

Aspects of SCFTs and their susy deformations

Ken Intriligator (UC San Diego)

20 years of F-theory workshop, Caltech

Thank the organizers, Caltech, and F-theory.
Would also like to thank my

Spectacular Collaborators



Clay
Córdova



Thomas
Dumitrescu

1506.03807: 6d conformal anomaly a from 't Hooft anomalies. 6d a-thm. for $N=(1,0)$ susy theories.

1602.01217: Classify susy-preserving deformations for $d>2$ SCFTs.

+ to appear & work in progress

“What is QFT?”



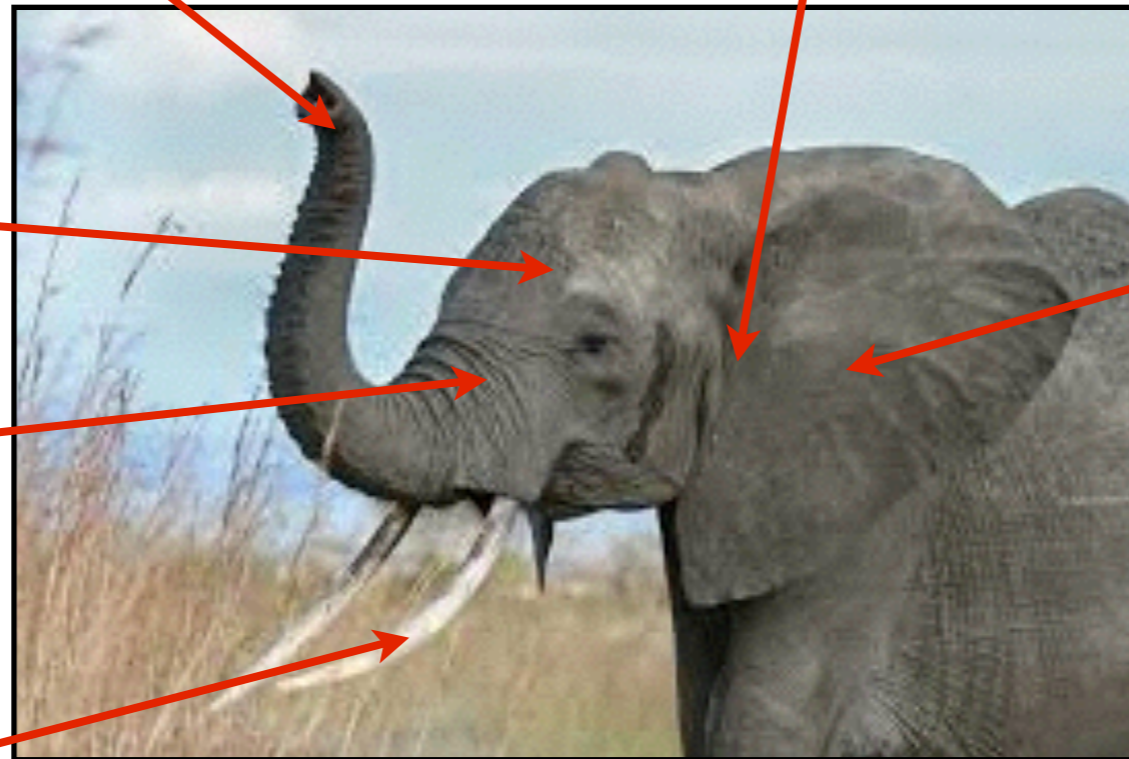
perturbation theory
around free field
Lagrangian theories

CFTs + perturbations

susy

non-Lagrangian

higher dim'l thy
compactified

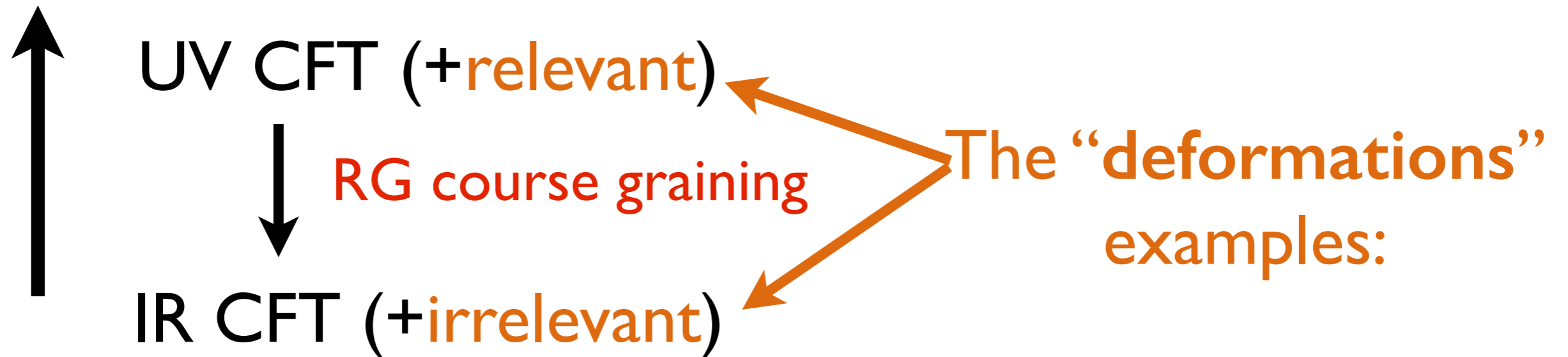


F-theory
realizations

unexplored...
something
crucial for
the future?

“# d.o.f.”

RG flows



- “ $\delta\mathcal{L}$ ” = $\sum_i g_i \mathcal{O}_i$ (OK even if SCFT is non-Lagrangian)
- Move on the moduli space of (susy) vacua.
- Gauge a (e.g. UV or IR free) global symmetry.
- Will here focus on RG flows that preserve supersymmetry.

