $\mathcal{N} = 6$, similar to the expected form of the holographic dual of the $\mathcal{N} = 3$ SCFTs we find.
  - In [Aharony, Evtikhiev '15] some properties of these theories were understood, assuming they existed, but no construction was known.

Outline

EYAWTK about the O3 plane

It will prove very illuminating to revisit the O3 plane (i.e. $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$) from multiple viewpoints, since it is the simplest case of a complex codimension four singularity with a F-theory lift, and is relatively well understood.

- Worldsheet CFT.
- F/M-theory.
- Holographic picture.
- Field theory.

EYAWTK about the O3 plane

It will prove very illuminating to revisit the O3 plane (i.e. $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$) from multiple viewpoints, since it is the simplest case of a complex codimension four singularity with a F-theory lift, and is relatively well understood.

- Worldsheet CFT.
- **F/M theory.**
- **Holographic picture.**
- **Field theory.**

Everything but the CFT approach potentially generalizes to $k = 3, 4, 6$.

Worldsheet description of the O3 plane

We start with IIB string theory on $\mathbb{R}^{10} = \mathbb{R}^4 \times \mathbb{C}^3$, and quotient by $\mathcal{I}(-1)^{F_L}\Omega$. Here $\mathcal{I}$ acts as reflection on the $\mathbb{C}^3$:

$$\mathcal{I}\colon (x, y, z) \to (-x, -y, -z) \tag{1}$$

while $(-1)^{F_L}\Omega$ acts on the worlsheet. Its induced effect on the spacetime fields is easily computed, for instance

$$(-1)^{F_L}\Omega\colon \begin{pmatrix} B_2 \\ C_2 \end{pmatrix} \to \begin{pmatrix} -B_2 \\ -C_2 \end{pmatrix} \tag{2}$$

Worldsheet description of the O3 plane

We start with IIB string theory on $\mathbb{R}^{10} = \mathbb{R}^4 \times \mathbb{C}^3$, and quotient by $\mathcal{I}(-1)^{F_L}\Omega$. Here $\mathcal{I}$ acts as reflection on the $\mathbb{C}^3$:

$$\mathcal{I}\colon (x, y, z) \to (-x, -y, -z) \tag{1}$$

while $(-1)^{F_L}\Omega$ acts on the worlsheet. Its induced effect on the spacetime fields is easily computed, for instance

$$(-1)^{F_L}\Omega\colon \begin{pmatrix} B_2 \\ C_2 \end{pmatrix} \to \begin{pmatrix} -B_2 \\ -C_2 \end{pmatrix} \tag{2}$$

If we have a stack of N D3 branes we need to choose an action on the Chan-Paton factors, which will project $U(N)$ down to an orthogonal or symplectic group:

$O3^-$ $\quad$ $\widetilde{O3}^-$ $\quad$ $O3^+$ $\quad$ $\widetilde{O3}^+$

Last three are related by Montonen-Olive duality. [Witten '98])

F(M)-theory description of the O3 plane

IIB without orientifold is given by M-theory on T^2 in the $\mathrm{vol}(T^2) \to 0$ limit, we wish to quotient this by the lift of $\mathcal{I}(-1)^{F_L}\Omega$.

F(M)-theory description of the O3 plane

IIB without orientifold is given by M-theory on T^2 in the $\mathrm{vol}(T^2) \to 0$ limit, we wish to quotient this by the lift of $\mathcal{I}(-1)^{F_L}\Omega$.

The $\mathcal{I}$ action on the IIB coordinates lifts trivially to a $\mathcal{I}$ action on six of the M-theory coordinates: $(x, y, z) \to (-x, -y, -z)$.

F(M)-theory description of the O3 plane

IIB without orientifold is given by M-theory on T^2 in the $\mathrm{vol}(T^2) \to 0$ limit, we wish to quotient this by the lift of $\mathcal{I}(-1)^{F_L}\Omega$.

