Nilpotent deformations in 3d

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Based on work with

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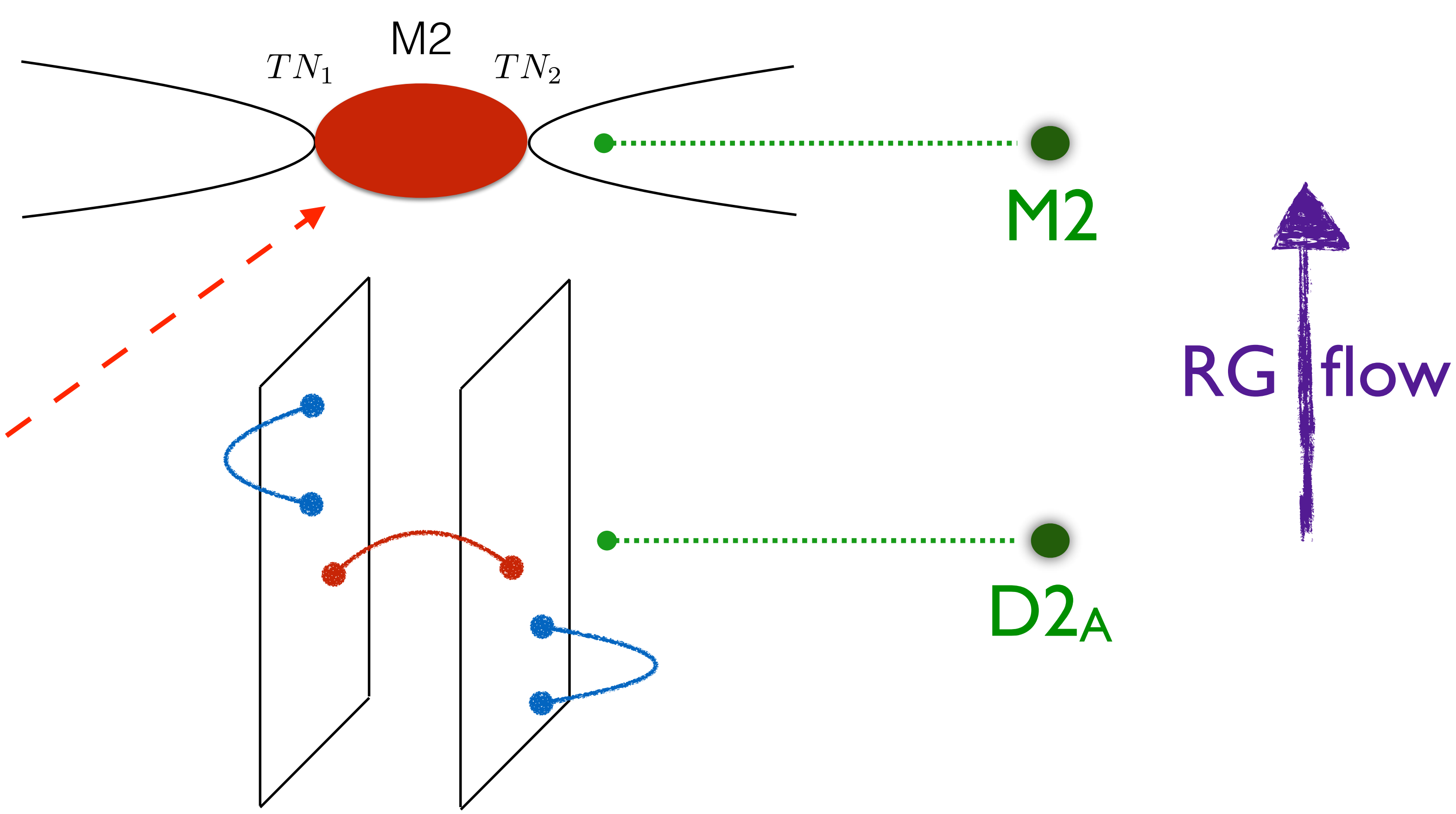
Motivation

