F-Theory at 20, Caltech
23rd of February, 2016

Novel matter structures and extremal transitions

Denis Klevers

arXiv:1602.nnnnn: D.K., W. Taylor (*to appear*)
arXiv:1507.05954: M. Cvetič, D.K., H. Piragua, W. Taylor
arXiv:1408.4808: D.K., D. Mayorga Peña, P. Oehlmann, H. Piragua, J. Reuter

Motivation

Importance of extremal transitions in F-theory

Recent theme in F-theory: Study of entire moduli space of F-theory vacua.

1. Classification of F-theory landscape.
2. Study of transition mechanisms.

Apply transition mechanisms to test completeness of mathematics/physics dictionary of F-theory and to discover new physics.

➡ Recent topics: SCFTs, gauge symmetry change (discrete symmetries, $U(1)$'s)…

Today: Unravel new matter structures in F-theory via extremal transitions.

Goals of this talk

Goal: Extend geometry/physics dictionary by classification of matter structures.

Two explicit classes of models

1. Global F-theory models with SU(3) and matter in $\mathrm{Sym}^2\mathbf{3}=\mathbf{6}$,
2. Global F-theory models with SU(2) and matter in $\mathrm{Sym}^3\mathbf{2}=\mathbf{4}$.

- Both models arise from unHiggsing of Abelian models with U(1)'s.
- Admit further unHiggsing to large gauge groups with conventional matter.
- Gauge symmetry realized on singular divisors, exotic matter at singularities.

➡ Inspiration for a still missing classification of Weierstrass models for singular divisors.

Outline

I. Review on matter structures & singularities

II. Two-index symmetric tensor representations

1) Field theory

2) The Abelian model

3) The unHiggsing

4) Novel matter structures & non-Tate WS-models

III. Three-index symmetric tensor representations

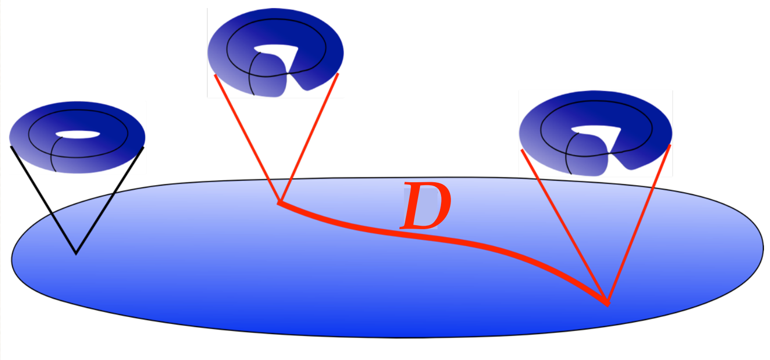
IV. Conclusions & Outlook

I. Review on matter structures & singularities

Conventional matter in F-theory

Normal form Weierstrass models: the UDF case
Tate's algorithm: [Bershadsky,Intriligator,Kachru,Morrison,Sadov,Vafa]
recent refinements: [Katz,Morrison,SchäferNameki,Sully]

Example: Singularity of Kodaira type I_n ($SU(n)$)
over divisor $D = \{t = 0\}$



Algebraic approach: [Morrison,Taylor]

- start with local expansions around divisor D

$$f = f_0 + f_1 t + \mathcal{O}(t^2) \qquad g = g_0 + g_1 t + \mathcal{O}(t^2)$$

- solve order by order the conditions imposed by $\Delta = 4f^3 + 27g^2 \sim t^n$
- for smooth D: (Auslander-Buchsbaum theorem)

$$\text{Local ring } R = \frac{\text{Ring of fcts. on } U \subset B}{\langle t \rangle} \quad \text{universal factorization domain (UFD)}$$

➡ Solutions e.g. $f_0 = \phi^2,\ g_0 = \phi^3$ (n=2); $f_0 = \phi^4,\ g_0 = \phi^6$ (n>2, split condition)

- Codim. 2 singularities worked out: fundamental, adjoint +anti-sym. tensor matter
[Morrison,Taylor]

Exotic matter & singular divisors

More exotic matter representations require non-UFD local rings:

- Divisor is D necessarily singular

Sources of exotic matter:

1. *D* irreducible, but singular:

Singular point P of D contributes to its arithmetic genus g as

$$g = p_g + \sum_P \frac{m_P(m_P - 1)}{2}$$

multiplicity m_P, geometric genus p_g

[Kumar,Park,Taylor; Morrison,Taylor]

Contribution to genus of rep. **R** implied by 6D anomalies

$$g = \sum x_{\mathbf{R}} g_{\mathbf{R}}$$

with $g_R = \frac{1}{12}(2C_R + B_R - A_R)$

➡ a) Smooth *D:* only adjoints [Witten]

$$g_{\mathbf{adj}} \equiv 1 \quad x_{\mathbf{adj}} \equiv p_g$$

b) singular *D:* exotic matter ($\mathbf{R} \neq \mathbf{adj}$)

$$g_{\mathbf{R}} \equiv \tfrac{1}{2} m_P(m_P - 1) \quad x_{\mathbf{R}} \equiv \sharp(P)$$

2. Tri- or multi-fundamental representations:
   $D=D_1+D_2+\ldots$ with more than two intersecting components
   - non-perturbative examples in $U(1)^3$ model [Cvetic,DK,Piragua,Song]

Strategy to find exotic matter models

Lesson: smooth D ⟺ "standard" WSF ⟺ standard representations

Question: How to systematically obtain exotic matter structures?

Math answer: Classify Weierstrass models for all singular divisors $D=\{t=0\}$.

Today: 1) Start with known models already exhibiting exotic matter.

2) Perform extremal transition/unHiggsing.

➡ Paradigm examples: well-studied Abelian models!

Key result: Interplay of structure of D and the form of the Weierstrass model.

➡ non-deformability of singularities of D.

➡ matter content can only change through (4,6,12) singularity (SCFT).

⟶ **talk by Lara Anderson**

II. Two-index symmetric tensor representations

1) Field Theory

SU(3) gauge theories with symmetrics

Field theory: 6D anomalies with only **3**'s and **8**'s of *SU(3)* yield multiplicities

$$x_3 = 3b\cdot(-3a-b) \qquad x_8 = 1+\tfrac{1}{2}b\cdot(b+a)$$

➡ Green-Schwarz-coefficients *a, b* determine spectrum.

