Bounding the number of tensor multiplets in 6D F-theory vacua

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6D String Universality

[Kumar/Taylor, Kumar/Morrison/Taylor]

Are all 6D $\mathcal{N} = 1$ supergravity theories embeddable into string theory?

- First superstring revolution [Alvarez-Gaumé/Witten, Green/Schwarz, Gross/Harvey/Martinec/Rohm]
- Swampland [Vafa, Ooguri/Vafa]

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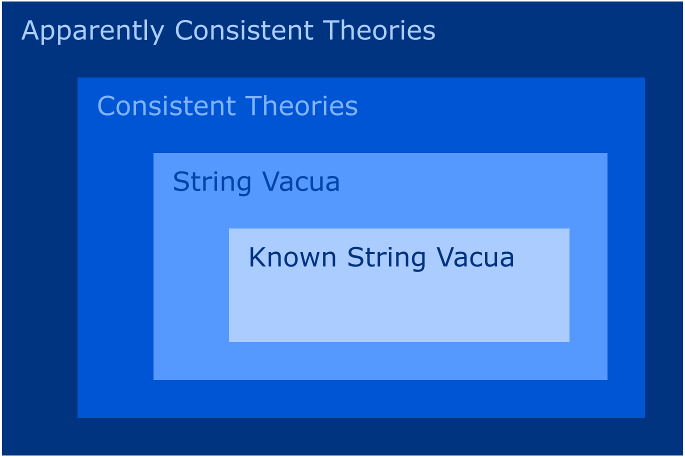
6D String Universality: Strategy



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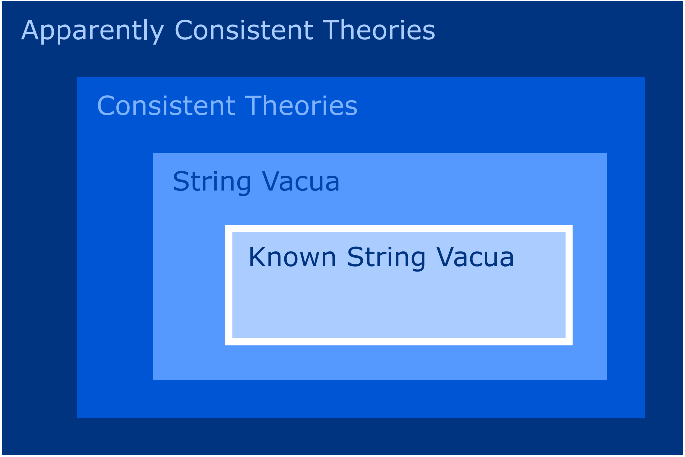

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- All known 6D $\mathcal{N} = 1$ string vacua can be embedded into geometric F-theory!* [Vafa, Morrison/Vafa]
  - Geometric F-theory vacuum $\Leftrightarrow$ Elliptically fibered CY3 $X \to B$
  - Finite number of families of X [Grassi, Gross]
- Bounds on physical parameters?

*Modulo discrete fluxes, which do not affect the bound we study in this talk.

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6D String Universality

Massless spectrum

- Number of tensor multiplets : T
- Gauge algebra : $\mathfrak{g} = \mathfrak{g}_1 \oplus \mathfrak{g}_2 \oplus \cdots \oplus \mathfrak{g}_n$
- Matter content : (R_i^I)
- Scalars vevs: $j \in SO(1,T)/SO(T)$

Multiplet	Field Content
Gravity	$(g_{\mu\nu}, \psi^+_\mu, B^+_{\mu\nu})$
Tensor	$(\phi, \chi^-, B^-_{\mu\nu})$
Vector	(A_μ, λ^+)
Hyper	$(4\varphi, \psi^-)$

Can we bound T?

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- $T = h^2(B) - 1$
  - Which surfaces B can be used as a base for a smooth elliptically fibered Calabi-Yau threefold? $\Rightarrow$ *"Admissible surface"*
- Some results on bounds on T exist in the literature.
  - B : semi-toric ($\supset$ toric) $\Rightarrow$ $T \leq 193$ [Morrison/Taylor, Martini/Taylor]
  - $h^{2,1}(X) \geq 150$ $\Rightarrow$ $T \leq 100$ [Taylor/Wang]

Main result: $T \leq 35908$

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- ∃ Infinite classes of theories with T unbounded, that satisfy all known low-energy consistency conditions.
  (cf. Green/Schwarz/West, Sagnotti, Kumar/Taylor, Seiberg/Taylor)

$$\begin{aligned} \mathfrak{g} &= \mathfrak{u}(1)^{\oplus 29k} \\ T &= k \quad (\geq 9) \\ \text{matter} &= 273 \times \cdot \end{aligned}$$

  - Not realizable in F-theory. ($\Rightarrow T \leq 9$ for all abelian theories.)
  - Do these theories violate unknown consistency conditions? (Unitarity? Causality?)