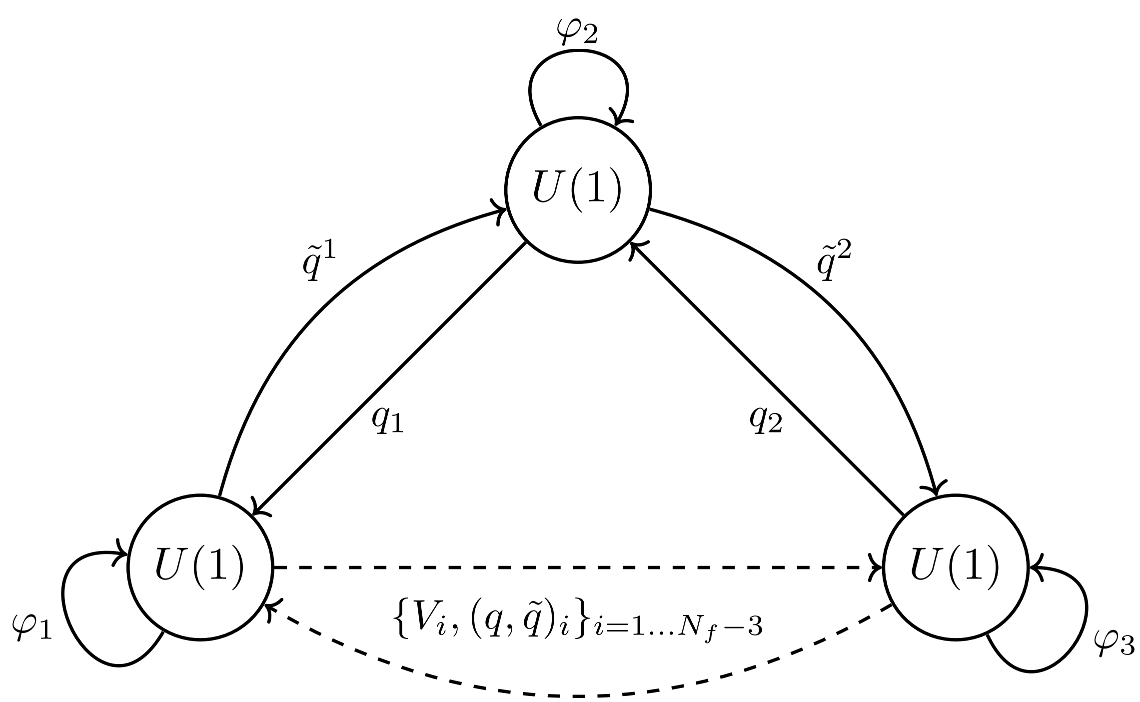
- Describe how T-branes look like in M/F-theory.
  - Existing proposals are more "geometrical": Try to track this data in the singularity of the internal space. [Anderson-Heckman-Katz `13]; [Collinucci-RS `14]
  - Here we look at T-branes through the eyes of a probe. [Heckman-Tachikawa-Vafa-Wecht `10]
  - ‣ Use 3d field theory to gain computational power and a transparent physical meaning.



T-branes in M/F-theory: coherent states of vanishing M2's

T-branes as monopole insertions

- D2$_B$ probes a singular ALF space. E.g. $SU(N_f)$ flavor:



$U(1)^{N_f-1}$ $\mathcal{N}=4$ quiver gauge theory.

N_f-1 global currents coupled to background vectors:

$$\int \mathrm{d}^3x\, J^{\mu}_{(i)} A^{b}_{\mu(i)} \subset \int \mathrm{d}^3x \mathrm{d}^2\theta \mathrm{d}^2\bar{\theta}\, \Sigma_{(i)} V^{b}_{(i)}$$

$$\int_{\mathbb{P}^1_{(i)} \times \mathbb{R}^3} C_3^{\mathrm{IIA}} \wedge *_3 J_{(i)} = \int_{\mathrm{D4}_{(\mathrm{i})}} C_3^{\mathrm{IIA}} \wedge F_{(i)}$$

