Fitting fermion masses in F-theory

fernando marchesano

Instituto de Física Teórica UAM-CSIC

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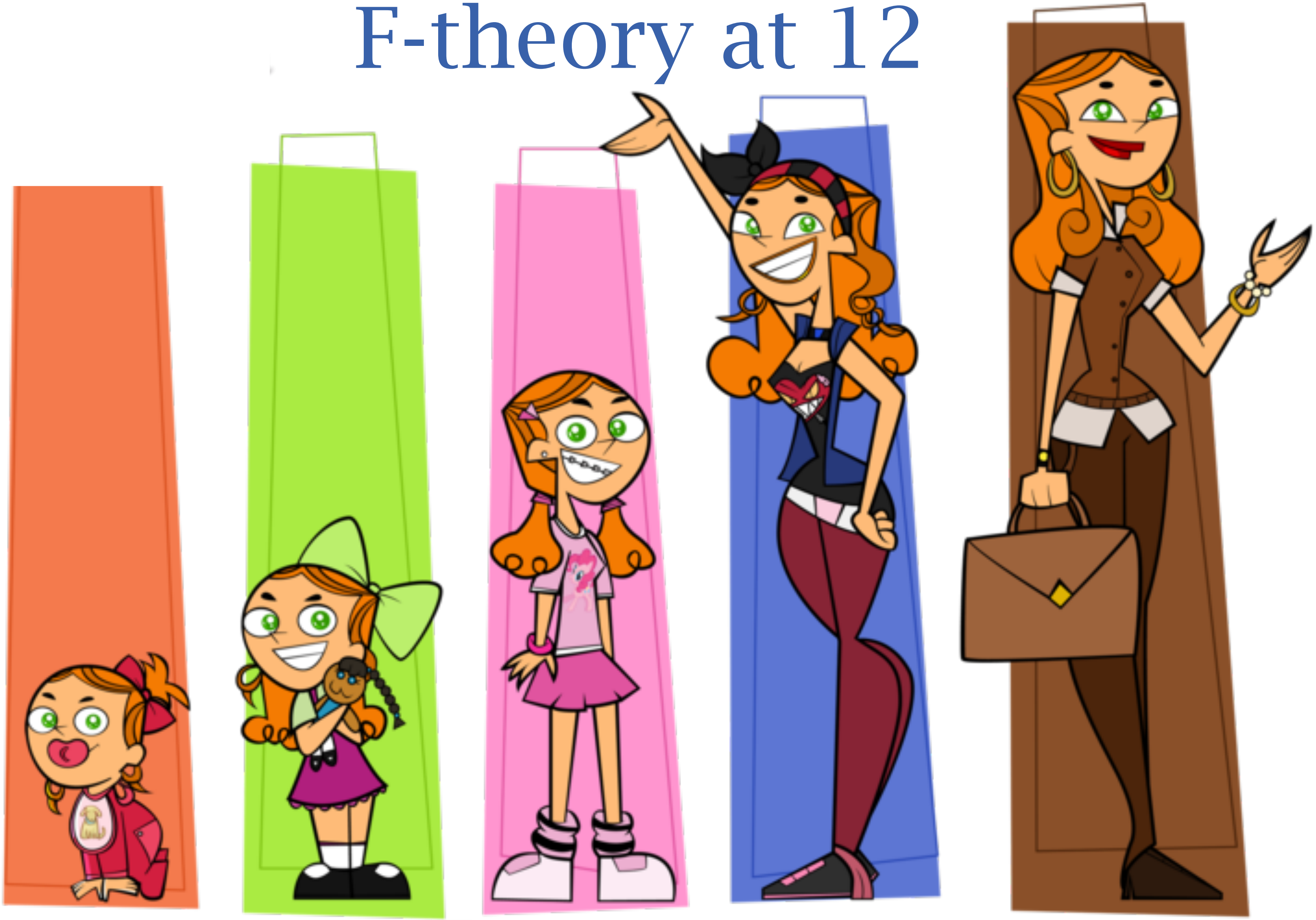
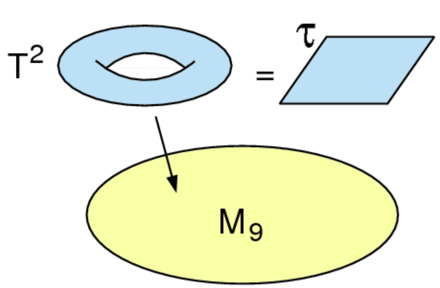
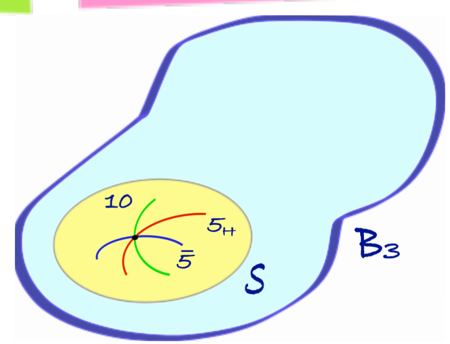
Based on:

