Dualities between (Non-)Geometric Heterotic String Vacua via F-Theory

joint work with A. Font, I. García-Etxebarria, D. Lüst and S. Massai:
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- Because (most probably) amount of such vacua is much larger than geometric ones;
- Step in this direction is understanding of following 6d heterotic vacua and dualities among them;

Heterotic String Theory on T^2 I

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  - complexified Kähler modulus: $\rho = \int_{T^2} B + \omega \wedge \bar{\omega}$;

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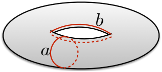

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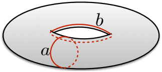

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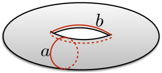

  - Wilson line moduli: $\beta^i = \int_a A^i + i \int_b A^i$;
- Moduli space of het. torus compactification is (Narain space):

$$O(2) \times O(2 + n_{WL}) \backslash O(2, 2 + n_{WL}) / O(2, 2 + n_{WL}, \mathbb{Z})\,;$$

[Narain '86]

Main case of interest: $n_{WL} = 1$ (and $n_{WL} = 0$);

Heterotic String Theory on T^2 II

- For $n_{WL} = 1$, above Narain space can be mapped to Siegel upper half plan of genus two

$$\mathbb{H}_2 = \left\{ \Omega = \begin{pmatrix} \tau & \beta \\ \beta & \rho \end{pmatrix} \middle| \Im(\det(\Omega)) > 0 \wedge \Im(\rho) > 0 \right\}$$

quotient by $Sp(4,\mathbb{Z})$-action $\Omega \to (A\Omega + B)(C\Omega + D)^{-1}$ with

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- Above moduli fields are entries of Ω;
- $\mathbb{H}_2 / Sp(4,\mathbb{Z})$ is (complex structure) moduli space of genus two curves;

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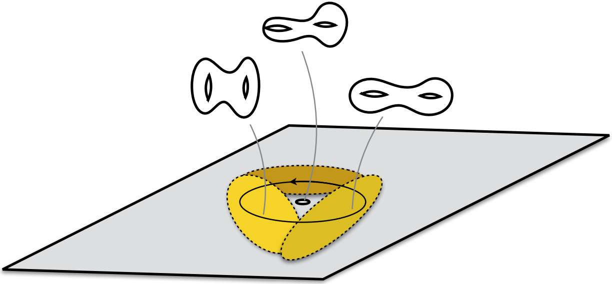

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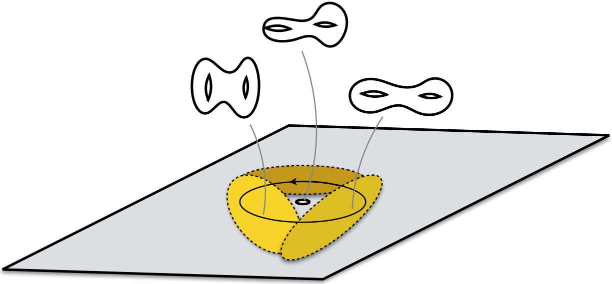

- That way end up with non-geometric compactification; Because allow for identifications with inverse of metric, or even total mixing of three moduli τ, ρ and β;

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- Natural question: can we find identification/interpretation of physical objects at all these degenerations?

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- But for $n_{WL} = 0$ and $n_{WL} = 1$, there is even identification in terms of moduli space; [Cardoso '96, McOrist et al. '10, Malmendier&Morrison '14]

Duality with F-Theory II

- For both cases ($n_{WL} = 0, 1$) F-Theory K3 given by

$$y^2 = x^3 + (a\,u^4 + c\,u^3)\,x + (b\,u^6 + d\,u^5 + e\,u^7) = 0\,;$$

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- K3 has II^* sing. at $u = \infty$ and III^* sing. (or II^* in case of $c = 0$) at $u = 0$; Therefore, Picard number of K3 is 17 ($n_{WL} = 1$) or 18 ($n_{WL} = 1$), respectively;

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- Moduli spaces agree with het. ones and can even be mapped:
  - $n_{WL} = 1$ ($e = 1$): $a = -\frac{1}{48}\psi_4(\Omega)$, $b = -\frac{1}{864}\psi_6(\Omega)$, $c = -4\chi_{10}(\Omega)$, $d = \chi_{12}(\Omega)$;
    Siegel modular forms ψ_4, ψ_6, χ_{10} and χ_{12} fix Ω uniquely;

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  - $n_{WL} = 0$ ($c = 0$): $j(\tau)j(\rho) = -1728^2\frac{a^3}{27d\,e}$, $(j(\tau) - 1728)(j(\rho) - 1728) = 1728^2\frac{b^2}{4d\,e}$ and $\beta = 0$;

Duality with F-Theory III

- Therefore, have identification of $E_8 \times E_7$ K3 with genus two curve, and identification of $E_8 \times E_8$ K3 with two tori glued together at one point (degenerated hyperelliptic curve);
- Further, if genus two curve is given in terms of sextic, i.e.

$$y^2 = c_6 x^6 + c_5 x^5 + \dots ,$$

  then a, b, c, d of K3 are simply given by Igusa-Clebsch invariants of sectic, i.e. polynomials of coefficients c_i;
- Fortunately, all degenerations of genus two curves are in this form; Therefore, can easily map them to (singularities of) K3;
- Note, to go from K3 to representation of hyperelliptic curve is much more involved;