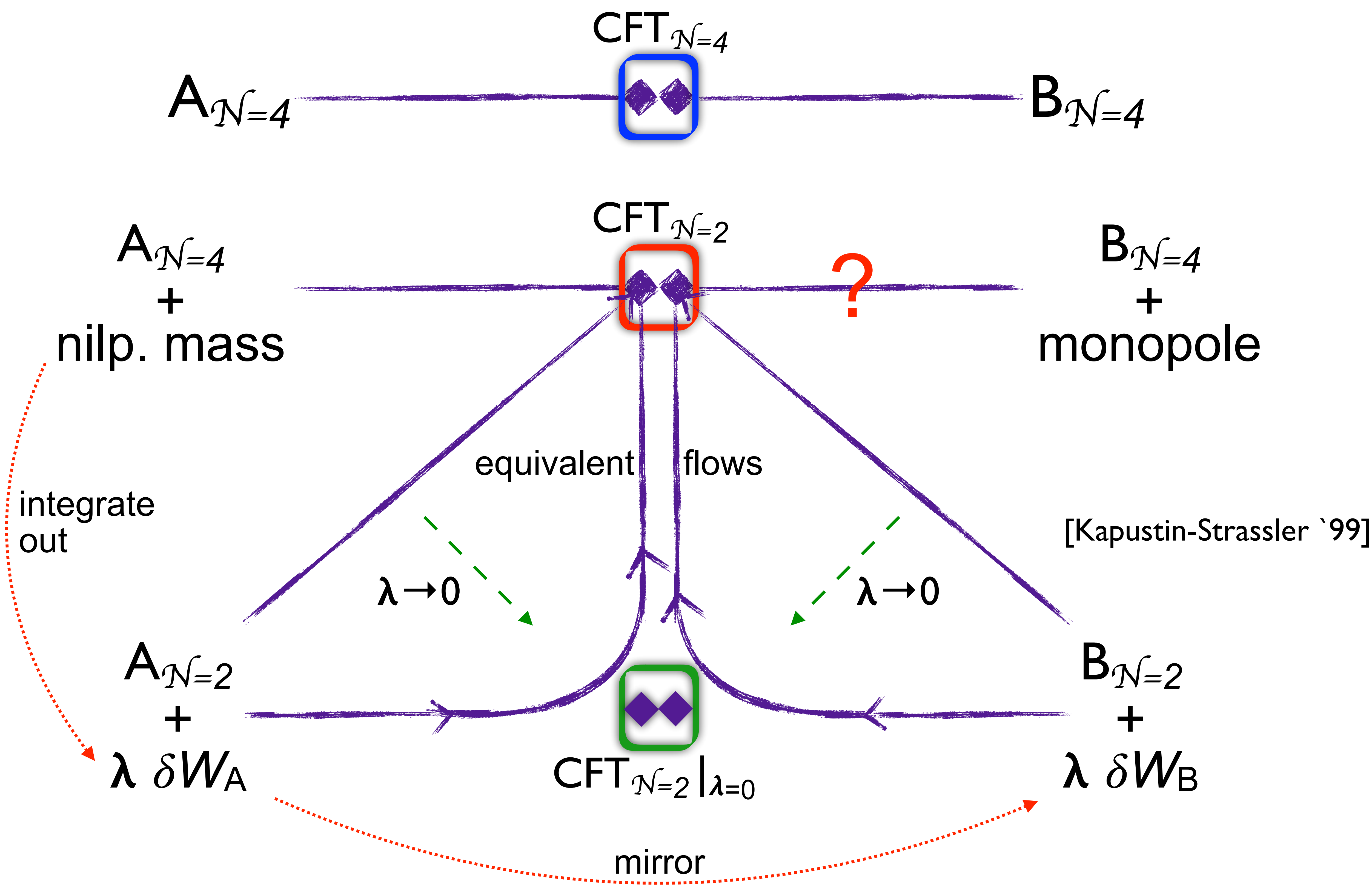
- "Electrons" associated to $J_{(i)}$ $\Rightarrow$ D2's on vanishing cycles (UV precursors of T-branes).

- $(J_{(i)}\,,\, W_{(i),\pm})$ form $\mathcal{N}=4$ multiplets. [Gaiotto-Witten `08]

  ➡ The monopoles $W_{(i),\pm}$ are the unique operators creating states with the same quantum numbers!

RG-flows

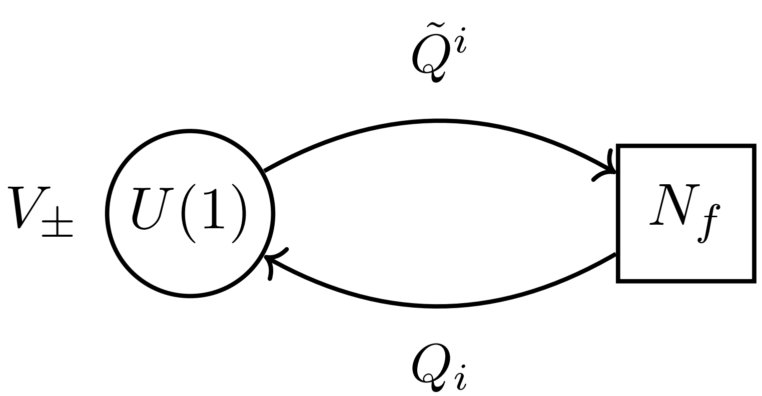
Exploit $\mathcal{N}=2$ mirror symmetry to treat nilpotent deformations.