F-theory: Maps to threefold with I_3-singularity $(\Delta = t^3\Delta')$ over genus *g* curve *D*:

$$x_3 = D\cdot([\Delta'] + 3K_B) \qquad x_8 = g = p_g$$

➡ standard identification $b = D,\ a = K_B$.

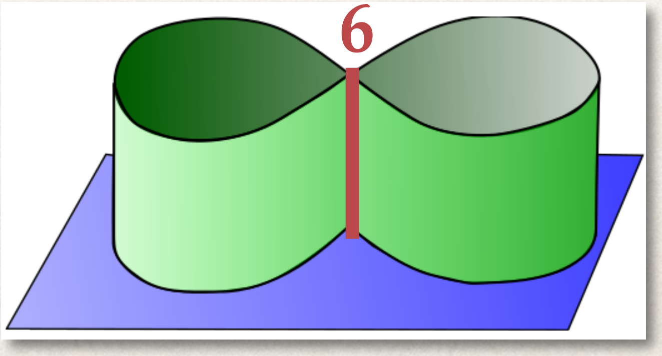
Change of matter: Anomalies allow replacement of adjoints

1 adjoint = symmetrics+ antisymmetric
$g_{\mathbf{8}} = 1 \quad g_{\mathbf{6}} = 1 \quad g_{\mathbf{3}} = 0$

⟷

Need singularity on *D*
with m_P=2: $\frac{1}{2}m_P(m_P-1)$=1

➡ *D* with non-deformable ordinary double point singularity (ODP)
Claim: **6** + **3** (not **8** + **1**)



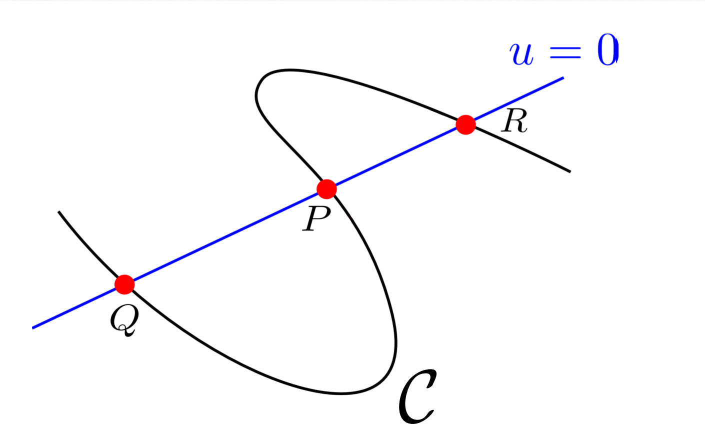
2) Abelian $U(1)^2$-models with exotic matter

Construction of non-toric model with $U(1)^2$

[Cvetič, DK, Piragua, Taylor]

Any elliptic fibration X with MW-rank two is fibration of special cubics in P^2

[Deligne;Borchmann,Mayrhofer,Palti,Weigand; Cvetič,DK,Piragua]



$$u f_2(u,v,w) + \prod_{i=1}^{3}(a_i v + b_i w) = 0$$

$$f_2 = s_1 u^2 + s_2 uv + s_3 v^2 + s_5 uw + s_6 vw + s_8 w^2$$

- Three sections have non-toric $[u{:}v{:}w]$-positions in elliptic fiber $\mathcal{C}$

$$P = [0 : -b_1 : a_1] \quad Q = [0 : -b_2 : a_2] \quad R = [0 : -b_3 : a_3]$$

➡ distance between rational points controlled by

$$\Delta_{ij} := a_i b_j - b_i a_j$$

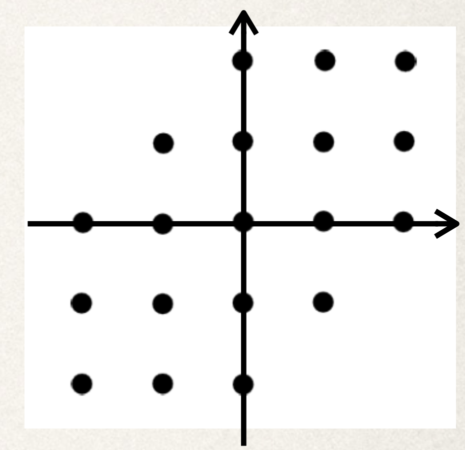
- Analysis of codimension 2 singularities: novel matter representations.

General low-energy effective theory

[Cvetič, DK, Piragua, Taylor]

Charge	Multiplicity	Locus
(-2,-2)	$x_{(-2,-2)} = [a_1] \cdot [b_1]$	$V_1 = \{a_1 = b_1 = 0\}$
(2,0)	$x_{(2,0)} = [a_2] \cdot [b_2]$	$V_2 = \{a_2 = b_2 = 0\}$
(0,2)	$x_{(0,2)} = [a_3] \cdot [b_3]$	$V_3 = \{a_3 = b_3 = 0\}$
(-2,-1)	$x_{(-2,-1)} = (2[b_3] + [s_3]) \cdot ([a_1] + [b_3]) - 2x_{(0,2)}$	$V_4 = \{\Delta_{12} = s_3 b_1^2 - s_6 a_1 b_1 + s_8 a_1^2 = 0\}$
(-1,-2)	$x_{(-2,-1)} = (2[b_1] + [s_3]) \cdot ([a_1] + [b_2]) - 2x_{(-1,1)}$	$V_5 = \{\Delta_{13} = s_3 b_1^2 - s_6 a_1 b_1 + s_8 a_1^2 = 0\}$
(-1,1)	$x_{(-1,1)} = (2[b_2] + [s_3]) \cdot ([a_3] + [b_2]) - 2x_{(2,0)}$	$V_6 = \{\Delta_{23} = s_3 b_2^2 - s_6 a_2 b_2 + s_8 a_2^2 = 0\}$
(1,1)	$x_{(1,1)} = ([a_1^4 a_2 b_3 s_8^2]) \cdot ([a_1^4 a_2^2 s_8^3]) - 2x_{(2,0)} - 8x_{(-2,-1)} - 4x_{(-1,-2)} - 20x_{(-2,-2)}$	V_7
(1,0)	$x_{(1,0)} = 4[b_1^3 b_2^3 s_3^3] \cdot ([a_1 b_2] - [K_B]) - 16x_{(2,0)} - 16x_{(-2,-1)} - x_{(-1,-2)} - 16x_{(-2,-2)} - x_{(-1,1)} - x_{(1,1)}$	V_8
(0,1)	$x_{(0,1)} = 4[b_1^3 b_3^3 s_3^3] \cdot ([a_1 b_3] - [K_B]) - x_{(-2,-1)} - 16x_{(0,2)} - 16x_{(-1,-2)} - 16x_{(-2,-2)} - x_{(-1,1)} - x_{(1,1)}$	V_9

U(1)xU(1) charge lattice



- nesting of matter loci: (2,2) matter at V_1 contained in locus V_4 of (-2,-1) matter
- ➡ crucial for appearance of exotic non-Abelian matter!