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- Need to blow up base to resolve such singularities; [Miranda '83, Grassi '93, Aspinwall&Morrison '97]
- To determine which base blow-ups must be done, take sort of toric approach; Write down f and g in terms of its (leading) monomials in u and t,

$$f = \sum_i f_i \, u^{m_i^1} t^{m_i^2} \,, \qquad g = \sum_i g_i \, u^{l_i^1} t^{l_i^2} \,,$$

and ask for allowed 'blow-up direction' **n** such that hypersurface

$$y^2 = x^3 + f\,x + g$$

is still CY;

Resolutions of F-Theory Side II

- Condition on vanishing first Chern class translates to:

$$(m_i^1 - 4)n_1 + (m_i^2 - 4)n_2 \geq -4 \quad \text{and} \quad (l_i^1 - 6)n_1 + (l_i^2 - 6)n_2 \geq -6$$

for all $\mathbf{m}^i$, $\mathbf{l}^i$ with $\mathbf{n} = (n_1, n_2)$ direction of blow-up, i.e. $t,\ u \to e^{n_1}t, e^{n_2}u$;

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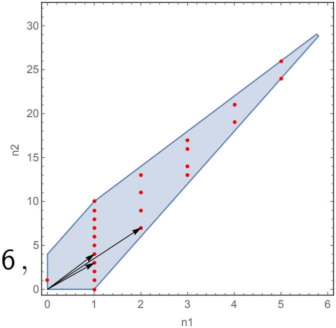
Ex. het. $[\mathrm{II}^* - \mathrm{I}_0]$ singularity: Solution set $\{\mathbf{n}^j\}$ to inequalities

$$(6, -1)\cdot\mathbf{n} \geq -4\,,\ (1, 0)\cdot\mathbf{n} \geq -4\,,$$

for f and for g:

$$(4, -1)\cdot\mathbf{n} \geq -6\,,\ (-1, 0)\cdot\mathbf{n} \geq -6\,,$$
$$(-6, 1)\cdot\mathbf{n} \geq -6\,;$$



Resolutions of F-Theory Side III

- Not all singularities which obtained from degenerations of genus two can be resolved on F-Theory side; Can give simple criterion for when can resolve in above way:

$$\mu(a) < 4 \quad \text{or} \quad \mu(b) < 6 \quad \text{or} \quad \mu(c) < 10 \quad \text{or} \quad \mu(d) < 12\,,$$

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- From $\{\mathbf{n}^j\}$ read off self-intersection numbers, because for adjacent $\mathbf{n}^j$ one has $a_j \mathbf{n}^j = \mathbf{n}^{j+1} + \mathbf{n}^{j-1}$ with a_j self-intersection number of $\mathbf{n}^j$;

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- Further vanishing orders of f, g and Δ along e_i's are immediately obtained;
- To work out gauge algebras and matter representations standard techniques have to be applied; [Bershadsky '96, Katz&Vafa '96,Grassi&Morrison '12,...]

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- From map, obtain following CY3 singularity (to leading orders):
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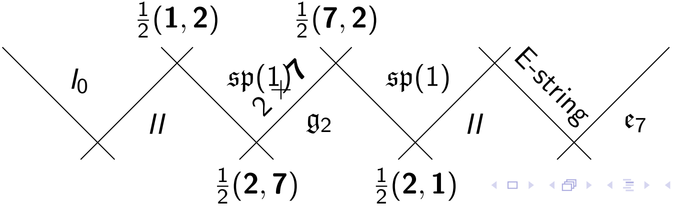
- Get following resolved geometry:



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- In subclass of elliptic models, obtain the following dual theories:

$\mu(I_{10})$	dual models
2	$[\mathrm{I}_0 - \mathrm{II}]_{0112}$
3	$[\mathrm{I}_0 - \mathrm{III}]_{0113}$
4	$[\mathrm{I}_0 - \mathrm{IV}]_{0224}$, $[\mathrm{II} - \mathrm{II}]_{0224}$
5	$[\mathrm{III} - \mathrm{II}]_{0225}$
6	$[\mathrm{I}_0 - \mathrm{I}_0^*]_{0226}$, $[\mathrm{III} - \mathrm{III}]_{0226}$, $[\mathrm{IV} - \mathrm{II}]_{0336}$
7	$[\mathrm{IV} - \mathrm{III}]_{0337}$
8	$[\mathrm{I}_0 - \mathrm{IV}^*]_{0448}$, $[\mathrm{IV} - \mathrm{IV}]_{0448}$, $[\mathrm{I}_0^* - \mathrm{II}]_{0338}$
9	$[\mathrm{I}_0 - \mathrm{III}^*]_{0339}$, $[\mathrm{I}_0^* - \mathrm{III}]_{0339}$
10	$[\mathrm{I}_0 - \mathrm{II}^*]_{0\,5\,5\,10}$, $[\mathrm{IV}^* - \mathrm{II}]_{0\,5\,5\,10}$, $[\mathrm{I}_0^* - \mathrm{IV}]_{0\,4\,4\,10}$
11	$[\mathrm{II} - \mathrm{III}^*]_{0\,4\,4\,11}$, $[\mathrm{IV}^* - \mathrm{III}]_{0\,5\,5\,11}$

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- Start from $\mathrm{III}-\mathrm{III}$ and apply following moves:

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- In-depth study of het. EOM and its solution for non-vanishing Wilson line (for $\beta = 0$ see D. Lüst's talk);

Thank you for your attention!