$\mathcal{N}$=2 mirror symmetry

- Idea:
  1. See $W_A^{effective}$ as deformed $\mathcal{N}$=2 superpotential.
  2. Use $\mathcal{N}$=2 mirror map to get $W_B^{effective}$.
- We only have such map for abelian theories: [Aharony et al. `97]

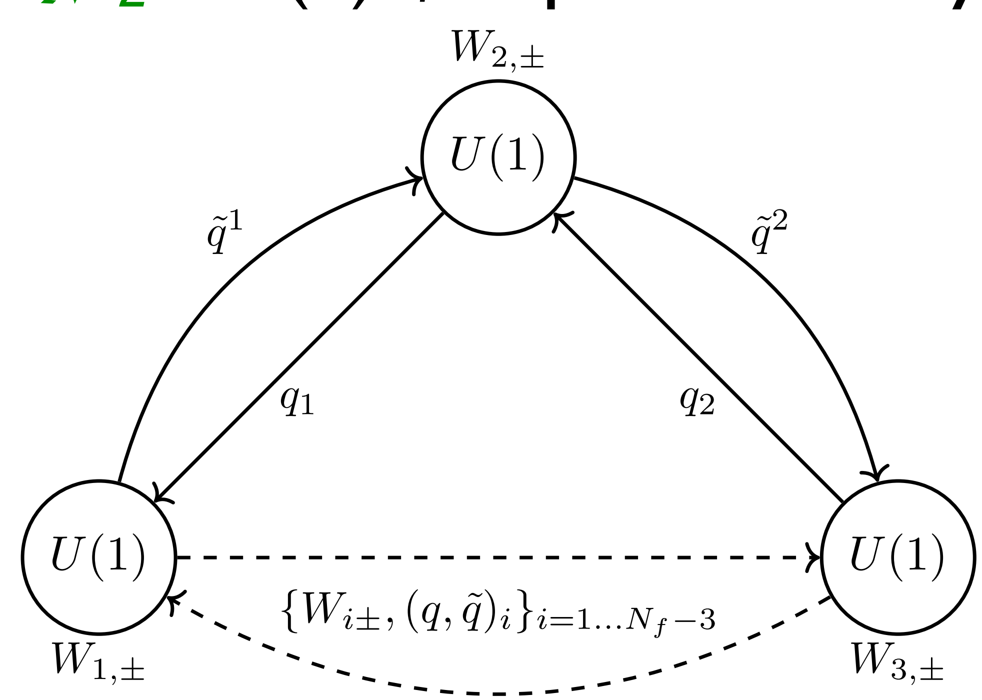
$A_{\mathcal{N}=2}$: SQED with N_f flavors



$W_A = 0$

V_+	$\longleftrightarrow$	$q_1 \cdots q_{N_f}$
V_-	$\longleftrightarrow$	$\tilde{q}_1 \cdots \tilde{q}_{N_f}$
$Q_i \tilde{Q}^i$	$\longleftrightarrow$	S_i
$Q_i \tilde{Q}^{i-1}$	$\longleftrightarrow$	$W_{i,+}$
$Q_{i-1} \tilde{Q}^i$	$\longleftrightarrow$	$W_{i,-}$

$B_{\mathcal{N}=2}$: $U(1)^{N_f-1}$ quiver theory



$W_B = \sum_i S_i q_i \tilde{q}^i$

The effective superpotential

- Introduce a mirror pair of neutral chiral fields $\Phi \leftrightarrow \Psi$.
- If deform with $\delta W_A = \sum_i Q_i \Phi \tilde{Q}^i \leftrightarrow \delta W_B = \Psi \sum_i S_i$

  ➡ the theory flows back to $\mathcal{N}=4$ SCFT. [Intriligator-Seiberg `96]

- Nilpotent deformations for $SU(N_f)$ flavor.

  1. Minimal orbit: $m\, Q_1 \tilde{Q}^2 \leftrightarrow m\, W_{2,-}$

     $$\mathbf{m} = \left(\begin{array}{cc|c} 0 & m & 0 \\ 0 & 0 & \\ \hline \multicolumn{2}{c|}{0} & 0 \end{array}\right)$$