3) The unHiggsing

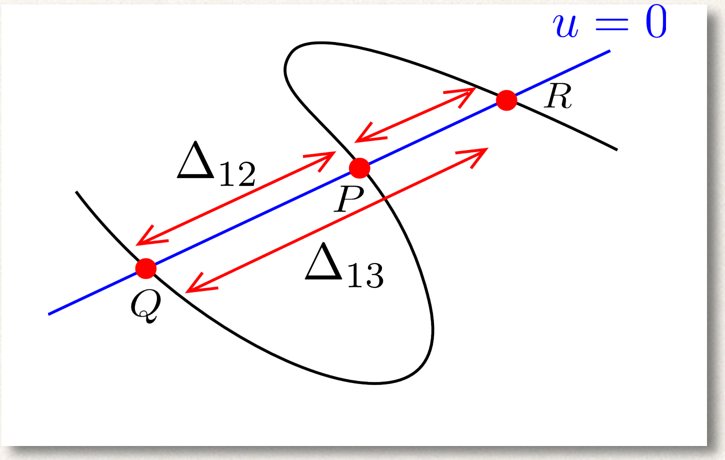
Geometry of unHiggsing U(1)'s

[Cvetič, DK, Piragua, Taylor]

<u>Reduction of Mordell-Weil group of X:</u>

- tune moduli of X so that rational points in ell. curve degenerate P=Q=R

- rk(MW)=2



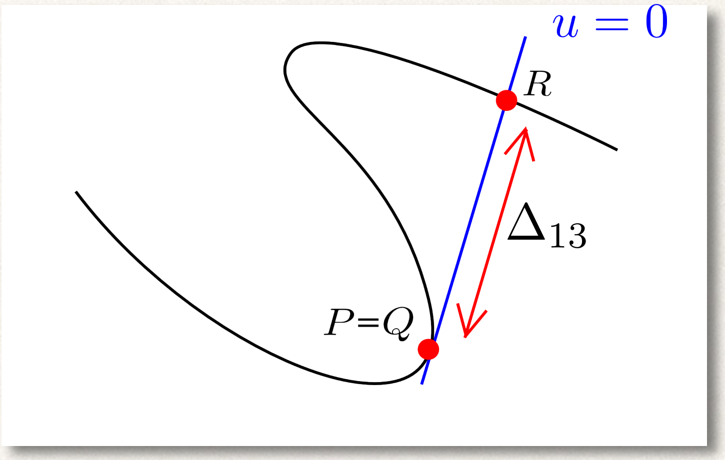
Geometry of unHiggsing U(1)'s

[Cvetič, DK, Piragua, Taylor]

Reduction of Mordell-Weil group of X:

- tune moduli of X so that rational points in ell. curve degenerate P=Q=R

- rk(MW)=2
- rk(MW)=1: $\Delta_{12} \to 0$



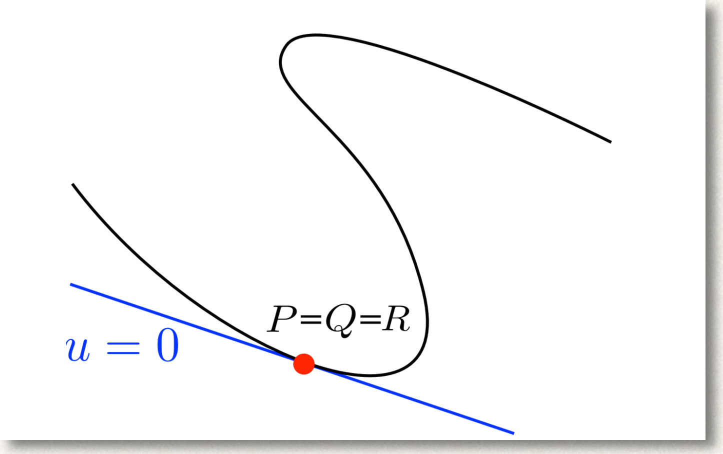
Geometry of unHiggsing U(1)'s

[Cvetič, DK, Piragua, Taylor]

Reduction of Mordell-Weil group of X:

- tune moduli of X so that rational points in ell. curve degenerate $P=Q=R$

- rk(MW)=2
- rk(MW)=1: $\Delta_{12} \to 0$
- rk(MW)=0: $\Delta_{13} \to 0$



Tuned geometry X: $$u f_2(u,v,w) + \lambda_1 \lambda_2 (a_1 v + b_1 w)^3 = 0$$

Gauge group: G =	$SU(2)$	x	$SU(2)$	x	$SU(3)$
on divisors: $U_1(1)$:	λ_1		-		t
$U_2(1)$:	-		λ_2		t

SU(3) divisor singular: $t = a_1^2 s_8 - a_1 b_1 s_6 + b_1^2 s_3$

➡ Generalizes to more U(1)'s.