The $\mathcal{I}$ action on the IIB coordinates lifts trivially to a $\mathcal{I}$ action on six of the M-theory coordinates: $(x, y, z) \to (-x, -y, -z)$.

The $(-1)^{F_L}\Omega$ action acts as

$$(-1)^{F_L}\Omega\colon \begin{pmatrix} B_2 \\ C_2 \end{pmatrix} \to \begin{pmatrix} -B_2 \\ -C_2 \end{pmatrix} \tag{3}$$

which when rewritten in terms of C_3 implies that

$$(-1)^{F_L}\Omega\colon (p, q) \to (-p, -q) \tag{4}$$

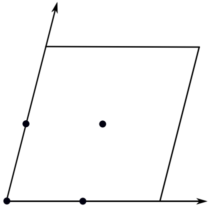
i.e. an inversion of the T^2: $u \to -u$. (Denoted by $-1 \in SL(2, \mathbb{Z})$)

F(M)-theory description of the O3 plane

Writing x, y, z, u for the $\mathbb{C}^3 \times T^2$ coordinates acted upon by the involution, we thus find

$$\mathcal{I}(-1)^{F_L}\Omega\colon (x,y,z,u) \to (-x,-y,-z,-u)$$

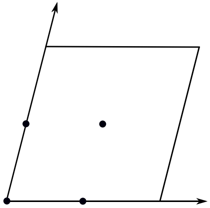
and the total geometry is $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$. This has four fixed points at $(x,y,z,u) = (0,0,0,p)$, with p a fixed point of the T^2 under the $\mathbb{Z}_2$.



F(M)-theory description of the O3 plane

Writing x, y, z, u for the $\mathbb{C}^3 \times T^2$ coordinates acted upon by the involution, we thus find

$$\mathcal{I}(-1)^{F_L}\Omega\colon (x, y, z, u) \to (-x, -y, -z, -u)$$



and the total geometry is $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$. This has four fixed points at $(x, y, z, u) = (0, 0, 0, p)$, with p a fixed point of the T^2 under the $\mathbb{Z}_2$.

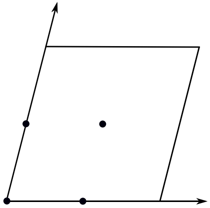
Various observations:

- The involution exists for any value of τ.

F(M)-theory description of the O3 plane

Writing x, y, z, u for the $\mathbb{C}^3 \times T^2$ coordinates acted upon by the involution, we thus find

$$\mathcal{I}(-1)^{F_L}\Omega \colon (x, y, z, u) \to (-x, -y, -z, -u)$$



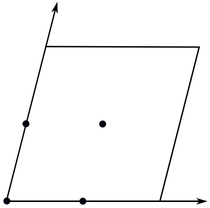
and the total geometry is $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$. This has four fixed points at $(x, y, z, u) = (0, 0, 0, p)$, with p a fixed point of the T^2 under the $\mathbb{Z}_2$.

Various observations:

- The involution exists for any value of τ.
- Close to each fixed point we have $\mathbb{C}^4/\mathbb{Z}_2$: this cannot be smoothed out in a CY way [Schlessinger '71] [Morrison, Plesser '98]. This agrees with the fact that the O3 has no light modes on it.

F(M)-theory description of the O3 plane

Writing x, y, z, u for the $\mathbb{C}^3 \times T^2$ coordinates acted upon by the involution, we thus find



$$\mathcal{I}(-1)^{F_L}\Omega\colon (x, y, z, u) \to (-x, -y, -z, -u)$$

and the total geometry is $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$. This has four fixed points at $(x, y, z, u) = (0, 0, 0, p)$, with p a fixed point of the T^2 under the $\mathbb{Z}_2$.

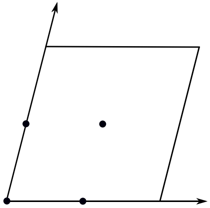
Various observations:

- The involution exists for any value of τ.
- Close to each fixed point we have $\mathbb{C}^4/\mathbb{Z}_2$: this cannot be smoothed out in a CY way [Schlessinger '71] [Morrison, Plesser '98]. This agrees with the fact that the O3 has no light modes on it.
- M2 branes probing $\mathbb{C}^4/\mathbb{Z}_k$: [Aharony, Bergman, Jafferis, Maldacena '08].