Carta, F.M., Zoccarato
1512.04846
and
F.M., Regalado, Zoccarato
1503.02683

F-theory at 12

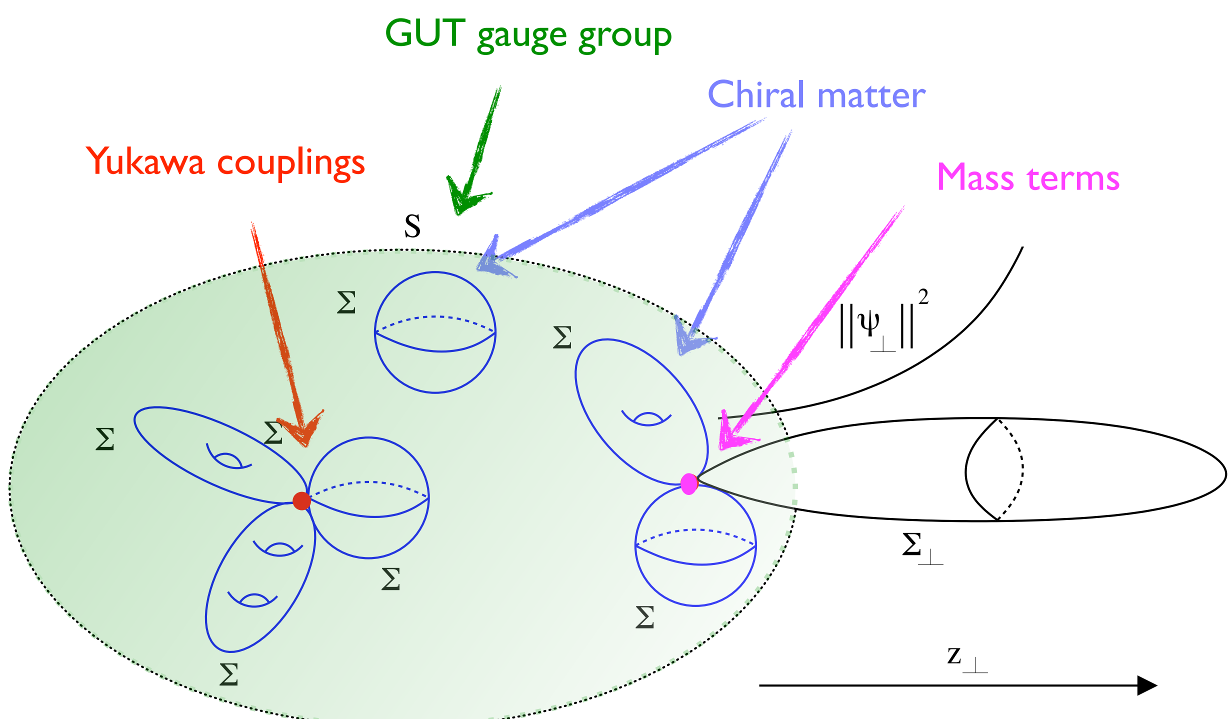






Key promises of F-theory GUTs

- Gauge coupling unification
- Realistic Yukawa couplings
  - ✦ Tree level top Yukawa (b.t. type II)
  - ✦ Local computation (b.t. heterotic)
- Doublet-triplet splitting via hypercharge flux



Taken from Beasley, Heckman, Vafa '08

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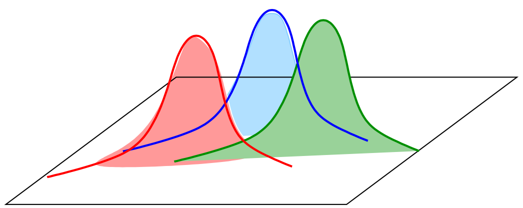
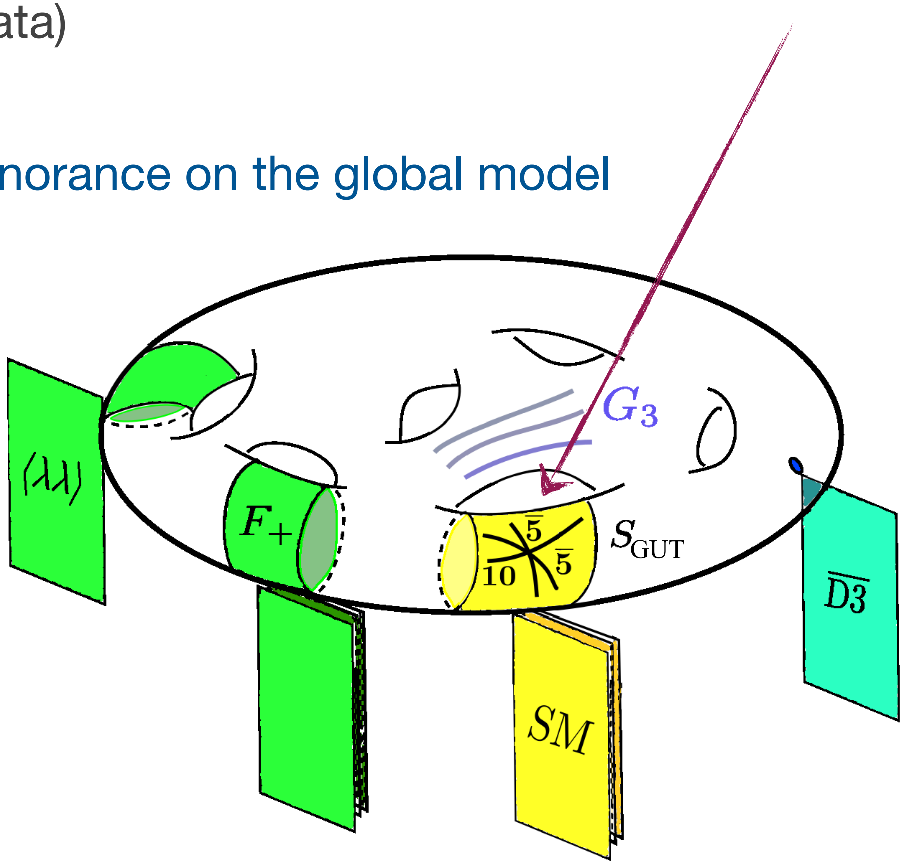
In addition:

- ✦ Good control over complex geometry
- ✦ Moduli stabilisation well developed

Key features of Yukawa couplings

- Computed via dim. red. of a 8d gauge theory on S_{GUT}
- Depend on ultra-local data around some points in S_{GUT} (holomorphic Yukawas on fewer data)
- Such local data parametrise our ignorance on the global model



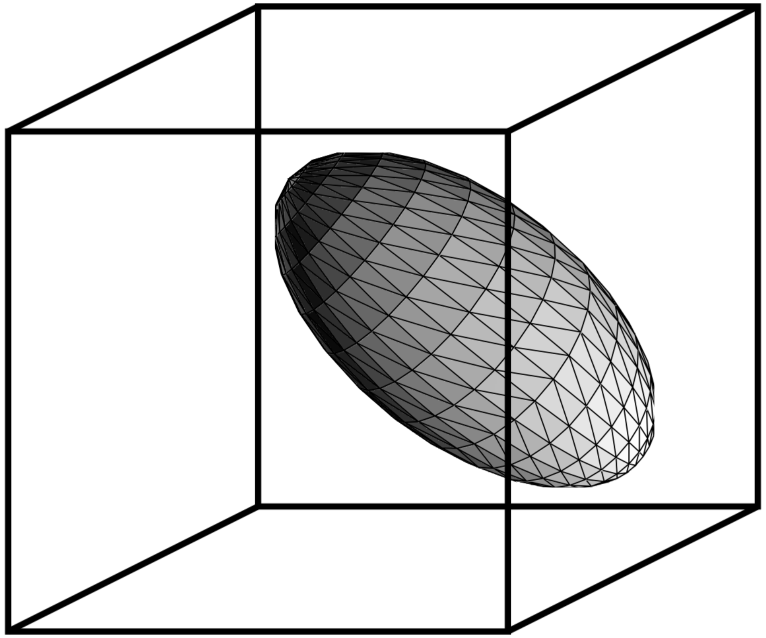


Taken from Camara, Ibañez, Valenzuela'14

Key features of Yukawa couplings

Questions:

How easy is it to get realistic Yukawas in terms of local parameters?