Geometry of unHiggsing U(1)'s

[Cvetič, DK, Piragua, Taylor]

G =	$SU(2)$ x	$SU(2)$ x	$SU(3)$
$U_1(1)$:	λ_1	-	t
$U_2(1)$:	-	λ_2	t

- Gauge symmetry from matter:

Charge	Multiplicity	Locus
(-2,-2)	$x_{(-2,-2)} = [a_1] \cdot [b_1]$	$V_1 = \{a_1 = b_1 = 0\}$
(2,0)	$x_{(2,0)} = [a_2] \cdot [b_2]$	$V_2 = \{a_2 = b_2 = 0\}$
(0,2)	$x_{(0,2)} = [a_3] \cdot [b_3]$	$V_3 = \{a_3 = b_3 = 0\}$
(-2,-1)	$x_{(-2,-1)} = (2[b_1] + [s_3]) \cdot ([a_1] + [b_2]) - 2x_{(-1,1)}$	$V_4 = \{\Delta_{12} = s_3 b_1^2 - s_6 a_1 b_1 + s_8 a_1^2 = 0\}$

- In tuning $\Delta_{12} \to 0$:

V_4 ⟶ SU(3) divisor $D=\{t = a_1^2 s_8 - a_1 b_1 s_6 + b_1^2 s_3 = 0\}$

- *(2,2)* matter at V_1 remains at its ODP singularity $a_1 = b_1 = 0$.

Geometry of unHiggsing U(1)'s

[Cvetič, DK, Piragua, Taylor]

G =	$SU(2)$ x	$SU(2)$ x	$SU(3)$
$U_1(1)$:	λ_1	-	t
$U_2(1)$:	-	λ_2	t

- Gauge symmetry from matter:

Charge	Multiplicity	Locus
(-2,-2)	$x_{(-2,-2)} = [a_1] \cdot [b_1]$	$V_1 = \{a_1 = b_1 = 0\}$
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W-boson → (-2,-1)

- In tuning $\Delta_{12} \to 0$:

V_4 ⟶ SU(3) divisor $D=\{t = a_1^2 s_8 - a_1 b_1 s_6 + b_1^2 s_3 = 0\}$

- *(2,2)* matter at V_1 remains at its ODP singularity a_1=b_1=0.

The unHiggsed model

[Cvetič, DK, Piragua, Taylor]

Non-Abelian matter spectrum by inspection of codim. two singularities

Representation	Multiplicity	
$(\mathbf{1},\mathbf{1},\mathbf{6})$	$x_{(\mathbf{1},\mathbf{1},\mathbf{6})} = [a_1]\cdot([t]+K_B-[a_1])$	$V_{sing}=\{a_1=b_1=0\}$
$(\mathbf{2},\mathbf{2},\mathbf{1})$	$x_{(\mathbf{2},\mathbf{2},\mathbf{1})} = [\lambda_1]\cdot[\lambda_2]$	$V_{bf}^{(1)}=\{\lambda_1=\lambda_2=0\}$
$(\mathbf{2},\mathbf{1},\mathbf{3})$	$x_{(\mathbf{2},\mathbf{1},\mathbf{3})} = [\lambda_1]\cdot[t]$	$V_{bf}^{(2)}=\{\lambda_1=t=0\}$
$(\mathbf{1},\mathbf{2},\mathbf{3})$	$x_{(\mathbf{1},\mathbf{2},\mathbf{3})} = [\lambda_2]\cdot[t]$	$V_{bf}^{(3)}=\{\lambda_2=t=0\}$
$(\mathbf{2},\mathbf{1},\mathbf{1})$	$x_{(\mathbf{2},\mathbf{1},\mathbf{1})} = [\lambda_1]\cdot(-8K_B-2[\lambda_1]-2[\lambda_2]-3[t])$	$V_f^{(1)}$
$(\mathbf{1},\mathbf{2},\mathbf{1})$	$x_{(\mathbf{1},\mathbf{2},\mathbf{1})} = [\lambda_2]\cdot(-8K_B-2[\lambda_1]-2[\lambda_2]-3[t])$	$V_f^{(2)}$
$(\mathbf{1},\mathbf{1},\mathbf{3})$	$x_{(\mathbf{1},\mathbf{1},\mathbf{3})} = [t]\cdot(-9K_B-2[\lambda_1]-2[\lambda_2]-3[t])+x_{(\mathbf{1},\mathbf{1},\mathbf{6})}$	$V_f^{(3)}$
$(\mathbf{3},\mathbf{1},\mathbf{1})$	$x_{(\mathbf{3},\mathbf{1},\mathbf{1})} = \frac{1}{2}[\lambda_1]\cdot([\lambda_1]+K_B)+1$	$D_1=\{\lambda_1=0\}$
$(\mathbf{1},\mathbf{3},\mathbf{1})$	$x_{(\mathbf{1},\mathbf{3},\mathbf{1})} = \frac{1}{2}[\lambda_2]\cdot([\lambda_2]+K_B)+1$	$D_2=\{\lambda_2=0\}$
$(\mathbf{1},\mathbf{1},\mathbf{8})$	$x_{(\mathbf{1},\mathbf{1},\mathbf{8})} = \frac{1}{2}[t]\cdot([t]+K_B)+1-x_{(\mathbf{1},\mathbf{1},\mathbf{6})}$	$D_3=\{t=0\}$

Claim: Two-index symmetric tensor at ODP of *SU(3)* divisor $D_3=\{t=0\}$.

➡ Provide two checks.

Matching of effective field theories

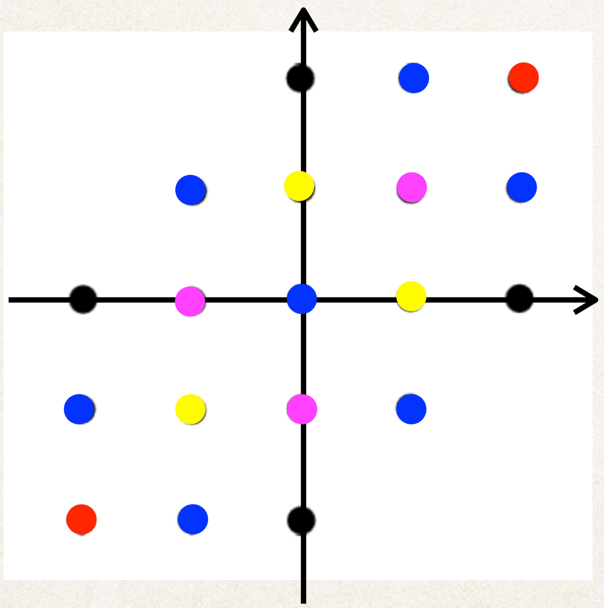
Higgsing back to Abelian theory by bifundamentals:

$U(1)^2$ ← **(2,1,2)** — $SU(2)xU(1)xSU(2)$ ← **(1,2,3)** — $G= SU(2)xSU(2)xSU(3)$

Special cases of smaller *G:* Higgsing by adjoints.