F(M)-theory description of the O3 plane

Writing x, y, z, u for the $\mathbb{C}^3 \times T^2$ coordinates acted upon by the involution, we thus find

$$\mathcal{I}(-1)^{F_L}\Omega\colon (x, y, z, u) \to (-x, -y, -z, -u)$$



and the total geometry is $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$. This has four fixed points at $(x, y, z, u) = (0, 0, 0, p)$, with p a fixed point of the T^2 under the $\mathbb{Z}_2$.

Various observations:

- The involution exists for any value of τ.
- Close to each fixed point we have $\mathbb{C}^4/\mathbb{Z}_2$: this cannot be smoothed out in a CY way [Schlessinger '71] [Morrison, Plesser '98]. This agrees with the fact that the O3 has no light modes on it.
- M2 branes probing $\mathbb{C}^4/\mathbb{Z}_k$: [Aharony, Bergman, Jafferis, Maldacena '08].
- Different O3 types: different discrete fluxes on the fixed points [Hanany, Kol '00].

F(IIB)-theory description of the O3 plane

A holography appetizer

In IIB string theory the $\mathbb{C}^3/\mathcal{I}$ orbifold is non-supersymmetric, while the O3 preserves 16 supercharges. I discuss the near horizon geometry, $AdS_5 \times (S^5/\mathbb{Z}_2)$, which naively is non-supersymmetric.

F(IIB)-theory description of the O3 plane

A holography appetizer

In IIB string theory the $\mathbb{C}^3/\mathcal{I}$ orbifold is non-supersymmetric, while the O3 preserves 16 supercharges. I discuss the near horizon geometry, $AdS_5 \times (S^5/\mathbb{Z}_2)$, which naively is non-supersymmetric.

From the M-theory picture, it is clear what is going on: near horizon what we have is F-theory on $AdS_5 \times ((S^5 \times T^2)/\mathbb{Z}_2)$, i.e. a non-trivial $SL(2,\mathbb{Z})$ bundle on the $S^5/\mathbb{Z}_2$ horizon.

So we do not have the vanilla orbifold, but in addition it has a non-trivial flat $SL(2,\mathbb{Z})$ duality bundle on top, acting with $-1 \in SL(2,\mathbb{Z})$ as we go round the non-trivial one-cycle in the $S^5/\mathbb{Z}_2$ horizon manifold. One can check that the $-1 \in SL(2,\mathbb{Z})$ acting on the sugra spinors restores susy as expected.

F(IIB)-theory description of the O3 plane

A holography appetizer

In IIB string theory the $\mathbb{C}^3/\mathcal{I}$ orbifold is non-supersymmetric, while the O3 preserves 16 supercharges. I discuss the near horizon geometry, $AdS_5 \times (S^5/\mathbb{Z}_2)$, which naively is non-supersymmetric.

From the M-theory picture, it is clear what is going on: near horizon what we have is F-theory on $AdS_5 \times ((S^5 \times T^2)/\mathbb{Z}_2)$, i.e. a non-trivial $SL(2,\mathbb{Z})$ bundle on the $S^5/\mathbb{Z}_2$ horizon.

So we do not have the vanilla orbifold, but in addition it has a non-trivial flat $SL(2,\mathbb{Z})$ duality bundle on top, acting with $-1 \in SL(2,\mathbb{Z})$ as we go round the non-trivial one-cycle in the $S^5/\mathbb{Z}_2$ horizon manifold. One can check that the $-1 \in SL(2,\mathbb{Z})$ acting on the sugra spinors restores susy as expected.

The different kinds of orientifolds in this language are classified by discrete flux: $[H_3], [F_3] \in H^3(S^5/\mathbb{Z}_2, \widetilde{\mathbb{Z}}) = \mathbb{Z}_2$. [Witten '98]

Field theory description of the quotient

A stack of N D3 branes in flat space gives 4d $\mathcal{N} = 4$ $U(N)$ SYM.