RG flow constraints

- d=even: 't Hooft anomaly matching for all global symmetries (including NGBs + WZW terms for spont. broken ones + Green-Schwarz contributions for reducible ones). Weaker d=odd analogs, e.g. parity anomaly matching in 3d.

- Reducing # of d.o.f. intuition. For d=2,4 (& d=6?) : a-theorem

$$a_{UV} \geq a_{IR} \quad a \geq 0$$

For any
unitary theory

d=even: $\langle T_{\mu}^{\mu} \rangle \sim a E_d + \sum_i c_i I_i$

(d=odd: conjectured analogs, from sphere partition function / entanglement entropy.)

- Additional power from supersymmetry.

6d a-theorem?

For spontaneous conf'l symm breaking: dilaton has derivative interactions to give Δa anom matching **Schwimmer, Theisen; Komargodski, Schwimmer**

6d case: $\mathcal{L}_{\text{dilaton}} = \frac{1}{2}(\partial\varphi)^2 - b\frac{(\partial\varphi)^4}{\varphi^3} + \Delta a\frac{(\partial\varphi)^6}{\varphi^6}$ (schematic)

Maxfield, Sethi; Elvang, Freedman, Hung, Kiermaier, Myers, Theisen.

Can show that $b > 0$ ($b = 0$ iff free) but b 's physical interpretation was unclear; no conclusive restriction on sign of Δa .

Clue: observed that, for case of (2,0) on Coulomb branch,

$$\Delta a \sim b^2 > 0.$$

Cordova, Dumitrescu, Kl: this is a general req't of $N=(1,0)$ susy, and b is related to an 't Hooft anomaly matching term.

Longstanding hunch

e.g. Harvey
Minasian,
Moore '98

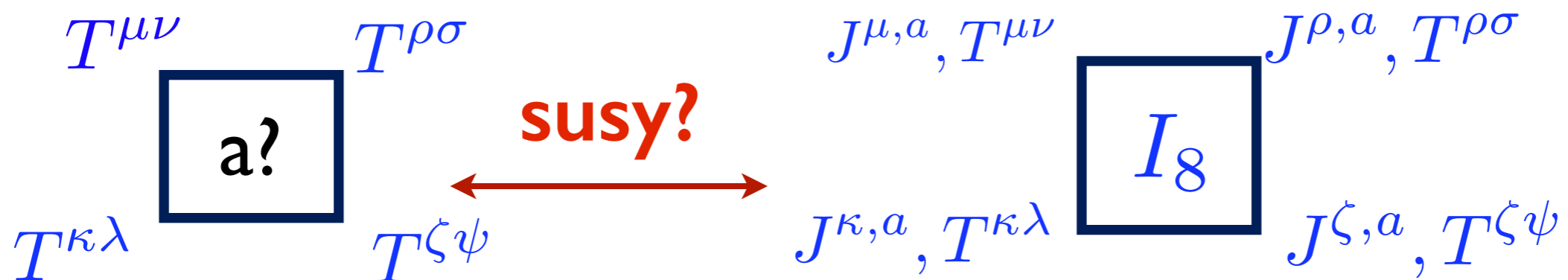
Susy multiplet of anomalies: should be able to relate a-anomaly to R-symmetry 't Hooft-type anomalies in 6d, as in 2d and 4d.

$$T^{\mu\nu} \leftrightarrow J_R^{\mu,a}$$

Stress-tensor supermultiplet

$$g_{\mu\nu} \leftrightarrow A_{R,\mu}^a$$

Sources = bkgrd SUGRA supermultiplet



4-point fn with too many indices. Hard to get a, and hard to compute.

Easier to isolate anomaly term, and enjoys anomaly matching

(1,0) 't Hooft anomalies

$$\mathcal{I}_8^{\text{origin}} = \frac{1}{4!} (\alpha c_2^2(R) + \beta c_2(R) p_1(T) + \gamma p_1^2(T) + \delta p_2(T))$$

$$c_2(R) \equiv \frac{1}{8\pi^2} \text{tr}(F_{SU(2)_R} \wedge F_{SU(2)_R})$$

Background gauge fields and metric
(\sim background SUGRA)

$$p_1(T) \equiv \frac{1}{8\pi^2} \text{tr}(R \wedge R)$$

Computed for (2,0) SCFTs + many (1,0) SCFTs

Harvey, Minasian, Moore; KI; Ohmori, Shimizu, Tachikawa; Ohmori, Shimizu, Tachikawa, Yonekura; Del Zotto, Heckman, Tomasiello, Vafa; Heckman, Morrison, Rudelius, Vafa.

E.g. for theory of N small E_8 instantons:

Ohmori,
Shimizu,
Tachikawa

$$\mathcal{E}_N : (\alpha, \beta, \gamma, \delta) = (N(N^2 + 6N + 3), -\frac{N}{2}(6N + 5), \frac{7}{8}N, -\frac{N}{2})$$

(Leading N^3 coeff. can be anticipated from Z_2 orbifold of A_{N-1} (2,0) case.)