*U(1)*²-theory

Charge spectrum



Non-Abelian theory

SU(2)xSU(2)xSU(3)-reps

(3,1,1) + **(1,3,1)**

(1,1,8)

(1,3,1) + **(1,1,3)**

(1,1,6)

Matching requires **6** of SU(3).

➡ Indirect check for presence of **6** + **3** instead of **8** + **1**.

4) Novel matter structures & non-TateWS-models

Non-Tate Weierstrass models of singular divisors

[Cvetič, DK, Piragua, Taylor]

Get Weierstrass model $y^2 = x^3 + fx + g$ ($\lambda_1 = \lambda_2 = 1$ no *SU(2)*'s) of the form

$$f = f_0 + f_1 t, \qquad g = g_0 + g_1 t + g_2 t^2$$

$$\text{for } f_0 = -\tfrac{1}{48}\left(s_6^2 - 4s_3 s_8\right)^2,\ g_0 = \tfrac{1}{864}\left(s_6^2 - 4s_3 s_8\right)^3$$

- Have WS-model with structure of I_2 singularity if t is formal parameter:

$$\Delta = t^2 \Delta'$$

- Identifying $t = a_1^2 s_8 - a_1 b_1 s_6 + b_1^2 s_3$ we get I_3 singularity by reducing Δ' in the quotient ring

$$R = \frac{\text{Ring of fcts. on } B}{\langle t \rangle}$$

- Note: R (or local rings) not UFD as $t=0$ has ODP singularity at $a_1=b_1=0$

- I_3 looks non-split: $f_0 \neq \phi^4$, $g_0 \neq \phi^6$ ⟶ only *SU(2)* gauge group?

➡ No: evasion of "standard" split condition due to special form of $t=0$.

Subtle split conditions for singular divisors

[Cvetič, DK, Piragua, Taylor]

Intertwined structure of Weierstrass form and $t = a_1^2 s_8 - a_1 b_1 s_6 + b_1^2 s_3$

$$f_0 = -\tfrac{1}{48}\left(s_6^2 - 4s_3 s_8\right)^2 \sim discr(t)^2$$
$$g_0 = \tfrac{1}{864}\left(s_6^2 - 4s_3 s_8\right)^3 \sim discr(t)^3$$

discr(t): discriminant of t as quadratic in (a_1,b_1)

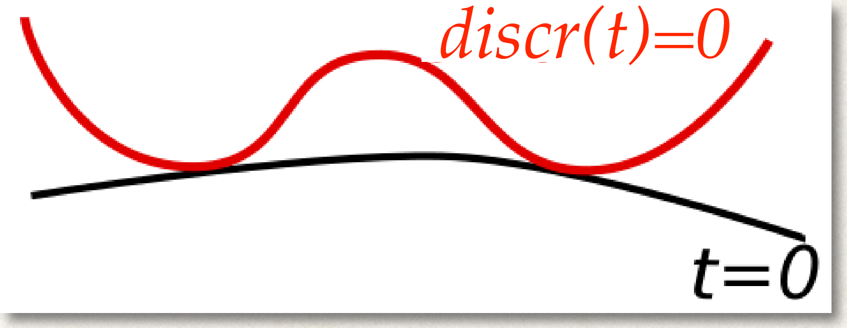
- Monodromy cover $\psi^2 + (9g_0/2f_0) = 0$: $\quad \psi^2 + \frac{1}{4}(s_6^2 - 4s_3 s_8) = 0$
  [Grassi,Morrison]

Generic divisor $t=0$: irreducible monodromy cover ➡ non-split?

Here: $t=0$ and $discr(t)=0$ intersect tangentially

➡ $discr(t)=x^2$ close to $t=0$

➡ split cover: I_3^s-fiber ($SU(3)$ gauge group)



- can not deform or smooth out $t=0$ without reducing gauge symmetry

➡ matter at $a_1=b_1=0$ is symmetric + antisymmetric matter.

Completeness & Generalizations

Completeness?

- Examples on $B = \mathbb{P}^2$: *[t]=5,6* and #(ODP)=1, 2
- ➡ *SU(3)* on quartic with up to two ODP's not covered (although its has adjoint Higgsing to $U(1)^2$).

Further unHiggsings:

- Additional tuning of *SU(3)* on *t=0* with ODP to smooth models

1) *SU(3)*x*SU(3)*

$$t \rightarrow (xa_1 + yb_1)(a_1 + zb_1)$$

➡ with bifundamental matter

2) *SU(6)*

$$t \rightarrow a_1^2(b^2 s_3 - b s_6 + s_8)$$

➡ with conventional matter

Generalization:

- start with $SU(3)^2$ or *SU(6)* and Higgs: field theory clear
- Geometric description = deformation of Weierstrass form is hard to find.

⟶ **talk by Nikhil Raghuram**

III. Three-index symmetric tensor representations

[DK, Taylor]: to appear soon

1) Field Theory

SU(2) gauge theories with three index symmetric tensors

6D anomaly-free theories with only **2**′s and **8**′s covered by F-theory with I_2-singularity on genus g divisor D:

$$x_2 = D \cdot (-4K_B - D) \quad x_3 = 1 + \frac{1}{2} D \cdot (D + K_B)$$

Change of matter: anomalies allow replace 6 adjoints by one **4** and two **2**

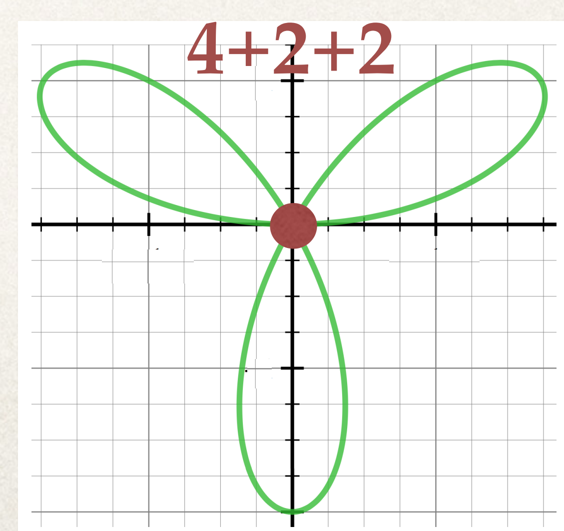
6 adjoint = Sym^3**2** + fund. + fund.
$g_{\mathbf{8}} = 1 \qquad g_{\mathbf{4}} = 6 \qquad g_{\mathbf{2}} = 0$