Field theory description of the quotient

A stack of N D3 branes in flat space gives 4d $\mathcal{N} = 4$ $U(N)$ SYM.

Rotations in the transverse $\mathbb{R}^6$ manifest themselves as the $SU(4)_R$ R-symmetry group. Implies that $\mathcal{I}$ acts as $-1 \in SO(6)_R$ in the field theory.

Field theory description of the quotient

A stack of N D3 branes in flat space gives 4d $\mathcal{N} = 4$ $U(N)$ SYM.

Rotations in the transverse $\mathbb{R}^6$ manifest themselves as the $SU(4)_R$ R-symmetry group. Implies that $\mathcal{I}$ acts as $-1 \in SO(6)_R$ in the field theory.

Similarly, IIB $SL(2,\mathbb{Z})$ descends straightforwardly to the $SL(2,\mathbb{Z})$ duality group of the field theory. In particular

$$-1 \in SL(2,\mathbb{Z})^{\text{IIB}} \to -1 \in SL(2,\mathbb{Z})^{\mathcal{N}=4} \tag{5}$$

Field theory description of the quotient

A stack of N D3 branes in flat space gives 4d $\mathcal{N}=4$ $U(N)$ SYM.

Rotations in the transverse $\mathbb{R}^6$ manifest themselves as the $SU(4)_R$ R-symmetry group. Implies that $\mathcal{I}$ acts as $-1 \in SO(6)_R$ in the field theory.

Similarly, IIB $SL(2,\mathbb{Z})$ descends straightforwardly to the $SL(2,\mathbb{Z})$ duality group of the field theory. In particular

$$-1 \in SL(2,\mathbb{Z})^{\text{IIB}} \to -1 \in SL(2,\mathbb{Z})^{\mathcal{N}=4} \tag{5}$$

A generic element of $SL(2,\mathbb{Z})^{\mathcal{N}=4}$ is *not* a symmetry, but -1 *is*: $(-1)(\tau) = \frac{-1\cdot\tau+0}{0\cdot\tau-1} = \tau$.

Field theory description of the quotient

A stack of N D3 branes in flat space gives 4d $\mathcal{N}=4$ $U(N)$ SYM.

Rotations in the transverse $\mathbb{R}^6$ manifest themselves as the $SU(4)_R$ R-symmetry group. Implies that $\mathcal{I}$ acts as $-1 \in SO(6)_R$ in the field theory.

Similarly, IIB $SL(2,\mathbb{Z})$ descends straightforwardly to the $SL(2,\mathbb{Z})$ duality group of the field theory. In particular

$$-1 \in SL(2,\mathbb{Z})^{\text{IIB}} \to -1 \in SL(2,\mathbb{Z})^{\mathcal{N}=4} \tag{5}$$

A generic element of $SL(2,\mathbb{Z})^{\mathcal{N}=4}$ is *not* a symmetry, but -1 *is*: $(-1)(\tau) = \frac{-1\cdot\tau+0}{0\cdot\tau-1} = \tau$.

So we can understand the orientifold projection as a quotient by a particular symmetry of $\mathcal{N}=4$ $U(N)$ SYM: $U(N)/(\mathbb{Z}_2^R \cdot \mathbb{Z}_2^{SL(2,\mathbb{Z})})$. (In this language we also have a choice of Chan-Paton factors.)

Recap and strategy

We have discussed four ways of viewing the action of an O3 plane on a stack of D3 branes:

- Worldsheet CFT: a projection of the CFT by $\mathcal{I}(-1)^{F_L}\Omega$, with a choice of Chan-Paton factors.
- M-theory: M2 branes probing $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$, with a choice of discrete torsion on the fixed points.
- IIB holography: An orbifold $AdS_5 \times (S^5/\mathcal{I})$ with a nontrivial flat $SL(2,\mathbb{Z})$ bundle, and choice of discrete $[F],[H]$ flux.
- Field theory: A quotient of $U(N)$ SYM by $(\mathbb{Z}_2^R \cdot \mathbb{Z}_2^{SL(2,\mathbb{Z})})$, with a choice of Chan-Paton factors.