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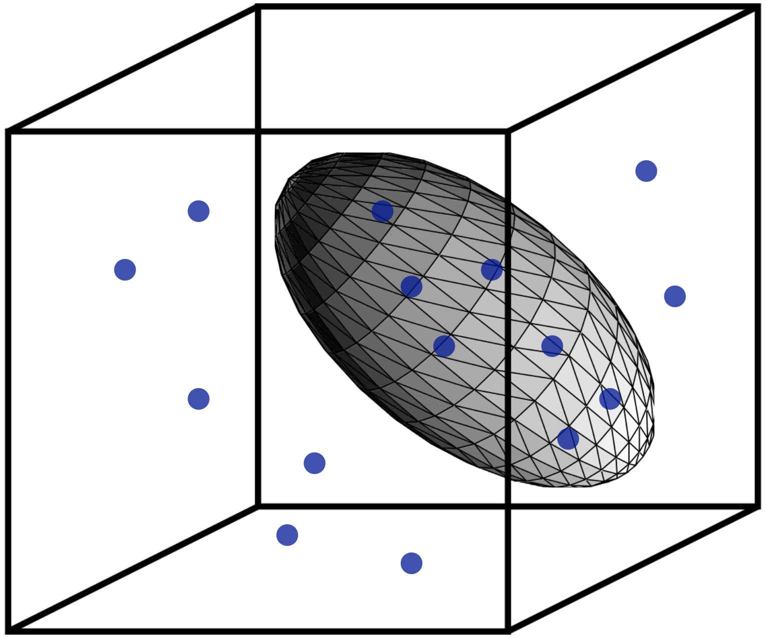
How generic are realistic Yukawas in the Landscape?



Key features of Yukawa couplings

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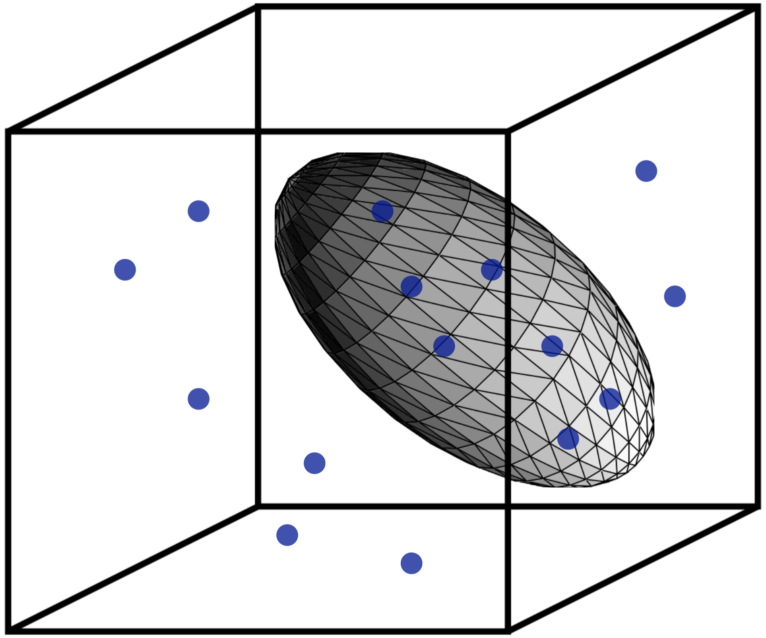


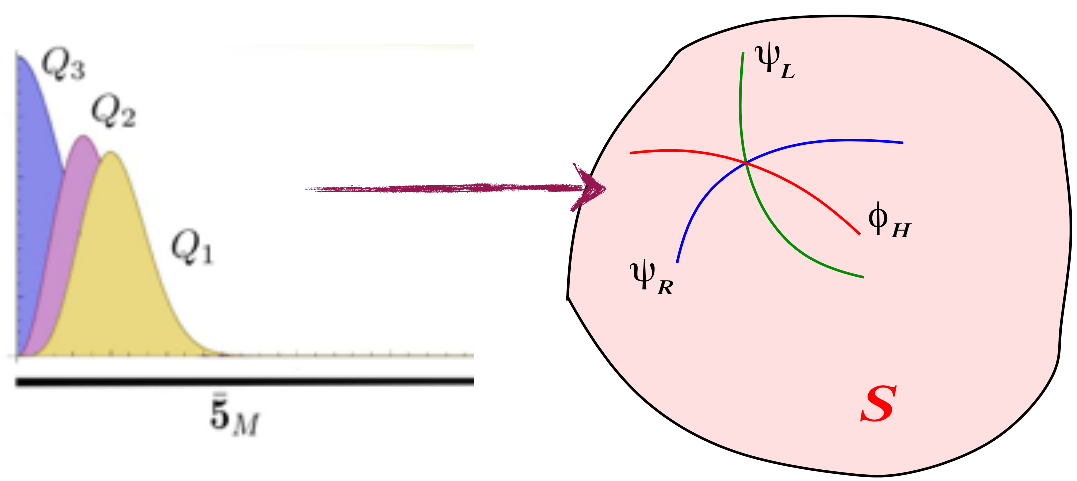
But generating a wide region of local data with realistic Yukawas is not as easy as it may seem…

First step: robust mechanism for family hierarchies

Rank one Yukawas

- F-theory comes with a mechanism to have one quark/lepton family much heavier than the other two
  - ✦ We may host several families of chiral fermions in a single matter curve by means of a worldvolume flux threading it
  - ✦ Holomorphic Yukawas independent of this flux. Their maximal rank only depends on the curves intersections