⟷

Need singularity on D
with m_P=3: $\frac{1}{2} m_P(m_P - 1)$ =1

- note that **4** is real rep: only one half-hyper at each triple point

➡ D with non-deformable ordinary triple point singularity (OTP)
Claim: **2**x**2**x**2**=**4** + **2** + **2**

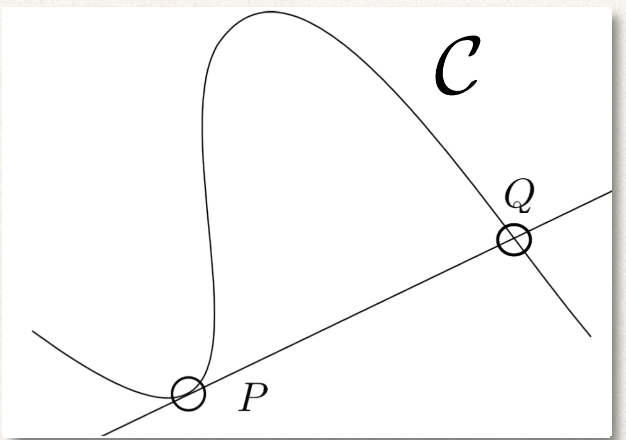


2) An Abelian U(1)-model with $q=3$

Abelian model with charge *q*=3

[DK,Mayorga-Pena,Oehlmann,Piragua,Reuter]

Elliptic fibration X with MW-rank by fibration of special cubics in P²



$$u f_2(u,v,w) + v(s_4 v^2 + s_7 vw + s_8 vw + s_9 w^2) = 0$$

$$f_2 = s_1 u^2 + s_2 uv + s_3 v^2 + s_5 uw + s_6 vw + s_8 w^2$$

- Two sections have the following *[u:v:w]*-coordinates

$$P = [0:0:1] \quad Q = [-s_9 \overline{PQ}, s_8 \overline{PQ}, s_9(s_3 s_8^2 - s_2 s_8 s_9 + s_1 s_9^2)]$$

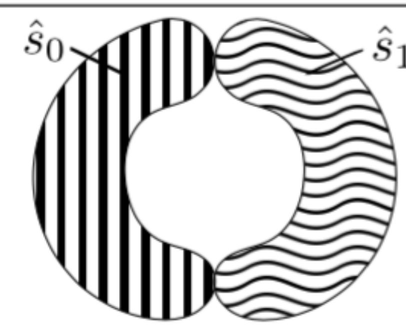
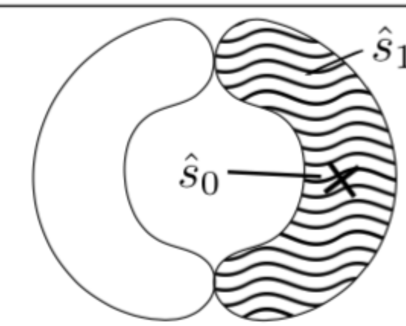
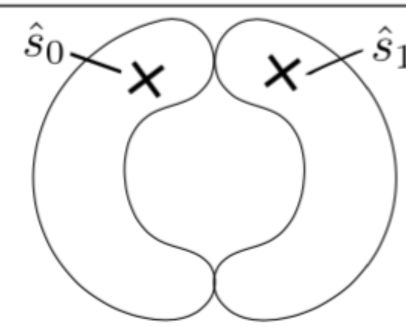
- distance between rational points controlled by

$$\overline{PQ} = s_7 s_8^2 - s_6 s_8 s_9 + s_5 s_9^2$$

General low-energy effective theory

[DK, Majorga-Pena,Piragua,Reuter,Oehlmann]

Matter spectrum: analysis of codimension two singularities

Representation	Multiplicity	Fiber	Locus
$\mathbf{1}_3$	$\mathcal{S}_9([K_B^{-1}] + \mathcal{S}_9 - \mathcal{S}_7)$		$V(I_{(3)}) := \{s_8 = s_9 = 0\}$
$\mathbf{1}_2$	$6[K_B^{-1}]^2 + [K_B^{-1}](4\mathcal{S}_9 - 5\mathcal{S}_7) + \mathcal{S}_7^2 + 2\mathcal{S}_7\mathcal{S}_9 - 2\mathcal{S}_9^2$		$V(I_{(2)}) := \{s_4 s_8^3 - s_3 s_8^2 s_9 + s_2 s_8 s_9^2 - s_1 s_9^3 = \overline{PQ} = 0\}$
$\mathbf{1}_1$	$12[K_B^{-1}]^2 + [K_B^{-1}](8\mathcal{S}_7 - \mathcal{S}_9) - 4\mathcal{S}_7^2 + \mathcal{S}_7\mathcal{S}_9 - \mathcal{S}_9^2$		$V(I_{(1)})$

- nesting structure: charge q=3 at singular locus of charge q=2 locus

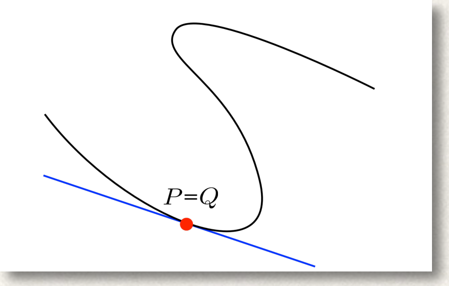
3) The unHiggsing

UnHiggsing: qualitative picture

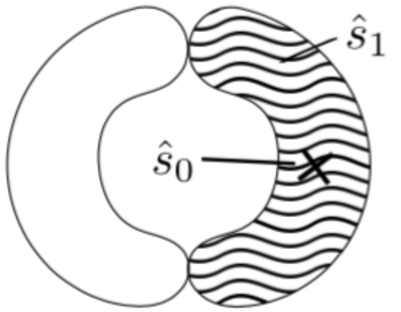
[DK,Taylor]