Recap and strategy

We have discussed four ways of viewing the action of an O3 plane on a stack of D3 branes:

- Worldsheet CFT: a projection of the CFT by $\mathcal{I}(-1)^{F_L}\Omega$, with a choice of Chan-Paton factors.
- M-theory: M2 branes probing $(\mathbb{C}^3 \times T^2)/\mathbb{Z}_2$, with a choice of discrete torsion on the fixed points.
- IIB holography: An orbifold $AdS_5 \times (S^5/\mathcal{I})$ with a nontrivial flat $SL(2,\mathbb{Z})$ bundle, and choice of discrete $[F], [H]$ flux.
- Field theory: A quotient of $U(N)$ SYM by $(\mathbb{Z}_2^R \cdot \mathbb{Z}_2^{SL(2,\mathbb{Z})})$, with a choice of Chan-Paton factors.

Strategy for generalization

Quotient by other possible symmetries of $\mathbb{C}^3 \times T^2$, S^5 or $U(N)$.

The generalization of the CFT approach seems less obvious.

OF3 planes from M-theory

We start by considering the M-theory picture, given by $\mathbb{Z}_k$ $(k > 2)$ quotients of $\mathbb{C}^3 \times T^2$ leaving isolated fixed points. It turns out that maximal supersymmetry ($\mathcal{N} = 3$) is preserved only for $k = 3, 4, 6$, with action [Font, López '04]

$$(x, y, z, u) \to (\omega_k x, \omega_k^{-1} y, \omega_k z, \omega_k^{-1} u) \tag{6}$$

with $\omega_k = \exp(2\pi i/k)$. (These are known to be terminal Gorenstein [Morrison, Stevens '84].) We focus on these.

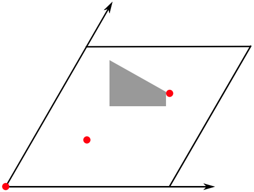
OF3 planes from M-theory

We start by considering the M-theory picture, given by $\mathbb{Z}_k$ $(k > 2)$ quotients of $\mathbb{C}^3 \times T^2$ leaving isolated fixed points. It turns out that maximal supersymmetry ($\mathcal{N} = 3$) is preserved only for $k = 3, 4, 6$, with action [Font, López '04]

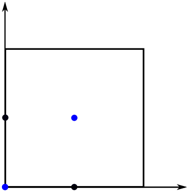
$$(x, y, z, u) \to (\omega_k x, \omega_k^{-1} y, \omega_k z, \omega_k^{-1} u) \qquad (6)$$

with $\omega_k = \exp(2\pi i/k)$. (These are known to be terminal Gorenstein [Morrison, Stevens '84].) We focus on these.

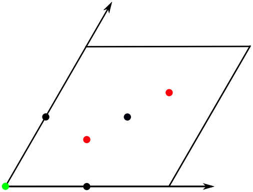
This action only maps the torus to itself for specific complex structures:



$\mathbb{Z}_3$: $\tau = e^{2\pi i/3}$
Three $\mathbb{C}/\mathbb{Z}_3$ points.



$\mathbb{Z}_4$: $\tau = i$
One $\mathbb{Z}_2$ and two $\mathbb{Z}_4$ points.



$\mathbb{Z}_6$: $\tau = e^{2\pi i/3}$
One $\mathbb{Z}_6$, one $\mathbb{Z}_2$ and one $\mathbb{Z}_3$ point.

Holographic perspective

There seems to be no obstruction to taking the F-theory limit, so we end up with a IIB background of the form $\mathbb{C}^3/\mathbb{Z}_k$. Putting D3 branes on the singularity, and taking the near horizon limit, this suggests a dual description for the field theories in terms of $AdS_5 \times (S^5/\mathbb{Z}_k)$, with a non-trivial flat $SL(2,\mathbb{Z})$ bundle. (Provides a microscopic realization of the setup proposed in [Ferrara,Porrati,Zaffaroni '98].)