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1. Motivation - 6D String Universality
2. Strategy
3. Baby problem
4. The general problem
5. Summary

Strategy

Smooth CY threefold $X \to B$:

$$y^2 = x^3 + Fx + G, \quad \Delta = 4F^3 + 27G^2$$

with $F \in \mathcal{O}(-4K)$, $G \in \mathcal{O}(-6K)$, $\Delta \in \mathcal{O}(-12K)$.

$\{C_i\}$: (Irreducible) curves with $C_i^2 \leq -2$.

- C_i must have genus zero. (Same for $C^2 = -1$.)
- Intersection patterns of C_i are restricted.
- $\exists$ minimal bounds on multiplicity of C_i in (F, G, Δ).

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Non-Higgsable Clusters

[Morrison/Taylor]

Name	Curves	Gauge	Matter	(f, g, δ)
12	(-12)	$\mathfrak{e}_8$	-	$(4, 5, 10)$
8	(-8)	$\mathfrak{e}_7$	-	$(3, 5, 9)$
7	(-7)	$\mathfrak{e}_7$	$\frac{1}{2}\mathbf{56}$	$(3, 5, 9)$
6	(-6)	$\mathfrak{e}_6$	-	$(3, 4, 8)$
5	(-5)	$\mathfrak{f}_4$	-	$(3, 4, 8)$
4	(-4)	$\mathfrak{so}_8$	-	$(2, 3, 6)$
3	(-3)	$\mathfrak{su}_3$	-	$(2, 2, 4)$
32	(-3)-(-2)	$\mathfrak{g}_2 \oplus \mathfrak{su}_2$	$(\mathbf{7} + \mathbf{1}, \frac{1}{2}\mathbf{2})$	$(2, 3, 6), (1, 2, 3)$
322	(-3)-(-2)-(-2)	$\mathfrak{g}_2 \oplus \mathfrak{su}_2$	$(\mathbf{7} + \mathbf{1}, \frac{1}{2}\mathbf{2})$	$(2, 3, 6), (2, 2, 4), (1, 1, 2)$
232	(-2)-(-3)-(-2)	$\mathfrak{su}_2 \oplus \mathfrak{so}_7 \oplus \mathfrak{su}_2$	$(\mathbf{1}, \mathbf{8}, \frac{1}{2}\mathbf{2}) + (\frac{1}{2}\mathbf{2}, \mathbf{8}, \mathbf{1})$	$(1, 2, 3), (2, 4, 6), (1, 2, 3)$
·	(-2) curves	-	-	$(0, 0, 0)$

Multiplicity bounds

Multiplicity m_i of C_i within effective divisor D:

$$(C_j \cdot C_i)\, m_i \leq [D] \cdot C_j\,, \quad m_i \geq 0\,.$$

Applied to $[D] = -nK$ for $n = 4, 6, 12$.

Residual divisors

$$R_F = -4K - \sum_i f_i C_i\,, \quad R_G = -6K - \sum_i g_i C_i\,, \quad R_\Delta = -12K - \sum_i \delta_i C_i\,.$$

R_F, R_G and R_Δ are effective. (cf. Cordova/Dumitrescu/Intriligator)

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Some manipulations

$$R_F^2 = (4K + \sum_i f_i C_i)^2 = 16(9 - T) + 8K \cdot f_i C_i + f_i f_j C_i \cdot C_j$$

$$9 = T + \frac{R_F^2}{16} + \sum_t \alpha_t n_t, \quad t \in \{12, 8, \cdots, 232\}$$

$$\alpha_t \equiv -\frac{1}{16}(8 f_i K \cdot C_i + f_i f_j C_i \cdot C_j), \quad C_i : \text{curves in NHC } t$$

Base equations

$$9 = T + \frac{R_F^2}{16} + \sum_t \alpha_t n_t \,, \quad 9 = T + \frac{R_G^2}{36} + \sum_t \beta_t n_t \,, \quad 9 = T + \frac{R_\Delta^2}{144} + \sum_t \gamma_t n_t \,.$$

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An assumption

The NHC condition

For generic F and G (i.e., in the maximally Higgsed phase) the non-abelian gauge/matter content of the theory is given by a direct sum of NHCs.

Gravitational anomaly constraint

$$\Rightarrow \quad T = \frac{273 - \nu}{29} + \sum_t \left(\frac{V_t - H_t}{29} \right) n_t$$

$$\nu = H_n + H_{ab} - V_{ab} = h^{2,1}(X) + 1 + (H_{ab} - V_{ab}) \leq h^{21}(X) + 1$$

Strategy

Linear programming problem?

1. Obtain lower bounds on R_F^2, R_G^2 and R_Δ^2 that are linear with respect to n_t.
2. Obtain lower bounds on T that are linear with respect to n_t.
3. Solve the linear programming problem to obtain maximum value of $T(\nu)$ for given ν. Vary ν in the allowed range $\nu \leq 150$ and find

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B-folds

Problem becomes simple with following assumption:

$$R_F^2\,, \quad R_G^2\,, \quad R_\Delta^2 \quad \geq 0$$

for generic $F \in \mathcal{O}(-4K)$, $G \in \mathcal{O}(-6K)$.