(1,0) on tensor branch

$$\mathcal{I}_8^{\text{origin}} = \frac{1}{4!} (\alpha c_2^2(R) + \beta c_2(R) p_1(T) + \gamma p_1^2(T) + \delta p_2(T))$$

't Hooft anomaly matching requires

$$\Delta \mathcal{I}_8 \equiv \mathcal{I}_8^{\text{origin}} - \mathcal{I}_8^{\text{tensor branch}} \sim X_4 \wedge X_4 \quad \text{must be a perfect square, match } \mathfrak{I}_8 \text{ via } X_4 \text{ sourcing } B:$$

$$\mathcal{L}_{GSWS} = -iB \wedge X_4 \quad \text{KI ; Ohmori, Shimizu, Tachikawa, Yonekura}$$

$$X_4 \equiv 16\pi^2 (x c_2(R) + y p_1(T)) \quad \text{for some real coefficients } x, y$$

Our classification of defs. gives: $\mathcal{L}_{\text{tensor}} = Q^8(\mathcal{O}) \supset \mathcal{L}_{\text{dilaton}} + \mathcal{L}_{GSWS}$

Then $b = \frac{1}{2}(y - x)$ **Adapting a SUGRA analysis of Bergshoeff, Salam, Sezgin '86 (!).**

Upshot:

$$a^{\text{origin}} = \frac{16}{7}(\alpha - \beta + \gamma) + \frac{6}{7}\delta$$

Change gears (1602.01217)

Classify susy-preserving deformations of SCFTs

- “ $\delta\mathcal{L}$ ” = $Q^{N_Q} \mathcal{O}_{\text{long}}$ “D-term” e.g. Kahler potential in 4d N=1.

SCFT unitarity, bound grows with dim d: $\Delta(\delta\mathcal{L}) > \frac{1}{2}N_Q + \Delta_{\text{min}}(\mathcal{O}_{\text{long}})$

Irrelevant. E.g. for 6d N=(1,0) such operators have $\Delta > \frac{1}{2}8 + 6 = 10$.

- “ $\delta\mathcal{L}$ ” = $Q^{n_{\text{top}}} \mathcal{O}_{\text{short}}$ $\Delta(\delta\mathcal{L}) = \frac{1}{2}n_{\text{top}} + \Delta(\mathcal{O}_{\text{short}})$ Constrained by SCFT unitarity.

e.g. F-terms, W in 4d N=1.

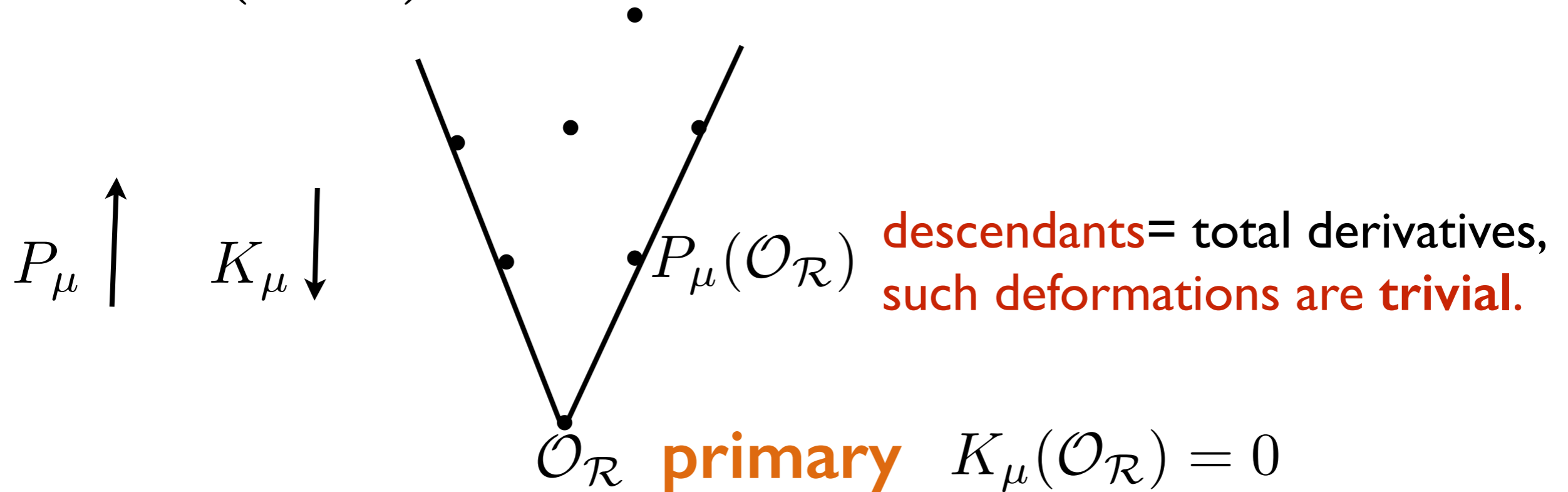
Short reps classified, in terms of the superconf’l primary operator at the bottom of the multiplet. Theory independent, just using SCFT rep constraints. We study the Q descendants, looking for Lorentz scalar “top” ops. Some oddball susy-preserving ops do exist, including in middle of multiplet(!) We had to be careful - it’s risky to claim a complete classification (embarrassing if something is overlooked)! Much more subtle and sporadic zoo than we originally expected (especially in 3d).

Some of our results:

- 6d (2,0): all 16 susy preserving deformations are irrel.
least irrelevant operator has $\dim = 12$.
- 6d (1,0): all 8 susy preserving deformations are irrel.
least irrelevant operator has $\dim = 10$. Also J. Luis, S. Lust.
- 5d: all susy preserving deformations are irrel., except
for real mass terms associated with global symmetries.
- 4d, N=3: no relevant or marginal deformations. Also O. Aharony
and M. Evtikhiev.
- 3d, N>3: all have universal, relevant, mass deformations
from stress-tensor; the only relevant deformations, and no
marginal. For N=4, also flavor current masses, no others.