Holographic perspective

There seems to be no obstruction to taking the F-theory limit, so we end up with a IIB background of the form $\mathbb{C}^3/\mathbb{Z}_k$. Putting D3 branes on the singularity, and taking the near horizon limit, this suggests a dual description for the field theories in terms of $AdS_5 \times (S^5/\mathbb{Z}_k)$, with a non-trivial flat $SL(2,\mathbb{Z})$ bundle. (Provides a microscopic realization of the setup proposed in [Ferrara,Porrati,Zaffaroni '98].)

Remarkably, the axio-dilaton τ is frozen to a $\mathcal{O}(1)$ value in these backgrounds. We learn that the theories on the branes no longer have the marginal deformation associated to changing the Yang-Mills coupling.

$\mathcal{N} = 4$ quotient perspective

In terms purely of the theory on the probe branes, we start from the observation that for particular (self-dual) values of τ_{YM}, certain $\mathbb{Z}_k$ subgroups of the $SL(2,\mathbb{Z})$ become symmetries. For instance, when $\tau = i$ we have that S-duality

$$S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \tag{7}$$

becomes a symmetry of the theory. $(-i^{-1} = i.)$

We can then construct appropriate quotients

$$\mathcal{Q}_k = \frac{\mathcal{N} = 4\ U(N)}{\mathbb{Z}_k^R \cdot \mathbb{Z}_k^{SL(2,\mathbb{Z})}}\,. \tag{8}$$

We choose $\mathbb{Z}_k^R$ to be the R-symmetry generator associated with the $\mathbb{Z}_k$ rotation in the transverse $\mathbb{R}^6$, in order to preserve susy.

Supersymmetry

We claim that these theories preserve (just) 12 supercharges for $n > 2$. We now show this in the $\mathcal{N} = 4$ SYM quotient perspective (the computation from the other viewpoints is essentially isomorphic). (Also in [Nishinaka, Tachikawa '16].)

Supersymmetry

We claim that these theories preserve (just) 12 supercharges for $n > 2$. We now show this in the $\mathcal{N} = 4$ SYM quotient perspective (the computation from the other viewpoints is essentially isomorphic). (Also in [Nishinaka, Tachikawa '16].)

The 16 supercharges arrange into four spacetime spinors Q^A_α, a spinor of $SU(4)_R$. Under the $\mathbb{Z}_k$ rotation these transform as ($\omega_k = \exp(2\pi i/k)$)

$$(Q^1, Q^2, Q^3, Q^4) \to (\omega_k^{\frac{1}{2}} Q^1, \omega_k^{\frac{1}{2}} Q^2, \omega_k^{\frac{1}{2}} Q^3, \omega_k^{-\frac{3}{2}} Q^4)\,. \quad (9)$$

Supersymmetry

We claim that these theories preserve (just) 12 supercharges for $n > 2$. We now show this in the $\mathcal{N} = 4$ SYM quotient perspective (the computation from the other viewpoints is essentially isomorphic). (Also in [Nishinaka, Tachikawa '16].)

The 16 supercharges arrange into four spacetime spinors Q^A_α, a spinor of $SU(4)_R$. Under the $\mathbb{Z}_k$ rotation these transform as ($\omega_k = \exp(2\pi i/k)$)

$$(Q^1, Q^2, Q^3, Q^4) \to (\omega_k^{\frac{1}{2}} Q^1, \omega_k^{\frac{1}{2}} Q^2, \omega_k^{\frac{1}{2}} Q^3, \omega_k^{-\frac{3}{2}} Q^4) \,. \tag{9}$$

The transformation of the supercharge generators under a $SL(2,\mathbb{Z})$ transformation is [Kapustin, Witten '06]

$$Q^A \to \gamma^{\frac{1}{2}} Q^A \qquad \text{with} \quad \gamma = \frac{|c\tau + d|}{c\tau + d} \,. \tag{10}$$

For the theories we are constructing, we have $\gamma = \omega_k^{-1}$, so only Q^A with $A = 1, 2, 3$ survive the quotient. (For $\mathbb{Z}_4$: $g_{SL(2,\mathbb{Z})} = S$, $\tau = i$, so $\gamma = -i$, while $\omega_4 = i$.) (Notice that for $k = 1, 2$ we preserve $\mathcal{N} = 4$.)