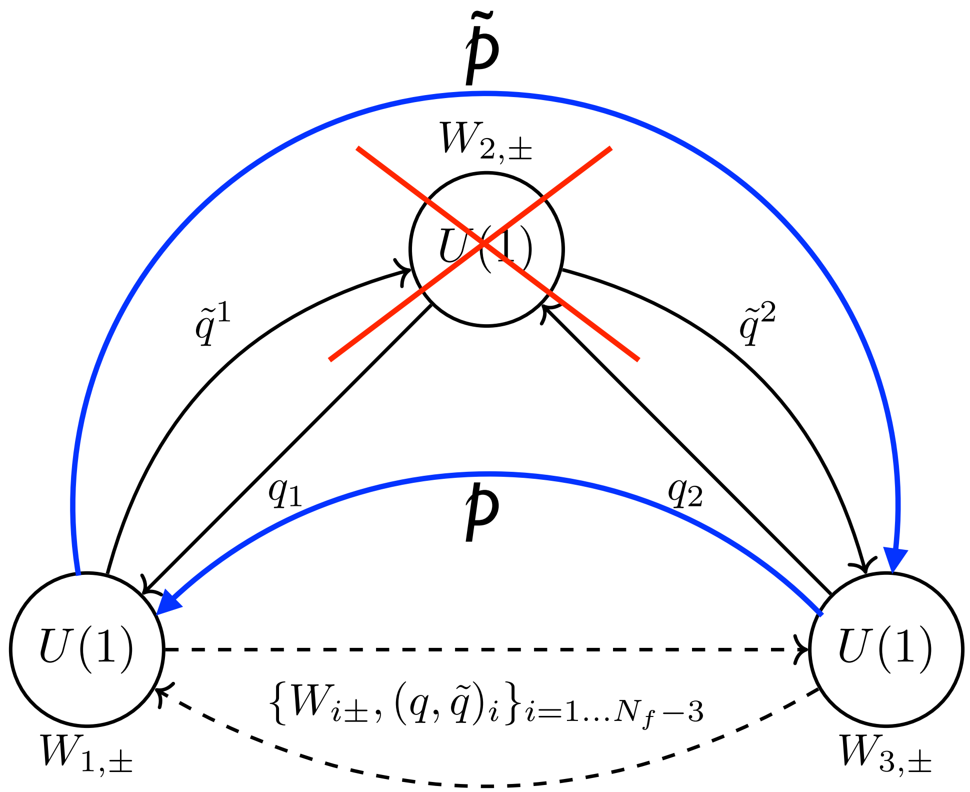
     integrating out the two heavy fields:

$$W_A^{eff} = \sum_{i=3}^{N_f} Q_i \Phi \tilde{Q}^i - \frac{\Phi^2}{m} P\tilde{P} \quad \xleftarrow{\;\mathcal{N}=2\;} \!\!\xrightarrow[\text{mirror}]{} \quad W_B^{eff} = \sum_{i=3}^{N_f} S_i (q_i \tilde{q}^i + \Psi) + S\left(p\tilde{p} - \frac{\Psi^2}{m}\right)$$

- Geometry of CB_A/HB_B is unchanged.
- HB_A/CB_B is partially lifted.

$$\boxed{V_+ V_- \sim \Psi^{N_f}}$$

Mutilated quiver



$\cancel{\mathrm{U(1)}}_2 : \int d^3x d^2\theta d^2\bar{\theta}\, V_{(2)} \Sigma^b_{(2)} \supset \int d^3x\, V_{(2)}\, \xi_2$

➡ 2nd node frozen to zero-size (obstructed blow-up).

[Anderson-Heckman-Katz `13]

2. Non-minimal orbits: sum of monopole operators.

$$\mathbf{m} = \begin{pmatrix} \begin{array}{ccccc|c} \multicolumn{5}{c|}{\overbrace{\hphantom{0 \quad m \quad 0 \quad \cdots \quad 0}}^{k}} & \\ 0 & m & 0 & \cdots & 0 & \\ & 0 & m & \cdots & 0 & \\ & & \ddots & \ddots & \vdots & 0 \\ & & & \ddots & m & \\ & & & & 0 & \\ \hline \multicolumn{5}{c|}{0} & 0 \end{array} \end{pmatrix}$$

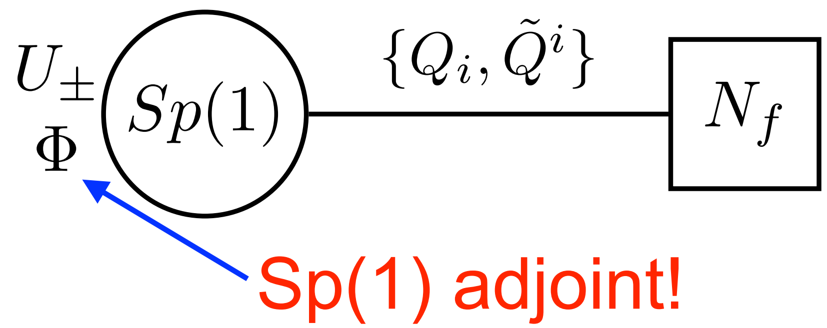
$$W_A^{eff} = \sum_{i=k+1}^{N_f} Q_i \Phi \tilde{Q}^i + (-1)^{k-1} \frac{\Phi^k}{m^{k-1}} P\tilde{P}$$

$\mathcal{N}$=2 mirror

$$W_B^{eff} = \sum_{i=k+1}^{N_f} S_i(q_i \tilde{q}^i + \Psi) + S\left(p\tilde{p} + (-1)^{k-1}\frac{\Psi^k}{m^{k-1}}\right)$$