Cecotti, Cheng, Heckman, Vafa '09



Single triple intersection

⇓

rank one Yukawas

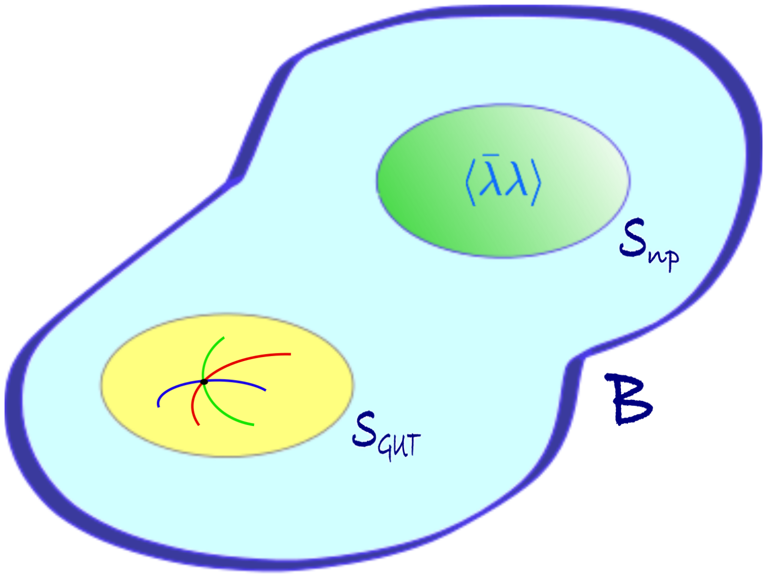
Adding non-perturbative effects

- Non-perturbative effects like E3-brane instantons will increase the rank of the Yukawa matrix while maintaining the family mass hierarchy

- In the case of plain D3-instantons we have

F.M. & Martucci '09

$$W_7 = W_7^{\text{tree}} + W^{\text{np}}$$



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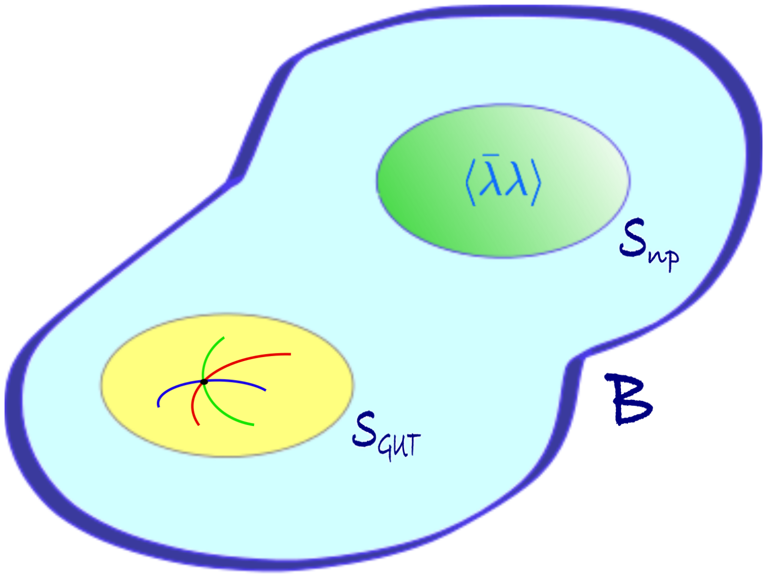
F.M. & Martucci '09

$$W_7 = W_7^{\rm tree} + W^{\rm np}$$

$$W_7 \; = \; \int_S {\rm Tr}\,(F \wedge \Phi) + \frac{\epsilon}{2} \int_S \theta_0 {\rm Tr}\,(F \wedge F)$$



$$W^{\rm np} \;\; = \;\; m_*^4 \epsilon \left[\int_S \theta_0 {\rm Tr} F^2 + \int_S \theta_1 {\rm Tr}(\Phi_{xy} F^2) + \int_S \theta_2 {\rm STr}(\Phi_{xy}^2 F^2) + \ldots \right]$$

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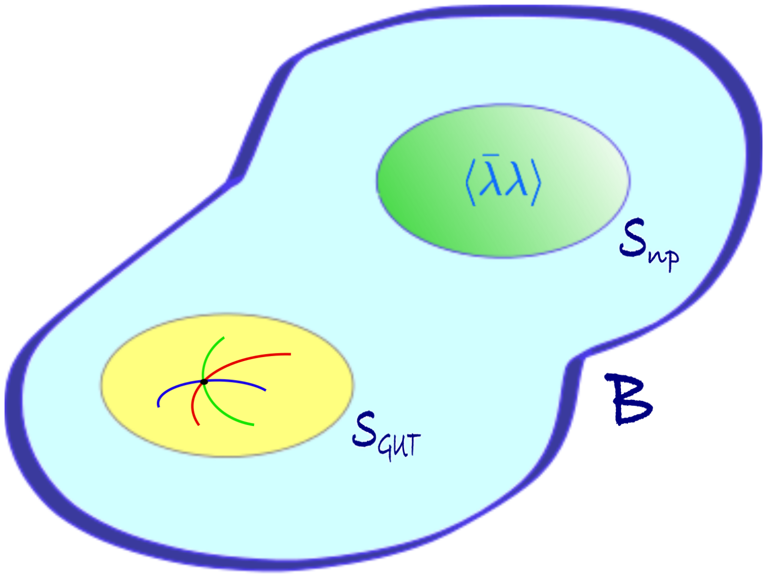
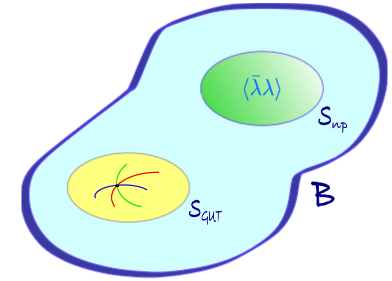


Similar effect not known for fluxed D3/M5-brane instantons

Grimm et al. '11

Martucci & Weigand '15

Adding non-perturbative effects



- The expression

$$W_7 = \int_S \mathrm{Tr}\,(F \wedge \Phi) + \frac{\epsilon}{2} \int_S \theta_0 \mathrm{Tr}\,(F \wedge F)$$

allows to carry the computation of np-corrected Yukawas at the local level

- Holomorphic Yukawas can also be computed via a residue formula. They depend on ϵ and θ_0 but not on worldvolume fluxes.

- Physical Yukawas are computed by solving for the MSSM fields internal wavefunctions and performing local dim. red. in the deformed theory.