Field theory properties

We have constructed new $\mathcal{N} = 3$ theories. What do we know about them?

Field theory properties

We have constructed new $\mathcal{N} = 3$ theories. What do we know about them?

During the last couple of months a beautiful set of results have appeared which (among other things) shed light on the behavior of $\mathcal{N} = 3$ SCFTs in 4d. [Aharony, Evtikhiev '15], [Nishinaka, Tachikawa '16], [Córdova, Dumitrescu, Intriligator '16], [Argyres, Lotito, Lü, Martone '16]

I'll give a very brief summary of what these works say about $\mathcal{N} = 3$ theories.

Why is $\mathcal{N} = 3$ not $\mathcal{N} = 4$?

A well known argument (in Weinberg's book, for example) asserts that $\mathcal{N} = 3$ supersymmetry in four dimensions, together with CPT invariance, automatically gives $\mathcal{N} = 4$.

Why is $\mathcal{N} = 3$ not $\mathcal{N} = 4$?

A well known argument (in Weinberg's book, for example) asserts that $\mathcal{N} = 3$ supersymmetry in four dimensions, together with CPT invariance, automatically gives $\mathcal{N} = 4$.

There is a loophole in this result [Aharony, Evtikhiev '15], [I.G.-E., Regalado '15]: it is based on looking to the field content of the Lagrangian description, so it assumes that the theory can be given a weakly coupled description. This is not the case for our theories.

Why is $\mathcal{N} = 3$ not $\mathcal{N} = 4$?

A well known argument (in Weinberg's book, for example) asserts that $\mathcal{N} = 3$ supersymmetry in four dimensions, together with CPT invariance, automatically gives $\mathcal{N} = 4$.

There is a loophole in this result [Aharony, Evtikhiev '15], [I.G.-E., Regalado '15]: it is based on looking to the field content of the Lagrangian description, so it assumes that the theory can be given a weakly coupled description. This is not the case for our theories.

(The enhancement to $\mathcal{N} = 4$ was always somewhat accidental in any case, purely $\mathcal{N} = 3$ Lagrangian supergravity theories are well known to exist.)

Relevant and marginal deformations

In [Aharony, Evtikhiev '15] and [Córdova,Dumitrescu,Intriligator '16] it is shown that truly $\mathcal{N}=3$ theories cannot have marginal or relevant deformations preserving $\mathcal{N}=3$.

(Seems to be in good agreement with our construction: $\mathcal{N}=4$ theories have no relevant deformations preserving $\mathcal{N}=4$, and just one marginal deformation preserving $\mathcal{N}=4$: the coupling, which we project out in our quotient.)

Rank one $\mathcal{N}=3$ theories

It was shown in [Nishinaka, Tachikawa '16] that for rank one $\mathcal{N}=3$ theories, the form of the moduli space is necessarily $\mathbb{C}^3/\mathbb{Z}_\ell$, with $\ell \in \{1,2,3,4,6\}$. Furthermore, for $\ell = 1,2$ one has enhancement to $\mathcal{N}=4$, while for $\ell = 3,4,6$ the theory is purely $\mathcal{N}=3$.

Rank one $\mathcal{N} = 3$ theories

It was shown in [Nishinaka, Tachikawa '16] that for rank one $\mathcal{N} = 3$ theories, the form of the moduli space is necessarily $\mathbb{C}^3/\mathbb{Z}_\ell$, with $\ell \in \{1, 2, 3, 4, 6\}$. Furthermore, for $\ell = 1, 2$ one has enhancement to $\mathcal{N} = 4$, while for $\ell = 3, 4, 6$ the theory is purely $\mathcal{N} = 3$.