$SO(2N_f)$ flavor symmetry

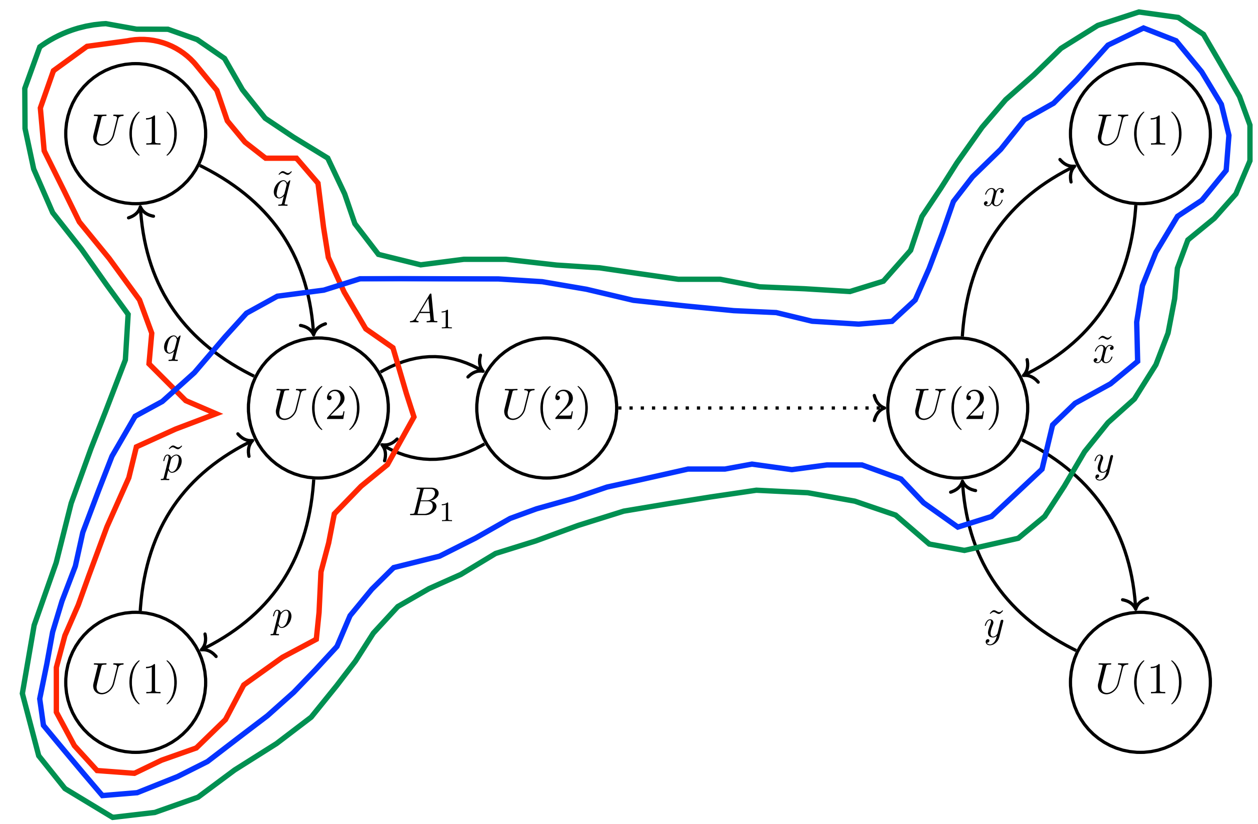
$A_{\mathcal{N}=4}$: probing D6's + O6



$$\psi^a = \frac{1}{\sqrt{2}} \left(\begin{array}{c} Q^a - \epsilon^{ab}\tilde{Q}_b \\ i[Q^a + \epsilon^{ab}\tilde{Q}_b] \end{array} \right) \Big\} 2N_f \quad a, b = 1, 2$$

$$\Longrightarrow \quad W_A = \sum_I \psi^a_I \epsilon_{ab} \Phi^b_c \psi^c_I$$

$B_{\mathcal{N}=4}$: affine D_{N_f} quiver



$W_B = W_B(h, \phi)$ [Borokhov `03]

- HB_A ($dim=4N_f-6$): $M_{IJ} = \psi^a_I \epsilon_{ab} \psi^b_J = -M_{JI}$, $rk=2$, $M^2=0$.

➡ CB_B: $M \in adj\ SO(2N_f)$ made of (R=1)-monopoles & $\mathbf{Tr}(\phi)$'s.

- CB_A/HB_B: $G^2 - R\,(B+R)^2 + R^{N_f-1} = 0$ $\quad D_{N_f}$ singularity.

T-branes and the HB_A/CB_B

- T-brane deformation: $\delta W = Tr\,(mM)$ w/ nilpotent m.
  - Brakes flavor symmetry. F-terms $\Rightarrow$ $[m\,, M] = 0$.