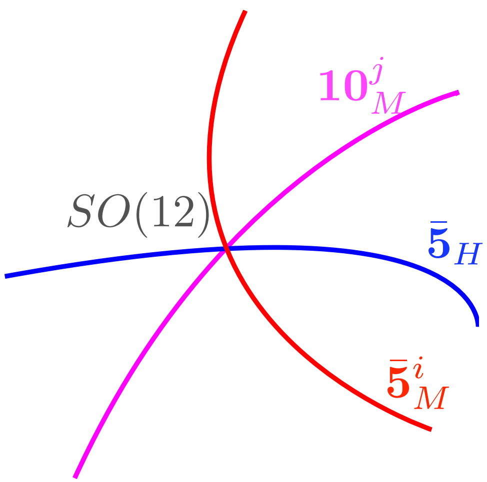
  ✦ SO(12) enhancement (down-type Yukawas) *Font, Ibañez, F.M., Regalado '12*

  ✦ E_6 enhancement (up-type Yukawas) *Font, F.M., Regalado, Zoccarato '13*

Down and Up-type Yukawas

Down-type

$$Y_D^{ij} : \bar{\mathbf{5}}_H \, \bar{\mathbf{5}}_M^i \, \mathbf{10}_M^j$$



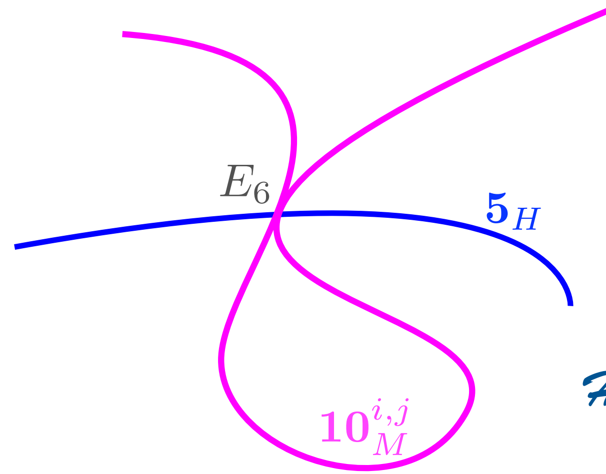
Intersecting branes, $[\langle\Phi\rangle, \langle\bar{\Phi}\rangle] = 0$

$$\langle\Phi\rangle \sim \begin{pmatrix} -x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & x-y \end{pmatrix} dx \wedge dy$$

$$\omega \wedge F = 0$$

Up-type

$$Y_U^{ij} : \mathbf{5}_H \, \mathbf{10}_M^i \, \mathbf{10}_M^j$$



Hayashi et al. '09

Cecotti et al. '10

T-branes, $[\langle\Phi\rangle, \langle\bar{\Phi}\rangle] \neq 0$

$$\langle\Phi\rangle \sim \begin{pmatrix} 0 & 1 & 0 \\ x & 0 & 0 \\ 0 & 0 & y \end{pmatrix} dx \wedge dy$$

$$\omega \wedge F + \frac{1}{2}[\Phi, \bar{\Phi}] = 0$$

Local model data

holomorphic

- ✦ $\langle \Phi \rangle$ contains the 7-brane intersection angles: μ, m
- ✦ Non-perturbative effect encoded in ϵ, θ_0

physical

- ✦ $\langle F \rangle$ generates chirality and family replication at matter curves, enters via flux densities: N_i, M_j
- ✦ $\langle F_Y \rangle$ breaks $G_{GUT} \to G_{MSSM}$, enters via densities N_Y, $\tilde{N}_Y$

Example: SU(5)

$$\begin{array}{c} 5_{H_u} \times 10 \times 10 \\ \bar{5}_{H_d} \times \bar{5} \times 10 \end{array} \xrightarrow{F_Y} \begin{array}{l} \lambda_u^{ij} Q^i U^j H_u \\ \lambda_d^{ij} Q^i D^j H_d + \lambda_l^{ij} L^i E^j H_d \end{array}$$

The presence of $\langle F \rangle$ also localises wavefunctions along matter curves and allows an ultra-local computation of Yukawa couplings

Not all of these parameters will be independent in a global model

General results

- Assuming $\theta_0 = i(\theta_{00} + x\, \theta_x + y\, \theta_y)$ one obtains, at the holomorphic level

$$\frac{Y^{\rm hol}}{Y^{\rm hol}_{33}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} + \mathcal{O}(\epsilon) \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

and so a family hierarchy $(1, \epsilon, \epsilon^2)$, still independent of worldvolume fluxes

- At the physical level the normalisation factors depend on family and hypercharge

$$Y^{ij}_{\rm phys} = \gamma_i \gamma_j \gamma_H Y^{ij}_{\rm hol}$$

$$\gamma_i^{-2} \propto \int dy\, e^{-\pi|F||y|^2} |f^i(y)|^2$$

- Higher hypercharge ⇒ thinner wavefunction

⇒ larger quotients

$$\gamma_i \propto \left(\frac{\pi}{\sqrt{2}} |F|, \sqrt{\pi} |F|^{1/2}, 1 \right)$$

$$\frac{m_\mu}{m_\tau} \simeq 3.3 \frac{m_s}{m_b} \quad \text{for} \quad \frac{N_Y}{N} \simeq 1.8$$

$$F = N + M_y$$

Unifying Yukawas

- Remarkably, local data are quite similar in both cases. We obtain realistic Yukawas for the third and second generation (including large Y_t) by taking
    - Small intersection angles O(0.1)
    - Not so small flux densities O(0.1)-O(0.5)
    - $\epsilon \sim 10^{-4}$
- This suggests that both Yukawa points could be very close to each other (even coincident) within S_{GUT} → enhancement to E_7 or E_8

Also motivated by CKM matrix, neutrino sector and computability of all relevant GUT couplings

Heckman, Tavanfar, Vafa '09
Palti '12

The E_8 story

F.M., Regalado, Zoccarato '15

- The goal is to build a local model where all Yukawas arise from a single patch of S_{GUT}, described by E_8 symmetry and assuming
  - ✦ SU(5) GUTs: $E_8 \to SU(5)_{GUT} \times SU(5)_{\perp}$
  - ✦ Reconstructible T-branes
  - ✦ More than two 5-curves
  - ✦ Rank one Yukawas at tree-level
  - ✦ Hierarchy $(1, \epsilon, \epsilon^2)$ for charged fermions after np effects

All these requirements are imposed at the holomorphic level

E_8 T-brane models

- We classify local models in terms of the T-brane structure of $\Phi \subset \mathfrak{su}(5)_\perp$
  We look at its block diagonal decomposition in the fund. representation

  - 4+1 or $\mathbb{Z}_4$ model → only two matter curves 
  - 3+2 or $\mathbb{Z}_3$ x $\mathbb{Z}_2$ models → down-type hierarchy $(1, \epsilon^2, \epsilon^2)$ 
  - 2+2+1 or $\mathbb{Z}_2$ x $\mathbb{Z}_2$ models → one good option 