The central charge has been computed: $a = c = (2\ell - 1)/4$. (For $\mathcal{N} = 3$ it is always the case that $a = c$. [Aharony, Evtikhiev '15])

Rank one $\mathcal{N} = 3$ theories

It was shown in [Nishinaka, Tachikawa '16] that for rank one $\mathcal{N} = 3$ theories, the form of the moduli space is necessarily $\mathbb{C}^3/\mathbb{Z}_\ell$, with $\ell \in \{1, 2, 3, 4, 6\}$. Furthermore, for $\ell = 1, 2$ one has enhancement to $\mathcal{N} = 4$, while for $\ell = 3, 4, 6$ the theory is purely $\mathcal{N} = 3$.

The central charge has been computed: $a = c = (2\ell - 1)/4$. (For $\mathcal{N} = 3$ it is always the case that $a = c$. [Aharony, Evtikhiev '15])

The associated 2d chiral algebras have been constructed.[Beem, Lemos,Liendo,Peelaers,Rastelli,van Rees '13], [Nishinaka, Tachikawa '16]

Rank one $\mathcal{N} = 3$ theories

It was shown in [Nishinaka, Tachikawa '16] that for rank one $\mathcal{N} = 3$ theories, the form of the moduli space is necessarily $\mathbb{C}^3/\mathbb{Z}_\ell$, with $\ell \in \{1, 2, 3, 4, 6\}$. Furthermore, for $\ell = 1, 2$ one has enhancement to $\mathcal{N} = 4$, while for $\ell = 3, 4, 6$ the theory is purely $\mathcal{N} = 3$.

The central charge has been computed: $a = c = (2\ell - 1)/4$. (For $\mathcal{N} = 3$ it is always the case that $a = c$. [Aharony, Evtikhiev '15])

The associated 2d chiral algebras have been constructed.[Beem, Lemos,Liendo,Peelaers,Rastelli,van Rees '13], [Nishinaka, Tachikawa '16]

$\mathcal{N} = 3$ theories are necessarily $\mathcal{N} = 2$. There is a proposed classification of rank-one $\mathcal{N} = 2$ theories by [Argyres, Lotito, Lü, Martone '16]. The possibilities allowed by the classification are very limited, and the $\mathcal{N} = 3$ theories we find seem to fit well in the classification.

Conclusions

- We have constructed the first known examples of $\mathcal{N} = 3$ SCFTs.
- We do so by a very natural F-theoretical generalization of the O3 plane, which freezes out the axio-dilaton, giving intrinsically strongly coupled backgrounds.
- The geometry involves rigid (neither deformable nor resolvable in a Calabi-Yau way) singularities.
- F-theoretical example of branes at singularities.
- The SCFTs we find have natural holographic descriptions as $AdS_5 \times X$, where X is a non-trivial smooth F-theory background with frozen axio-dilaton.
- The M-theory picture suggests that upon compactification on a circle we flow to $\mathcal{N} \geq 6$ ABJM theories.

Open questions

Many!

- **Classify torsion variants:**
  - In the M-theory picture. (Some subtleties with charges induced by torsion [Aharony, Hashimoto, Hirano, Ouyang '09].) And not clear which cases will end up trivial in the F-theory limit.
  - In the IIB/holographic picture. (Something like [Douglas, Park, Schnell '14] perhaps?)
- Make concrete the notion of Chan-Patons in the $\frac{U(N)\,\mathcal{N}=4}{\text{symmetry}}$ description. BPS states? SCI?
- Other $\mathcal{N}=4$ starting points, beyond $U(N)$?
- Relating the SCI of the $\mathcal{N}=3$ theories to $\mathcal{N}=6$ ABJM partition functions.
- And other geometries, of course. Any case with non-trivial dynamics without D3 branes?