Let's call $X \to B$ satisfying this property a B-fold.[†]

$$9 \geq \quad T + \sum_t \alpha_t n_t\,, \quad T + \sum_t \beta_t n_t\,, \quad T + \sum_t \gamma_t n_t\,.$$

[†]These manifolds are named after Andreas Braun, champion of the "F-Theory Mini Golf Tournament" held in Aspen in the summer of 2015. A bet was made that an elliptic fibration would be named after the winner of the tournament.

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- $N_C \equiv |\{C_i\}|$: Total number of all components of the NHCs
- N_{NHC} : Total number of all NHCs
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$$\sum_t (\alpha_t + N_t + 1)\, n_t\,,\ \sum_t (\beta_t + N_t + 1)\, n_t\,,\ \sum_t (\gamma_t + N_t + 1)\, n_t \leq 65$$

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$$T = \frac{273 - \nu}{29} + \sum_t \left(\frac{V_t - H_t}{29} \right) n_t\,, \quad \nu \leq 150$$

$$\Rightarrow T \leq 1454^{\ddagger}$$

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Outline

The general problem

$$R = \sum_c m_c c \quad \Rightarrow \quad R^2 = \sum_c m_c (c \cdot R)$$

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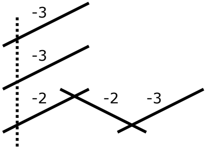
Negatively contributing curves

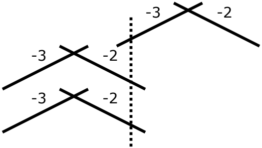
		12	8	7	6	5	4	3	32			322			232	
									$\dot{3}2$	$3\dot{2}$	$\dot{3}\dot{2}$	$\dot{3}22$	$3\dot{2}2$	$32\dot{2}$	$\dot{2}32$	$2\dot{3}2$
3					7	6										
32	$\dot{3}2$					12		×								11
	$3\dot{2}$		2	1				×	×		3					
	$\dot{3}\dot{2}$							×	×	×						
322	$\dot{3}22$					14		×	×	×	×					13
	$3\dot{2}2$				9	8		×	×	×	×	×				
	$32\dot{2}$	0						×	×	×	×	×	×			
232	$\dot{2}32$		5	4				×	×	×	×	×	×	×		
	$2\dot{3}2$						10	×	×	×	×	×	×	×	×	15

Negatively contributing curves

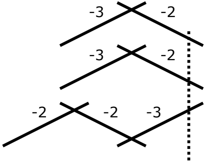
16.



17.



18.



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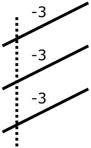
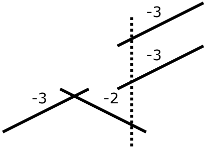


20.



1. Classify how a single NHC can connect to multiple (-1) curves.

Eg. The 7 cluster $\Rightarrow$ no. 1 or no. 4 curve

$$\begin{aligned} n_7 \quad \Rightarrow \quad & n_{7,(\cdot)}, \\ & n_{7,(1)}, n_{7,(1,1)}, n_{7,(1,1,1)}, n_{7,(1,1,1,1)}, n_{7,(1,1,1,1,1)}, \\ & n_{7,(4)}, n_{7,(1,4)}, n_{7,(1,1,4)}, n_{7,(1,1,1,4)}, \\ & n_{7,(4,4)}, n_{7,(1,4,4)} \\ & n_{7,(4,4,4)}. \end{aligned}$$

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2. Bound R_F^2, R_G^2 and R_Δ^2 as a linear functional of the 246 variables.

Eg. (-1) curve C: $7 - 1 - 3\dot{2}$.

$$R_G = mC + \cdots\,, \quad C \cdot R_G = -1$$

with $m \geq 1$. Then

$$R_G^2 = -m + \sum_{c \neq C} m_c (c \cdot R_G)$$

Is R_G^2 unbounded below?

Eg. Minus-one curve no. 1.

A (-7)-curve C_7 contributes to R_G^2 positively: $C_7 \cdot R_G = 5$.
$\widetilde{m}_{(-7)}$: degenerecy of the (-7) in G:

$$\widetilde{m}_7 \geq \frac{\sum_{c:(c\cdot C_7)=1} m_c + 30}{7}$$

The contribution of C_7 to R_G^2:

$$m_7 C_7 \cdot R_G = (\widetilde{m}_7 - 5) C_7 \cdot R_G \geq \frac{5}{7} \sum_{c:(c\cdot C_7)=1} m_c - \frac{25}{7}\,.$$

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$(-2)/(-3)$-curve C_2/C_3 of 32 cluster: $C_2 \cdot R_G = C_3 \cdot R_G = 1$.
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$$m_2 C_2 \cdot R_G + m_3 C_3 \cdot R_G \geq \frac{3}{5} \sum_{c:(c \cdot C_2)=1} m_c + \frac{4}{5} \sum_{c:(c \cdot C_3)=1} m_c - \frac{7}{5}$$

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Exceptions

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