The options depend on which matter curves
host the SM fermions

The E_8 model

- We take the following 8d Higgs profile

$$\langle \Phi_{xy} \rangle = \lambda_1 Q_1 + \lambda_2 Q_2 + m(E_1^+ + mxE_1^-) + \tilde{m}(E_2^+ + \tilde{m}yE_2^-)$$

$$\mathbf{\Phi_5} = \begin{pmatrix} \lambda_1 & m & 0 & 0 & 0 \\ m^2 x & \lambda_1 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & \tilde{m} & 0 \\ 0 & 0 & \tilde{m}^2 y & \lambda_2 & 0 \\ 0 & 0 & 0 & 0 & -2(\lambda_1 + \lambda_2) \end{pmatrix}$$

$$\lambda_1 = \mu_1^2(ax - y)$$

$$\lambda_2 = \mu_2^2(bx - y) + \kappa$$

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Matter curves:

$$\mathbf{10_a : \lambda_1^2 - m^3 x = 0}, \quad \mathbf{10_b : \lambda_2^2 - m^3 x = 0}, \quad \mathbf{10_c : \lambda_1 + \lambda_2 = 0}$$

$$\mathbf{5_a : \lambda_1 = 0}, \quad \mathbf{5_b : \lambda_2 = 0}, \quad \mathbf{5_c : (\lambda_1 + 2\lambda_2)^2 - m^3 x = 0},$$

$$\mathbf{5_d : (2\lambda_1 + \lambda_2)^2 - \tilde{m}^3 x = 0}, \quad \mathbf{5_e : (\lambda_1 + \lambda_2)^4 - 2(\lambda_1 + \lambda_2)^2 (m^3 x + \tilde{m}^3 y) + (m^3 x - \tilde{m}^3 y)^2 = 0}$$

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Model A: $\bar{\mathbf{5}}_M = \mathbf{5}_c$

Model B: $\bar{\mathbf{5}}_M = \mathbf{5}_b$

Model A vs. Model B

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Model A: $\bar{\mathbf{5}}_M = \mathbf{5}_c$

Model B: $\bar{\mathbf{5}}_M = \mathbf{5}_b$

coupling with singlet

$$W \supset \lambda H_u L S$$

Dirac neutrino masses if S = N_R

$$W \supset \lambda H_u H_d S$$

coupling with vector pairs in $\mathbf{5_e}$

$$W \supset \frac{\tilde{S}^2}{\Lambda} H_u H_d$$

effective μ-term

Model A vs. Model B

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Model A more suggestive

Computing physical Yukawas

- We take model A and for simplicity set a=b=1 in $\begin{cases} \lambda_1 = \mu_1^2(ax - y) \\ \lambda_2 = \mu_2^2(bx - y) + \kappa \end{cases}$

- We add worldvolume fluxes to localise wavefunctions around Yukawa point

- The eigenvalues for the physical Yukawas read

$$Y_t = \frac{\pi^2\,\gamma_U\,\gamma_{10,3}^{Q}\gamma_{10,3}^{U}}{2\rho_m\rho_\mu}, \qquad Y_c = \tilde{\epsilon}\,\frac{\pi^2\,\gamma_U\,\gamma_{10,2}^{Q}\gamma_{10,2}^{U}}{4\rho_m\rho_\mu^2}, \qquad Y_u = \mathcal{O}(\tilde{\epsilon}^2)$$

$$Y_b = \frac{\pi^2\,\gamma_D\,\gamma_{10,3}^{Q}\gamma_{5,3}^{D}}{2d\rho_m\rho_\mu}, \qquad Y_s = \tilde{\epsilon}\,\frac{\pi^2\,\gamma_D\,\gamma_{10,2}^{Q}\gamma_{5,2}^{D}}{4d^2\rho_m\rho_\mu^2}, \qquad Y_d = \mathcal{O}(\tilde{\epsilon}^2)$$

$$Y_\tau = \frac{\pi^2\,\gamma_D\,\gamma_{10,3}^{E}\gamma_{5,3}^{L}}{2d\rho_m\rho_\mu}, \qquad Y_\mu = \tilde{\epsilon}\,\frac{\pi^2\,\gamma_D\,\gamma_{10,2}^{E}\gamma_{5,2}^{L}}{4d^2\rho_m\rho_\mu^2}, \qquad Y_e = \mathcal{O}(\tilde{\epsilon}^2)$$

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Holomorphic parameters $\quad \tilde{\epsilon} = \epsilon(\theta_x + b\theta_y), \quad \rho_m = \frac{m^2}{m_*^2}, \quad \rho_\mu = \frac{\mu_1^2}{m_*^2}, \quad d = \frac{\mu_2^2}{\mu_1^2}$

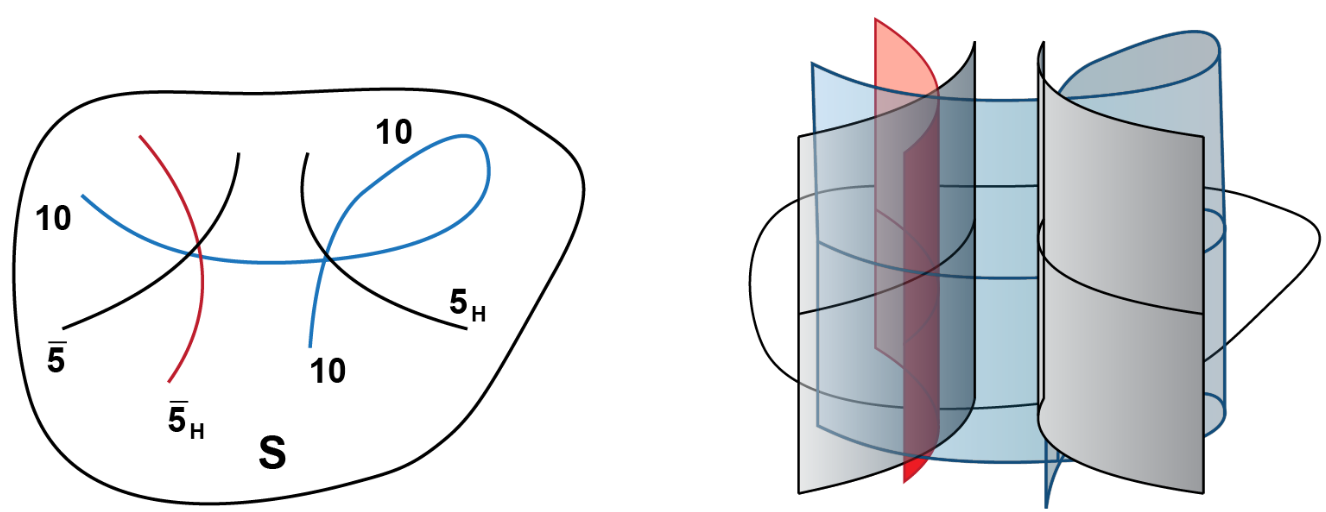
Flux densities enter the norm factors γ's in a complicated way $\quad c, N_1, N_2, M_1, M_2, N_Y, \tilde{N}_Y$