- Recall for $SU(N_f)$, $[m\,, M]=0$ $\Rightarrow$ $mM=0$, because rk M = 1.

➡ Still, on HB_A, $\boldsymbol{\Phi} = 0$.

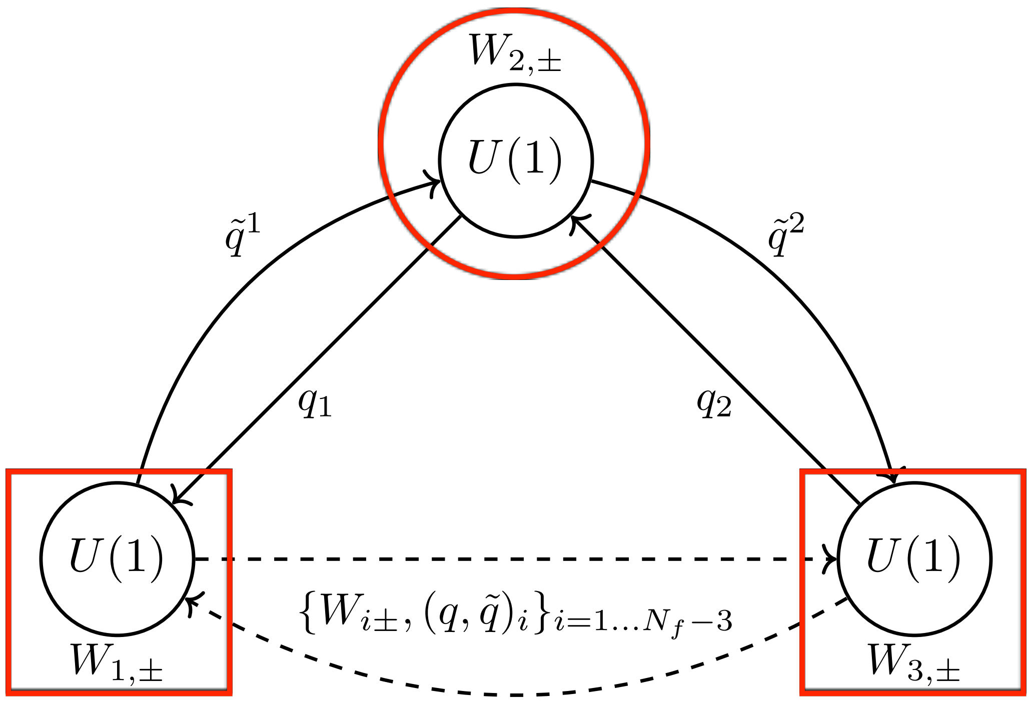
- For $SO(2N_f)$, F-terms $m_{IJ}\psi^a_J = \Phi^a_b \psi^b_I$ allow $mM \neq 0$.
  - But these vacua violate the D-term $[\,\boldsymbol{\Phi}\,, \boldsymbol{\Phi}^\dagger] = 0$.

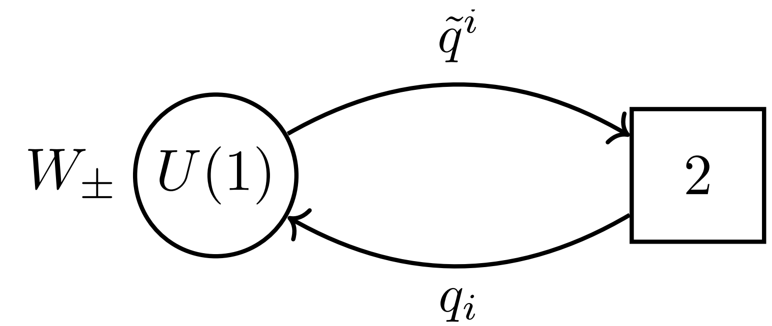
➡ Still, after any T-brane, on HB_A: $\boldsymbol{\Phi} = mM = 0$.

- This tells us which are the coordinates (monopoles & ϕ's) parametrizing the residual CB_B.

"Local" mirror symmetry

- Focus on one gauge node, treating all others as flavor nodes.