CFTs, first w/o susy

$SO(d, 2)$ Operators form representations



Unitarity: primary + all descendants must have + norm, e.g.

$$|P_\mu|\mathcal{O}\rangle|^2 \sim \langle\mathcal{O}|[K_\mu, P_\mu]|\mathcal{O}\rangle \geq 0$$

Zero norm, null states = set to zero. Nulls = both primary and descendant.

$$[P_\mu, K_\nu] \sim \eta_{\mu\nu}D + M_{\mu\nu}$$

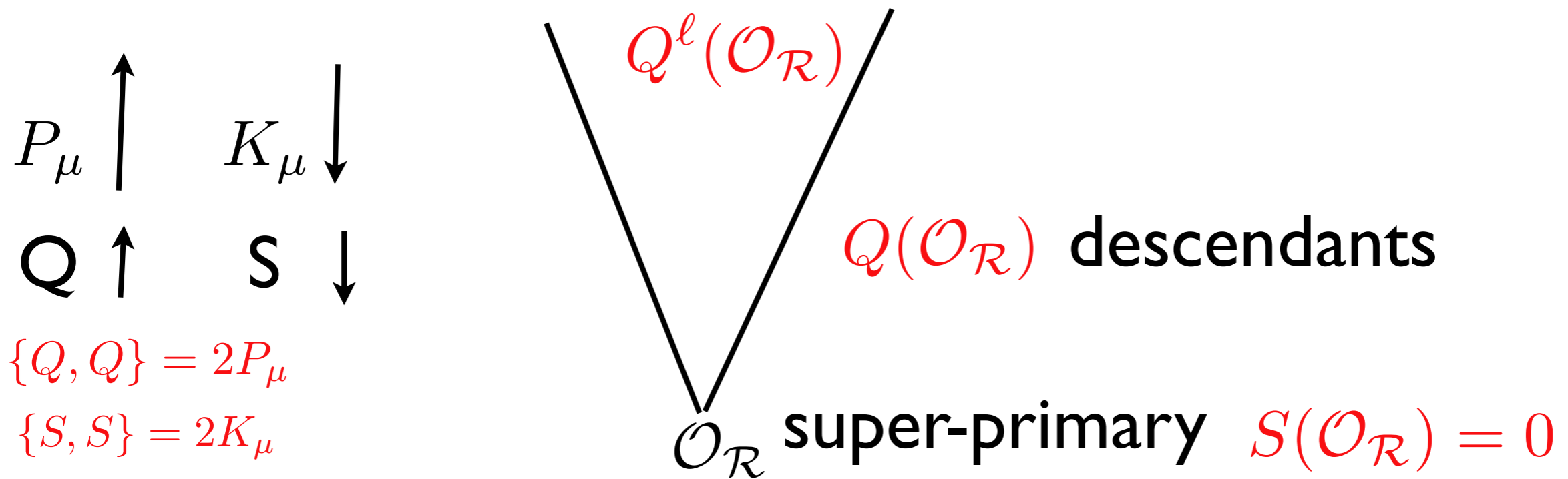
SCFT super-algebras

complete classification

$d > 6$ no SCFTs can exist

$d = 6$	$OSp(6, 2 \mathcal{N}) \supset SO(6, 2) \times Sp(\mathcal{N})_R$	$(\mathcal{N}, 0)$ $8\mathcal{N}Q_s$
$d = 5$	$F(4) \supset SO(5, 2) \times Sp(1)_R$	$8Q_s$
$d = 4$	$Su(2, 2 \mathcal{N} \neq 4) \supset SO(4, 2) \times SU(\mathcal{N})_R \times U(1)_R$	$4\mathcal{N}Q_s$
$d = 4$	$PSU(2, 2 \mathcal{N} = 4) \supset SO(4, 2) \times SU(4)_R$	$4\mathcal{N}Q_s$
$d = 3$	$OSp(4 \mathcal{N}) \supset SO(3, 2) \times SO(\mathcal{N})_R$	$2\mathcal{N}Q_s$
$d = 2$	$OSp(2 \mathcal{N}_L) \times OSp(2 \mathcal{N}_R)$	$\mathcal{N}_L Q_s + \mathcal{N}_R \bar{Q}_s$