Tune moduli of X so that $P=Q$

$$\overline{PQ} = s_7 s_8^2 - s_6 s_8 s_9 + s_5 s_9^2 \to 0$$



Matter to gauge symmetry:

$\mathbf{1}_3$	$x_{\mathbf{1}_3} = \mathcal{S}_9([K_B^{-1}] + \mathcal{S}_9 - \mathcal{S}_7)$		$V(I_{(3)}) := \{s_8 = s_9 = 0\}$
$\mathbf{1}_2$	$x_{\mathbf{1}_2} = 6[K_B^{-1}]^2 + [K_B^{-1}](4\mathcal{S}_9 - 5\mathcal{S}_7) + \mathcal{S}_7^2 + 2\mathcal{S}_7\mathcal{S}_9 - 2\mathcal{S}_9^2$		$V(I_{(2)}) := \{s_4 s_8^3 - s_3 s_8^2 s_9 + s_2 s_8 s_9^2 - s_1 s_9^3 = \overline{PQ} = 0\}$

- Codimension two to codimension one:

$$V(I_2) \longrightarrow SU(2) \text{ divisor } D =: \{t := s_4 s_8^3 - s_3 s_8^2 s_9 + s_2 s_8 s_9^2 - s_1 s_9^3 = 0\}$$

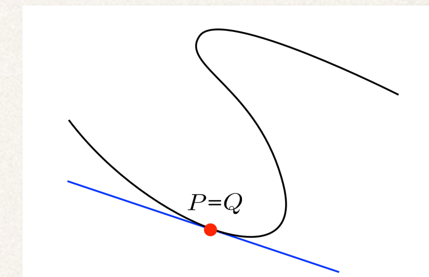
- $q=3$ matter at $V(I_3)$ becomes ordinary triple point singularity of D.

UnHiggsing: qualitative picture

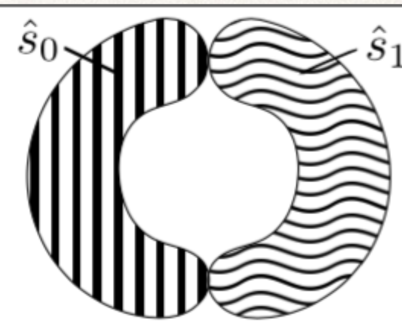
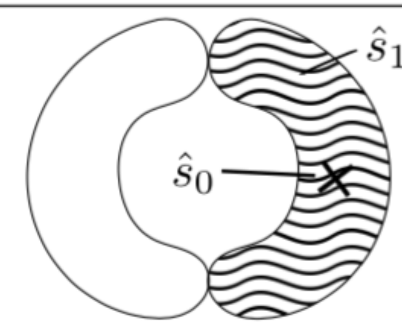
[DK,Taylor]

Tune moduli of X so that $P=Q$

$$\overline{PQ} = s_7 s_8^2 - s_6 s_8 s_9 + s_5 s_9^2 \to 0$$



Matter to gauge symmetry:

$\mathbf{1}_3$	$x_{\mathbf{1}_3} = \mathcal{S}_9([K_B^{-1}] + \mathcal{S}_9 - \mathcal{S}_7)$		$V(I_{(3)}) := \{s_8 = s_9 = 0\}$
$\mathbf{1}_2$ W-boson	$x_{\mathbf{1}_2} = 6[K_B^{-1}]^2 + [K_B^{-1}](4\mathcal{S}_9 - 5\mathcal{S}_7) + \mathcal{S}_7^2 + 2\mathcal{S}_7\mathcal{S}_9 - 2\mathcal{S}_9^2$		$V(I_{(2)}) := \{s_4 s_8^3 - s_3 s_8^2 s_9 + s_2 s_8 s_9^2 - s_1 s_9^3 = \overline{PQ} = 0\}$

- Codimension two to codimension one:

$$\text{V}(I_2) \longrightarrow SU(2) \text{ divisor } D =: \{t := s_4 s_8^3 - s_3 s_8^2 s_9 + s_2 s_8 s_9^2 - s_1 s_9^3 = 0\}$$

- $q=3$ matter at V(I_3) becomes ordinary triple point singularity of D.

UnHiggsing: details

[DK,Taylor]

1) Tuning : $\overline{PQ} = s_7 s_8^2 - s_6 s_8 s_9 + s_5 s_9^2 \to 0$

- In UFD, solve: $s_5 = s_8 \sigma_5 \,, \qquad s_6 = s_8 \sigma_7 + s_9 \sigma_5 \,, \qquad s_7 = s_9 \sigma_7$
- Special tuning: $s_5 = s_6 = s_7 \equiv 0$

2) Gauge group: $G \;=\; SU(2)$

on singular divisors: $D = \{s_4 s_8^3 - s_3 s_8^2 s_9 + s_2 s_8 s_9^2 - s_1 s_9^3 = 0\}$

3) Non-Abelian matter

SU(2)-rep	Multiplicity	Fiber	Locus
4	$x_{\mathbf{4}} = \frac{1}{2}\mathcal{S}_9 \cdot (-K_B + \mathcal{S}_9 - \mathcal{S}_7)$	I_0^{*ns}	$V_{\text{Sing}} = \{s_8 = s_9 = 0\}$
3	$x_{\mathbf{3}} = \frac{1}{2}[t] \cdot ([t] + K_B) + 1 - 6x_4$	I_2	$D = \{t = 0\}$
2	$x_{\mathbf{2}} = 2(3K_B^2 - K_B \cdot (2\mathcal{S}_7 - \mathcal{S}_9) - \mathcal{S}_7^2 + \mathcal{S}_7 \cdot \mathcal{S}_9 - \mathcal{S}_9^2) + 2m_{\mathbf{4}}$	I_3	$V(\mathfrak{p}_1) \cup V_{\text{Sing}}$

Matching of effective field theories

[DK,Taylor]

Higgsing back to Abelian theory by adjoints: see also: [Morrison,Taylor]

$$U(1) \xleftarrow{\ \mathbf{3}\ } G = SU(2)$$

- Number of adjoints is

$$x_{\mathbf{3}} = p_g = 1 + \tfrac{1}{2} D \cdot (D + K_B) - 3[s_8]\cdot[s_9] \geq 1$$

- Full matching of Abelian spectrum through Higgsing

$$\mathbf{4} \to \mathbf{1}_3 \oplus \mathbf{1}_{-3} \oplus \mathbf{1}_1 \oplus \mathbf{1}_{-1}\,, \qquad \mathbf{3} \to \mathbf{1}_2 \oplus \mathbf{1}_{-2} \oplus \mathbf{1}_0\,, \qquad \mathbf{2} \to \mathbf{1}_1 \oplus \mathbf{1}_{-1}\,.$$

$$x_{\mathbf{1}_3} = 2x_{\mathbf{4}}\,, \qquad x_{\mathbf{1}_2} = 2(x_{\mathbf{1}_3} - 1)\,, \qquad x_{\mathbf{1}_1} = 2(x_{\mathbf{1}_4} + x_{\mathbf{1}_1})$$

➡ Matching requires presence of three-index symmetric matter.

