Extremal Transitions with Enhanced Gauge Symmetry in M-theory on Calaby–Yau Fourfolds

M. Ronen Plesser

F Theory at 20 – Dave Day
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H. Jockers, S. Katz, D.R. Morrison, MRP, arXiv:1602.xxxx

Outline

1. Motivation
2. M-Theory Compactifications, Fluxes, and Superpotentials
3. Codimension-two Degenerations and Gauge Theory Predictions
4. The Geometric Transition
5. An Example

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- In M-theory on CY fourfolds fourform flux G plays an essential role, can use transitions to better understand it Intriligator, Jockers, Katz, Mayr, Morrison,MRP

M-theory Compactified on CY Fourfold X

- Spectrum ($\mathcal{N} = 2$):
  - $U(1)^{h^{1,1}(X)}$ gauge symmetry - scalars parameterize Kähler class $J = \sum x_i e^i$
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- Superpotential can be corrected by fivebrane instantons Witten

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In general we will not be able to set $G^\sharp = G^\flat = 0$ and $M^\sharp = M^\flat$

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- There are also mixed branches. In codimension $N - p$ in Coulomb branch we have unbroken

$$SU(k_1) \times \cdots \times SU(k_p) \times U(1)^{p-1} \quad (5)$$

- Higgs branch emanating from this has dimension

$$\dim_{\mathbb{C}} \mathcal{H}_{(k_1,\ldots,k_p)} = (p_g - 1)\sum_{i=1}^{p}(k_i^2 - 1) + p_g(p-1) \quad (6)$$

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- Along Coulomb branch turn on generic expectation values for $p_g(N-1)$ neutral chiral multiplets
- Now when $\phi \to 0$ unbroken gauge symmetry is $U(1)^{N-1}$ and massless charged matter is $(p_g-1)N(N-1)$ multiplets with charges of adjoints
- When these acquire expectation values Higgsing symmetry completely we find Higgs branch of dimension

$$\dim_{\mathbb{C}} \mathcal{H} = (p_g-1)N(N-1)-(N-1)+p_g(N-1) = (p_g-1)(N^2-1) \tag{7}$$

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- For $p_g \geq 1$ there will be no corrections to the superpotential
- In M-theory fivebrane instantons will be suppressed deep in the cone σ unless they wrap the shrinking divisors E_i. These can generate a superpotential if the divisor has $\chi(E_i, \mathcal{O}_{E_i}) = 1$. Fibration gives $\chi(E_i, \mathcal{O}_{E_i}) = p_g - q + 1$ here so (trivial) agreement. cf. Diaconescu, Gukov

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- X_0 is intersection of a quintic and the singular quadric $y_0 y_1 - y_2^2 = 0$

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- X_0 is intersection of a quintic and the singular quadric $y_0 y_1 - y_2^2 = 0$
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- What is the $G^\flat$ with which we obtain $X^\flat$?

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Thanks for Listening

Thanks, Dave, for all you've taught us so far. Happy Birthday!!!