SCFT operator reps

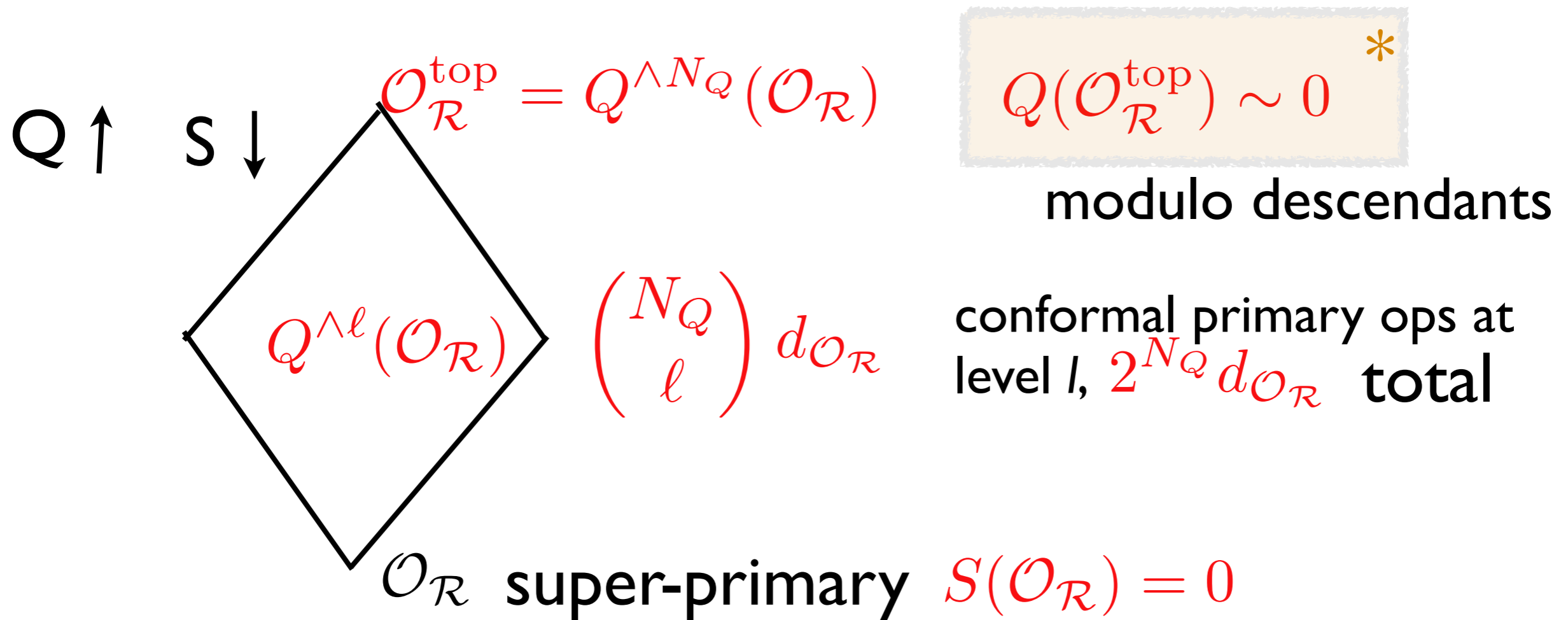


“ $\delta\mathcal{L}$ ” = $\sum_i g_i \mathcal{O}_i$ primary, modulo descendants.

$\{Q, Q\} \sim P_\mu \sim 0$ Clifford algebra.

Level $Q^{\wedge l}(\mathcal{O}_R)$ $l = 0 \dots l_{max} \leq N_Q$

Typical, long multiplets



Can generate multiplet from bottom up, via Q , or from top down, via S . **Reflection symmetry**. Unique op at bottom, so unique op at the top. Operator at top = susy preserving deformation. No other susy preserving operators in long multiplets. Easy cases. D-terms. Unitarity bounds at bottom of give bounds at top.

Unitary bounds

All Q-descendants must have non-negative norm.

E.g. at Q-level one:

$$0 \leq |Q|\mathcal{O}\rangle|^2 \sim \langle \mathcal{O}^\dagger | SQ|\mathcal{O}\rangle \sim \langle \mathcal{O}^\dagger | \{S, Q\}|\mathcal{O}\rangle$$

$$\{S, Q\} \sim D - (M_{\mu\nu} + R)$$

→ $\Delta \geq c(\text{Lorentz}) + c(\text{R} - \text{symmetry}) + \text{shift}$

Saturated iff there is a **null state**: a Q-descendant that is also a superconformal primary:

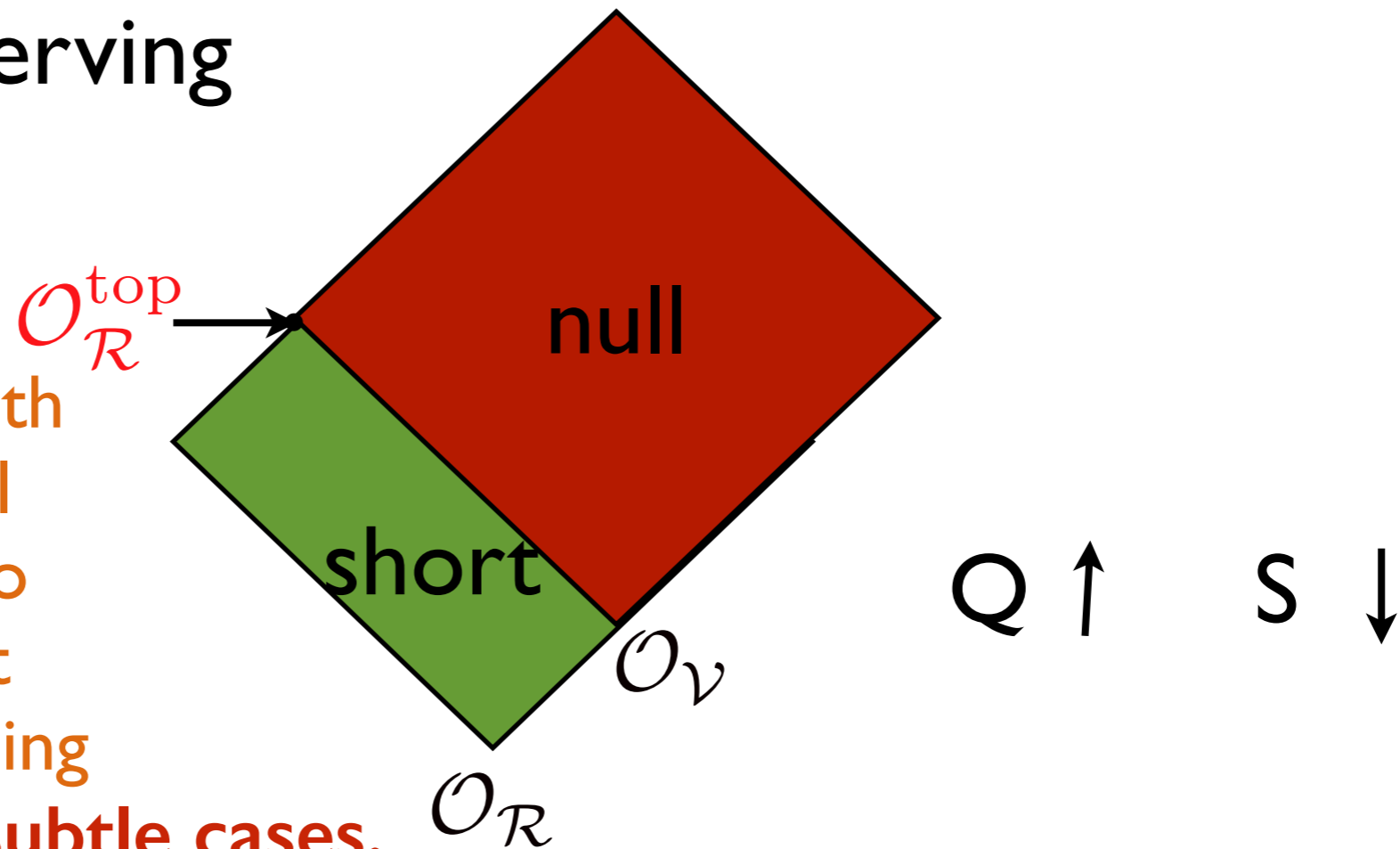
$$\mathcal{O}_\nu = Q(\mathcal{O}') \quad \text{and} \quad S(\mathcal{O}_\nu) = 0$$

Set $\mathcal{O}_\nu = 0$ along with all its Q-descendants.