CKM matrix

- Switching on the parameter κ in we separate the Yukawa points p_{up} and p_{down} *and induce a source of family mixing*

$$\begin{cases} \lambda_1 = \mu_1^2(ax - y) \\ \lambda_2 = \mu_2^2(bx - y) + \kappa \end{cases}$$

comparing V_{tb} with the experimental value sets the separation of points ~ $R_{GUT}/100$



Taken from Aparicio et al. '12

Fitting E_8 Yukawas

- Putting all together one is able to fit charged fermion masses for the 3rd and 2nd families at the GUT scale assuming an MSSM scheme

Ross & Serna '07

$\tan\beta$	10	38	50
m_d/m_s	$5.1 \pm 0.7 \times 10^{-2}$	$5.1 \pm 0.7 \times 10^{-2}$	$5.1 \pm 0.7 \times 10^{-2}$
m_s/m_b	$1.9 \pm 0.2 \times 10^{-2}$	$1.7 \pm 0.2 \times 10^{-2}$	$1.6 \pm 0.2 \times 10^{-2}$
m_e/m_μ	$4.8 \pm 0.2 \times 10^{-3}$	$4.8 \pm 0.2 \times 10^{-3}$	$4.8 \pm 0.2 \times 10^{-3}$
m_μ/m_τ	$5.9 \pm 0.2 \times 10^{-2}$	$5.4 \pm 0.2 \times 10^{-2}$	$5.0 \pm 0.2 \times 10^{-2}$
m_b/m_τ	0.73 ± 0.03	0.73 ± 0.03	0.73 ± 0.04
Y_τ	0.070 ± 0.003	0.32 ± 0.02	0.51 ± 0.04
Y_b	0.051 ± 0.002	0.23 ± 0.01	0.37 ± 0.02
Y_t	0.48 ± 0.02	0.49 ± 0.02	0.51 ± 0.04

Fitting E_8 Yukawas

- Putting all together one is able to fit charged fermion masses for the 3^{rd} and 2^{nd} families at the GUT scale assuming an MSSM scheme

HOWEVER

- No large region of parameters reproduces such realistic values for the choices made

- This is partly because local flux densities need to satisfy certain inequalities to induce the appropriate local chirality in matter curves

- In particular these inequalities are incompatible with vanishing local chirality for Higgs triplets when we set a=b

The E_7 story

Carta, F.M., Zoccarato '15

- The case of E_7 has less possibilities since $\mathfrak{g}_\perp = \mathfrak{su}(3) \oplus \mathfrak{u}(1)$
  - ✦ Φ_3 diagonal → no T-brane

  - ✦ Φ_3 2+1 block diagonal → promising $(1, \epsilon, \epsilon^2)$ hierarchy

  - ✦ Φ_3 single block → vanishing up-type Yukawas

Again we have different options when
assigning the SM fermions to matter curves

The E_7 model

- We take the following 8d Higgs profile

$$\langle \Phi_{xy} \rangle = \lambda_1 Q_1 + \lambda_2 Q_2 + m(E_1^+ + mxE_1^-) \quad \begin{cases} \lambda_1 = \mu_1^2(ax - y) \\ \lambda_2 = \mu_2^2(bx - y) + \kappa \end{cases}$$

Matter curves:

$$\mathbf{10_a} : \lambda_\mathbf{1}^\mathbf{2} - \mathbf{m^3 x = 0}, \quad \mathbf{10_b} : \lambda_\mathbf{1} - \lambda_\mathbf{2} = \mathbf{0}$$

$$\mathbf{5_a} : \lambda_\mathbf{1} = \mathbf{0}, \quad \mathbf{5_b} : \lambda_\mathbf{1} + \lambda_\mathbf{2} = \mathbf{0}, \quad \mathbf{5_c} : \lambda_\mathbf{2}^\mathbf{2} - \mathbf{m^3 x = 0}$$

The E_7 model

- We take the following 8d Higgs profile

$$\langle \Phi_{xy} \rangle = \lambda_1 Q_1 + \lambda_2 Q_2 + m(E_1^+ + mxE_1^-) \qquad \begin{cases} \lambda_1 = \mu_1^2(ax - y) \\ \lambda_2 = \mu_2^2(bx - y) + \kappa \end{cases}$$

Matter curves:

$$\mathbf{10_a} : \lambda_1^2 - m^3 x = 0, \qquad \mathbf{10_b} : \lambda_1 - \lambda_2 = 0$$

$$\mathbf{5_a} : \lambda_1 = 0, \qquad \mathbf{5_b} : \lambda_1 + \lambda_2 = 0, \qquad \mathbf{5_c} : \lambda_2^2 - m^3 x = 0$$

Model A: $\bar{\mathbf{5}}_M = \mathbf{5}_c$

Model B: $\bar{\mathbf{5}}_M = \mathbf{5}_b$

The E8 model

- We take the following 8d Higgs profile

$$\langle \Phi_{xy} \rangle = \lambda_1 Q_1 + \lambda_2 Q_2 + m(E_1^+ + mxE_1^-) + \tilde{m}(E_2^+ + \tilde{m}yE_2^-)$$

$$\Phi_{\mathbf{5}} = \begin{pmatrix} \lambda_1 & m & 0 & 0 & 0 \\ m^2 x & \lambda_1 & 0 & 0 & 0 \\ 0 & 0 & \lambda_2 & \tilde{m} & 0 \\ 0 & 0 & \tilde{m}^2 y & \lambda_2 & 0 \\ 0 & 0 & 0 & 0 & -2(\lambda_1 + \lambda_2) \end{pmatrix}$$

$$\lambda_1 = \mu_1^2(ax - y)$$

$$\lambda_2 = \mu_2^2(bx - y) + \kappa$$

Matter curves:

$$\mathbf{10_a : \lambda_1^2 - m^3 x = 0}, \quad \mathbf{10_b : \lambda_2^2 - m^3 x = 0}, \quad \mathbf{10_c : \lambda_1 + \lambda_2 = 0}$$

$$\mathbf{5_a : \lambda_1 = 0}, \quad \mathbf{5_b : \lambda_2 = 0}, \quad \mathbf{5_c : (\lambda_1 + 2\lambda_2)^2 - m^3 x = 0},$$

$$\mathbf{5_d : (2\lambda_1 + \lambda_2)^2 - \tilde{m}^3 x = 0}, \quad \mathbf{5_e : (\lambda_1 + \lambda_2)^4 - 2(\lambda_1 + \lambda_2)^2(m^3 x + \tilde{m}^3 y) + (m^3 x - \tilde{m}^3 y)^2 = 0}$$