4) Novel matter structures & non-TateWS-models

Non-Tate Weierstrass model

[DK, Taylor]

Get a Weierstrass model $y^2 = x^3 + fx + g$ of the form

$$f = f_0 \qquad g = g_0 + g_1 t$$

Properties:

- Leading terms f_0, g_0 seemingly unrelated,
- No $O(t)$ term in f,
- If t is formal variable, we have no vanishing of Δ.
- Chosen Weierstrass parametrization exhibits

$$\Delta|_{t=0} \sim s_4 s_8^3 - s_3 s_8^2 s_9 + s_2 s_8 s_9^2 - s_1 s_9^3 \quad \Rightarrow \quad \begin{array}{l} \Delta|_{t=0} = 4f_0^3 + 27 g_0^2 \sim t \\ \text{if } \; t \equiv s_4 s_8^3 - s_3 s_8^2 s_9 + s_2 s_8 s_9^2 - s_1 s_9^3 \end{array}$$

- Get additional cancellation at order one so that

$$\Delta = t^2 \Delta'$$

➡ I_2-structure does not allow to change t: OTP non-deformable

⟺ presence of **4** representation of SU(2).

Completeness & Generalizations

[DK,Taylor]

Completeness?

- Constructed examples with $B = \mathbb{P}^2$, *[t]>8* and #(OTP)=0,…, 18.

➡ *e.g.* *SU(2)* on quintic with one OTP is missing (although it has adjoints)

Further unHiggsings:

- Tune/unHiggs *SU(2)* on $t = s_4 s_8^3 - s_3 s_8^2 s_9 + s_2 s_8 s_9^2 - s_1 s_9^3 = 0$ with OTP to larger gauge group on smooth divisor

1) *SU(2)xSU(2)xSU(2)*

$$t \to \prod_{i=1}^{3} (n_i s_8 + m_i s_9)$$

➡ non-standard WS-model with tri-fundamental matter

2) *SU(2)xG₂*: $s_8 \to a s_9$

$$t \to (-s_1 + a s_2 - a^2 s_3 + a^3 s_4) s_9^3$$

➡ standard WS-model with conventional matter

Generalizations:

- start with $SU(2)^3$ or $SU(2)\mathrm{x}G_2$ and Higgs/deform WS-model

Deformation of G_2x$SU(2)$ singularities

[DK,Taylor]

Start with *SU(2)*-model with three-index symmetric matter (at OTP)

- Set $s_8 = a s_9$: get I_0^*-singularity (G_2) on $s_9=0$ and I_2-singularity on $\tilde{s}_1 = 0$

$$f = (-\tfrac{1}{3}\tilde{s}_2^2 + \tilde{s}_3\tilde{s}_1)s_9^2\,, \quad g = (-\tfrac{2}{27}\tilde{s}_2^3 + \tfrac{1}{3}\tilde{s}_2\tilde{s}_3\tilde{s}_1 - s_4\tilde{s}_1^2)s_9^3\,, \quad \Delta = \tilde{s}_1^2 s_9^6 \Delta'$$

➡ Conventional Weierstrass model with I_2- and I_0^*-singularity: conventional matter representations

- Define deformation parameter $\epsilon := s_8 - a s_9$.

- Rewrite *f*, *g* of original *SU(2)* model in terms of ϵ

$$f_\epsilon = f + (\tfrac{1}{3}\tilde{s}_2\tilde{s}_3 - 3\tilde{s}_1\tilde{s}_4)s_9\epsilon + (\tilde{s}_2\tilde{s}_4 - \tfrac{1}{3}\tilde{s}_3^2)\epsilon^2$$

$$g_\epsilon = g + (\tilde{s}_1(\tilde{s}_2\tilde{s}_4 - \tfrac{2}{3}\tilde{s}_3^2) + \tfrac{1}{9}\tilde{s}_2^2\tilde{s}_3)s_9^2\epsilon + (\tfrac{1}{9}\tilde{s}_2\tilde{s}_3^2 - \tfrac{2}{3}\tilde{s}_2^2\tilde{s}_4 + \tilde{s}_1\tilde{s}_3\tilde{s}_4)s_9\epsilon^2 + (\tfrac{1}{3}\tilde{s}_2\tilde{s}_3\tilde{s}_4 - \tfrac{2}{27}\tilde{s}_3^3 - \tilde{s}_1\tilde{s}_4^2)\epsilon^3$$

➡ Found deformation of Weierstrass model corresponding to Higgsing G_2x$SU(2)$ ➡ $SU(2)$.

III. Conclusions & Outlook

Summary

1. Used extremal transitions/unHiggsing to generate exotic non-Abelian matter from exotic Abelian matter.
2. First explicit and concrete realization of
   - *SU(3)* with two-index symmetric tensor **6**: unHiggs $U(1)^2$ with (2,2) matter.
   - *SU(2)* with three-index symmetric tensor **4**: unHiggs *U(1)* with q=3 matter.
3. Further unHiggsing to
   - models with larger gauge group and both conventional and non-conventional matter (tri-fundamentals)
   - ➡ constructed deformations of Weierstrass models ⟷ Higgsing.

Outlook

- Window into new and mainly unexplored field of F-theory with exotic matter

➡Generalization? **work in progress…**

➡Systematic classification of Weierstrass models with singular divisors (cusp…)

- Physical applications to phenomenology, bounding max. *U(1)*-charge, new Tate-Shafarevich groups?

Thank you
for your attention!