Long - null = short

Specific operator dimensions, in terms of Lorentz + R-symmetry + shifts, to get null states. Set null states to zero: a short multiplet. **Simplest cases** also have the reflection symmetry, unique operator at bottom and top = susy preserving deformation:

Act on bottom op. with all Q's, setting the null linear combinations to zero. But can also act with R-symmetry raising and lowering. **Some subtle cases.**



Multiple top op. cases

(Unique bottom operator, so no reflection symmetry.)

E.g. $T_{\mu\nu}$ multiplet of 4d N=4, top ops = $T_{\mu\nu}, \mathcal{O}_\tau, \mathcal{O}_{\bar{\tau}}$

Conserved $J_\mu^{a,\text{global}}$ of 5d N=1, top ops = $J_\mu^{a,\text{global}}, \mathcal{O}_{m^a}$

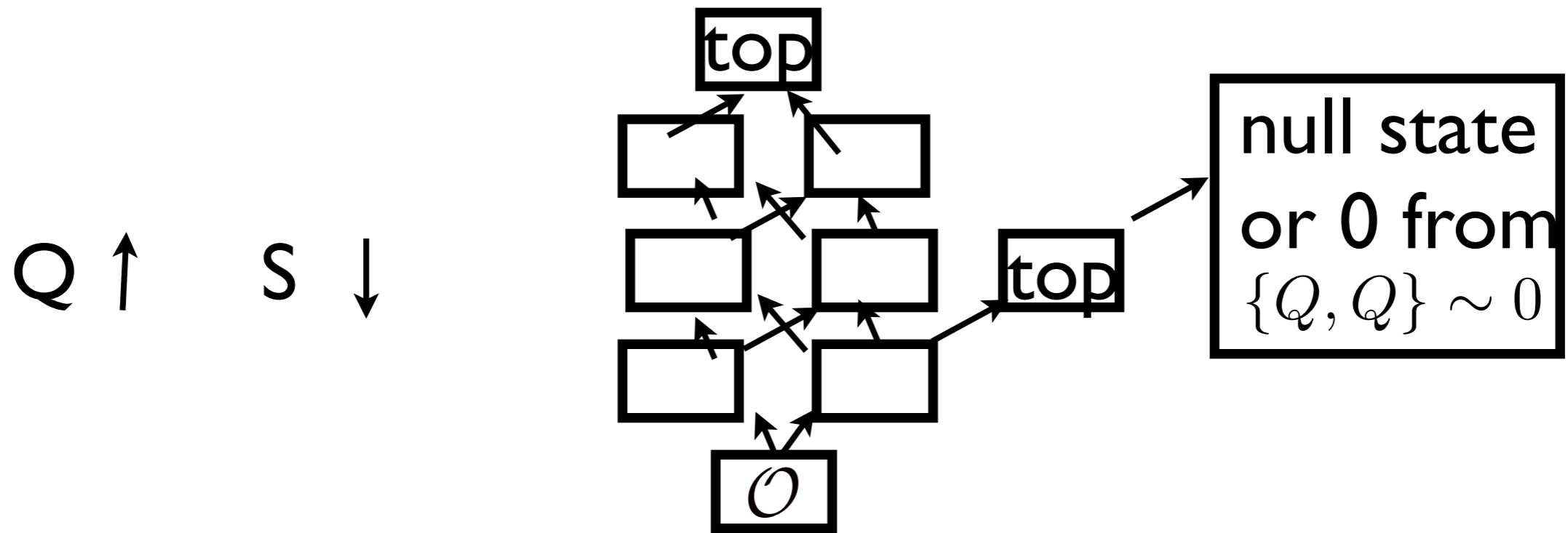
Many examples, especially with conserved currents; in such cases, setting $\{Q, \bar{Q}\} \sim P_\mu \sim 0$ requires care, since current cons. laws are null, both primary and descendant. **But** also examples of multiple top operators without conserved currents, e.g. in 4d N=2,

$$\mathcal{O}^{\text{bottom}} = A_2 \bar{A}_2 [0; 0]_{\Delta=3}^{R=1, r=0} \quad \mathcal{O}^{\text{top}} = Q^3 \bar{Q}^2 \mathcal{O}^{\text{bottom}}$$

No conserved currents
in this multiplet, yet 2 tops:

and $\mathcal{O}^{\text{top}'} = \bar{Q}^3 Q^2 \mathcal{O}^{\text{bottom}}$

Mid-level susy tops(!)



3d $\mathcal{N} \geq 4$ $T_{\mu\nu}$ multiplet: the stress-tensor is at top, at level 4.

Another top, at level 2, Lorentz scalar. Gives susy-preserving “universal mass term” relevant deformations. First found in 3d $\mathcal{N}=8$ (KI '98, Bena & Warner '04; Lin & Maldacena '05). Seems special to 3d. Indeed, these examples give a deformed susy algebra with a “non-central extension” with R-symm gens R_{ij} playing role of central term (=3d loophole to Haag-Lopuszanski-Sohnius theorem).