Model A: $\bar{\mathbf{5}}_M = \mathbf{5}_c$

Model B: $\bar{\mathbf{5}}_M = \mathbf{5}_b$

The E_7 model

- We take the following 8d Higgs profile

$$\langle \Phi_{xy} \rangle = \lambda_1 Q_1 + \lambda_2 Q_2 + m(E_1^+ + mxE_1^-) \qquad \begin{cases} \lambda_1 = \mu_1^2(ax-y) \\ \lambda_2 = \mu_2^2(bx-y)+\kappa \end{cases}$$

Matter curves:

$$\mathbf{10_a}: \lambda_1^2 - m^3 x = 0, \qquad \mathbf{10_b}: \lambda_1 - \lambda_2 = 0$$

$$\mathbf{5_a}: \lambda_1 = 0, \qquad \mathbf{5_b}: \lambda_1 + \lambda_2 = 0, \qquad \mathbf{5_c}: \lambda_2^2 - m^3 x = 0$$

Model A: $\bar{\mathbf{5}}_M = \mathbf{5}_c$ Model B: $\bar{\mathbf{5}}_M = \mathbf{5}_b$

But now there is no criterion regarding the neutrino sector.
We analyse both models on equal footing

Fitting E_7 Yukawas

- We apply the same analysis made for E_8 to these E_7 models, computing the physical Yukawas for the 2nd and 3rd families for models A and B

- We open new regions in parameter space by allowing arbitrary a, b. The latter allows to maintain vanishing local chirality for Higgs triplets (locally we have vector-like triplets)

- Both models have a complicated dependence on worldvolume flux densities through the normalisation factors γ, but:

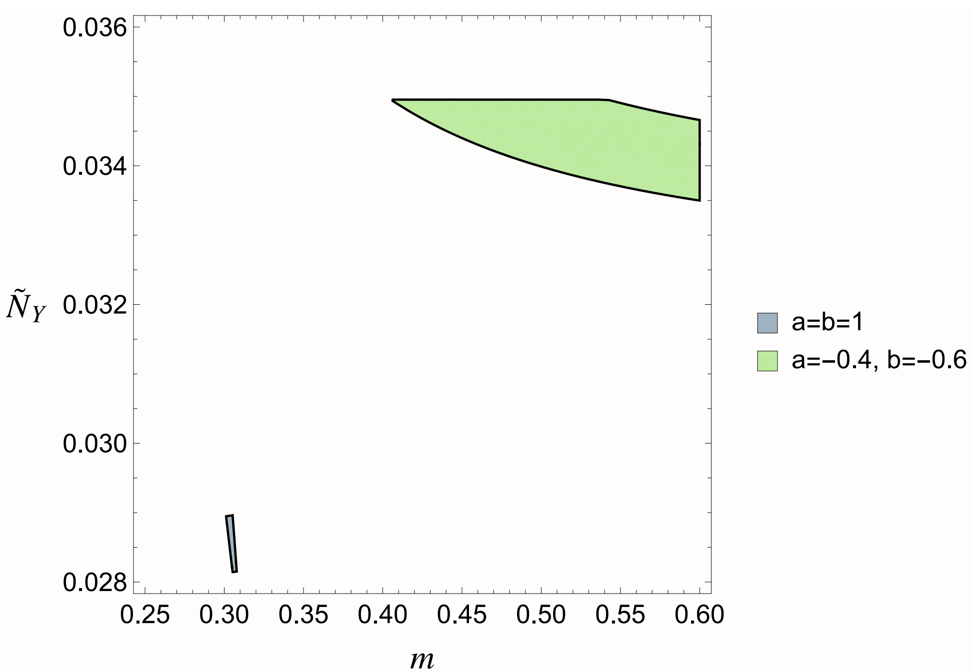
  ✦ For Model A, ratios of mass ratios have simpler expressions

$$\frac{m_\mu/m_\tau}{m_s/m_b} = \sqrt{\frac{(x-1)\left(y-\frac{1}{2}\right)}{\left(x-\frac{1}{6}\right)\left(y-\frac{1}{3}\right)}}\,, \qquad x = -\frac{M_1}{\tilde{N}_Y}\,, \quad y = -\frac{M_2}{\tilde{N}_Y}$$

  ✦ For Model B this is not true, and we cannot satisfy $\frac{m_\mu/m_\tau}{m_s/m_b} = 3.3 \pm 1$ for fluxes that induce the appropriate local chirality

Fitting E_7 Yukawas

- Model A displays large regions in local parameter space where we achieve
  - ✦ Appropriate local chirality (vanishing for Higgs triplets)
  - ✦ Realistic fermion masses
- Compared to previous E_8 analysis



Bonus: more diluted fluxes!!

typical value:
tan β ~ 10-20

Fitting E_7 and E_8 Yukawas

- Model A displays large regions in local parameter space where we achieve
  - ✦ Appropriate local chirality (vanishing for Higgs triplets)
  - ✦ Realistic fermion masses
- In fact, the structure of Yukawa couplings is identical to the E_8 model discussed previously, despite the more complicated T-brane structure of the latter → we scan over the same Yukawa values
- More precisely Models A and B correspond to each other in both cases. So in the E_8 case Model A is selected for phenomenological reasons even ignoring the neutrino sector and the generation of a μ-term.

Conclusions

- Precise computation of Yukawa couplings is so far limited to ultra-local computation via dimensional reduction of the 7-brane 8d gauge theory
- Such ultra-local models depend on many parameters which may or may not be independent or even realisable in a global completion
- Even so, reproducing realistic fermion masses and mixing is hard to achieve. Such fitting becomes simpler family hierarchies are naturally generated by some mechanism. We have explored the scenario $[(1, \epsilon, \epsilon^2)]$ of rank one Yukawas + non-perturbative effects, in which T-branes are key ingredient.
- This proposal leads naturally to models of E_7 or E_8 enhancement. We have analysed both of them and appropriate fitting of fermion masses have led us to a unique structure of matter curves in both cases.
- How this structure may be embedded in a global completion remains so far a challenge, hopefully to be overcome before F-theory turns 30.