Classify susy preserving deformations of SCFTs



Many multiplets have mid-level Lorentz scalars, in all dimensions. We do many cross checks that we're not overlooking any exotic susy deformations (e.g. verify that Q can map to an operator at the next level, check Bose-Fermi degeneracy, recombination rules, etc).

Detailed tables

Give all susy-preserving deformations, relevant, marginal, and all irrelevant deformations, for all N , $d > 2$

E.g.

3d, $N=8$:

L=long,
A,B,C,..
=short.

33

Primary \mathcal{O}	Deformation $\delta\mathcal{L}$	Comments
$B_1 \left\{ \begin{array}{l} (0, 0, 2, 0) \\ \Delta_{\mathcal{O}} = 1 \end{array} \right\}$	$Q^2 \mathcal{O} \in \left\{ \begin{array}{l} (0, 0, 0, 2) \\ \Delta = 2 \end{array} \right\}$	Stress Tensor (T)
$B_1 \left\{ \begin{array}{l} (0, 0, 0, 2) \\ \Delta_{\mathcal{O}} = 1 \end{array} \right\}$	$Q^2 \mathcal{O} \in \left\{ \begin{array}{l} (0, 0, 2, 0) \\ \Delta = 2 \end{array} \right\}$	Stress Tensor (T)
$B_1 \left\{ \begin{array}{l} (0, 0, R_3 + 4, 0) \\ \Delta_{\mathcal{O}} = 2 + \frac{1}{2}R_3 \end{array} \right\}$	$Q^8 \mathcal{O} \in \left\{ \begin{array}{l} (0, 0, R_3, 0) \\ \Delta = 6 + \frac{1}{2}R_3 \end{array} \right\}$	F -Term (\tilde{T})
$B_1 \left\{ \begin{array}{l} (0, 0, 0, R_4 + 4) \\ \Delta_{\mathcal{O}} = 2 + \frac{1}{2}R_4 \end{array} \right\}$	$Q^8 \mathcal{O} \in \left\{ \begin{array}{l} (0, 0, 0, R_4) \\ \Delta = 6 + \frac{1}{2}R_4 \end{array} \right\}$	F -Term (\tilde{T})
$B_1 \left\{ \begin{array}{l} (0, 0, R_3 + 2, R_4 + 2) \\ \Delta_{\mathcal{O}} = 2 + \frac{1}{2}(R_3 + R_4) \end{array} \right\}$	$Q^{10} \mathcal{O} \in \left\{ \begin{array}{l} (0, 0, R_3, R_4) \\ \Delta = 7 + \frac{1}{2}(R_3 + R_4) \end{array} \right\}$	—
$B_1 \left\{ \begin{array}{l} (0, R_2 + 2, R_3, R_4) \\ \Delta_{\mathcal{O}} = 2 + R_2 + \frac{1}{2}(R_3 + R_4) \end{array} \right\}$	$Q^{12} \mathcal{O} \in \left\{ \begin{array}{l} (0, R_2, R_3, R_4) \\ \Delta = 8 + R_2 + \frac{1}{2}(R_2 + R_3) \end{array} \right\}$	—
$B_1 \left\{ \begin{array}{l} (R_1 + 2, R_2, R_3, R_4) \\ \Delta_{\mathcal{O}} = 2 + R_1 + R_2 + \frac{1}{2}(R_3 + R_4) \end{array} \right\}$	$Q^{14} \mathcal{O} \in \left\{ \begin{array}{l} (R_1, R_2, R_3, R_4) \\ \Delta = 9 + R_1 + R_2 + \frac{1}{2}(R_3 + R_4) \end{array} \right\}$	—
$L \left\{ \begin{array}{l} (R_1, R_2, R_3, R_4) \\ \Delta_{\mathcal{O}} > 1 + R_1 + R_2 + \frac{1}{2}(R_3 + R_4) \end{array} \right\}$	$Q^{16} \mathcal{O} \in \left\{ \begin{array}{l} (R_1, R_2, R_3, R_4) \\ \Delta > 9 + R_1 + R_2 + \frac{1}{2}(R_3 + R_4) \end{array} \right\}$	D -Term

universal
mass

all others
irrelev.

Table 16: Deformations of three-dimensional $\mathcal{N} = 8$ SCFTs. The R -charges of the deformation are denoted by the $\mathfrak{so}(8)_R$ Dynkin labels $R_1, R_2, R_3, R_4 \in \mathbb{Z}_{\geq 0}$.

4d, N=3 (all irrelevant)

Primary \mathcal{O}	Deformation $\delta\mathcal{L}$	Comments
$B_1\bar{B}_1 \left\{ \begin{array}{l} (R_1 + 4, 0; 2R_1 + 8) \\ \Delta_{\mathcal{O}} = 4 + R_1 \end{array} \right\}$	$Q^4\bar{Q}^2\mathcal{O} \in \left\{ \begin{array}{l} (R_1, 0; 2R_1 + 6) \\ \Delta = 7 + R_1 \end{array} \right\}$	F -Term (*)
$B_1\bar{B}_1 \left\{ \begin{array}{l} (0, R_2 + 4; -2R_2 - 8) \\ \Delta_{\mathcal{O}} = 4 + R_2 \end{array} \right\}$	$Q^2\bar{Q}^4\mathcal{O} \in \left\{ \begin{array}{l} (0, R_2; -2R_2 - 6) \\ \Delta = 7 + R_2 \end{array} \right\}$	F -Term (*)
$B_1\bar{B}_1 \left\{ \begin{array}{l} (R_1 + 2, R_2 + 2; 2(R_1 - R_2)) \\ \Delta_{\mathcal{O}} = 4 + R_1 + R_2 \end{array} \right\}$	$Q^4\bar{Q}^4\mathcal{O} \in \left\{ \begin{array}{l} (R_1, R_2; 2(R_1 - R_2)) \\ \Delta = 8 + R_1 + R_2 \end{array} \right\}$	—
$L\bar{B}_1 \left\{ \begin{array}{l} (0, 0; r + 6), r > 0 \\ \Delta_{\mathcal{O}} = 1 + \frac{1}{6}r \end{array} \right\}$	$Q^6\mathcal{O} \in \left\{ \begin{array}{l} (0, 0; r), r > 0 \\ \Delta = 4 + \frac{1}{6}r > 4 \end{array} \right\}$	F -term (*)
$B_1\bar{L} \left\{ \begin{array}{l} (0, 0; r - 6), r < 0 \\ \Delta_{\mathcal{O}} = 1 - \frac{1}{6}r \end{array} \right\}$	$\bar{Q}^6\mathcal{O} \in \left\{ \begin{array}{l} (0, 0; r), r < 0 \\ \Delta = 4 - \frac{1}{6}r > 4 \end{array} \right\}$	F -Term (*)
$L\bar{B}_1 \left\{ \begin{array}{l} (R_1 + 2, 0; r + 4), r > 2R_1 + 6 \\ \Delta_{\mathcal{O}} = 2 + \frac{2}{3}R_1 + \frac{1}{6}r \end{array} \right\}$	$Q^6\bar{Q}^2\mathcal{O} \in \left\{ \begin{array}{l} (R_1, 0; r), r > 2R_1 + 6 \\ \Delta = 6 + \frac{2}{3}R_1 + \frac{1}{6}r > 7 + R_1 \end{array} \right\}$	(†)
$B_1\bar{L} \left\{ \begin{array}{l} (0, R_2 + 2; r - 4), r < -2R_2 - 6 \\ \Delta_{\mathcal{O}} = 2 + \frac{2}{3}R_2 - \frac{1}{6}r \end{array} \right\}$	$Q^2\bar{Q}^6\mathcal{O} \in \left\{ \begin{array}{l} (0, R_2; r), r < -2R_2 - 6 \\ \Delta = 6 + \frac{2}{3}R_2 - \frac{1}{6}r > 7 + R_2 \end{array} \right\}$	(†)
$L\bar{B}_1 \left\{ \begin{array}{l} (R_1, R_2 + 2; r + 2), r > 2(R_1 - R_2) \\ \Delta_{\mathcal{O}} = 3 + \frac{2}{3}(R_1 + 2R_2) + \frac{1}{6}r \end{array} \right\}$	$Q^6\bar{Q}^4\mathcal{O} \in \left\{ \begin{array}{l} (R_1, R_2; r), r > 2(R_1 - R_2) \\ \Delta = 8 + \frac{2}{3}(R_1 + 2R_2) + \frac{1}{6}r > 8 + R_1 + R_2 \end{array} \right\}$	(‡)
$B_1\bar{L} \left\{ \begin{array}{l} (R_1 + 2, R_2; r - 2), r < 2(R_1 - R_2) \\ \Delta_{\mathcal{O}} = 3 + \frac{2}{3}(2R_1 + R_2) - \frac{1}{6}r \end{array} \right\}$	$Q^4\bar{Q}^6\mathcal{O} \in \left\{ \begin{array}{l} (R_1, R_2; r), r < 2(R_1 - R_2) \\ \Delta = 8 + \frac{2}{3}(2R_1 + R_2) - \frac{1}{6}r > 8 + R_1 + R_2 \end{array} \right\}$	(‡)
$L\bar{L} \left\{ \begin{array}{l} (R_1, R_2; r) \\ \Delta_{\mathcal{O}} > 2 + \max \left\{ \begin{array}{l} \frac{2}{3}(2R_1 + R_2) - \frac{1}{6}r \\ \frac{2}{3}(R_1 + 2R_2) + \frac{1}{6}r \end{array} \right\} \end{array} \right\}$	$Q^6\bar{Q}^6\mathcal{O} \in \left\{ \begin{array}{l} (R_1, R_2; r) \\ \Delta > 8 + \max \left\{ \begin{array}{l} \frac{2}{3}(2R_1 + R_2) - \frac{1}{6}r \\ \frac{2}{3}(R_1 + 2R_2) + \frac{1}{6}r \end{array} \right\} \end{array} \right\}$	D -Term

Table 25: Deformations of four-dimensional $\mathcal{N} = 3$ SCFTs. The $\mathfrak{su}(3)_R$ Dynkin labels $R_1, R_2 \in \mathbb{Z}_{\geq 0}$ and the $\mathfrak{u}(1)_R$ charge $r \in \mathbb{R}$ denote the R -symmetry representation of the deformation.

d=5, 6 = simpler

No exotic susy deformations (but not a 100% proof).

5d, N=1: $Q^2 C_1 [0, 0]^{R=2} = [0, 0]_4^{R=0}$ mass terms via flavor symms

(E.g. gauge kinetic terms) $Q^4 C_1 [0, 0]^{R+4} = [0, 0]_{8+\frac{3}{2}R}^R$ irrel. F-terms

$Q^8 L_1 [0, 0]^R = [0, 0]_{\Delta > 8+\frac{3}{2}R}^R$ irrel. D-terms

6d, N=(1,0):

$Q^4 D_1 [0, 0, 0]^{R+4} = [0, 0, 0]_{\Delta=10+2R}^R$ irrel. F-terms

$Q^8 L [0, 0, 0]^R = [0, 0, 0]_{\Delta > 10+2R}^R$ irrel. D-terms

Conclude

- QFT is vast, expect still much to be found.
- susy QFTs and RG flows are rich, useful testing grounds for exploring QFT. Strongly constrained: unitarity, a-thm., etc. Can rule out some things. Exact results for others.
- Thank you !
- Happy birthdays, F-theory and Dave!