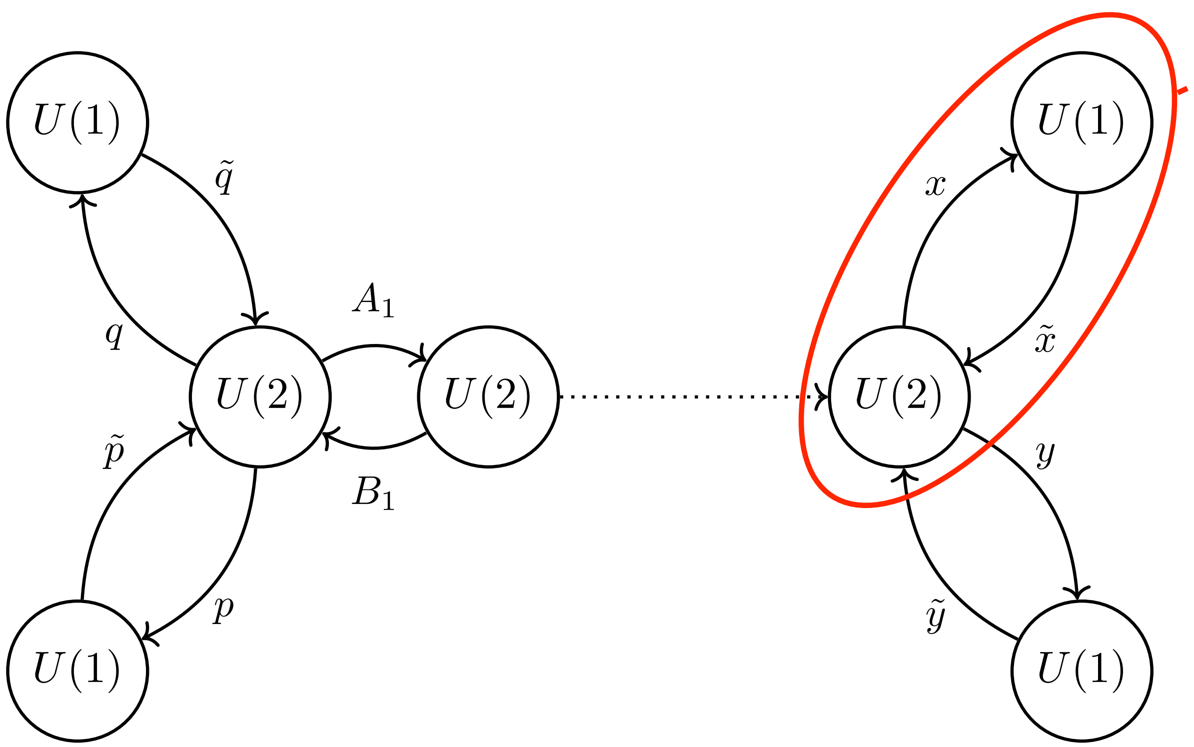
B_{loc}

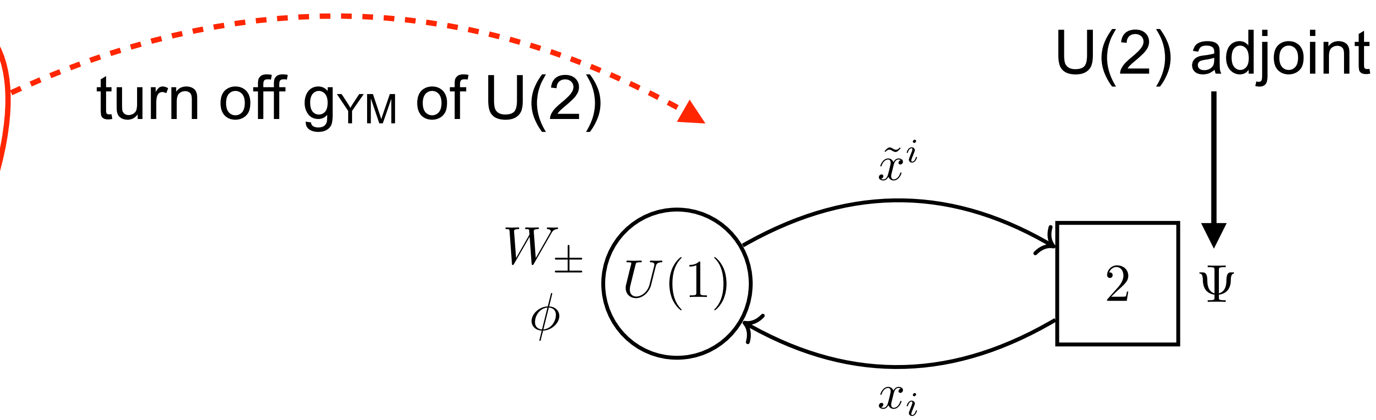
- Take mirror $B_{loc} \rightarrow A_{loc}$ $\Rightarrow$ monopole → off-diagonal mass.
- Integrate out heavy fields $\Rightarrow$ W_A^{eff} & mirror back → W_B^{eff}.
- Couple back the effective B_{loc} (no gauge fields!) into the quiver.

✓ We successfully tested this strategy by iteration for any nilpotent deformation of $SU(N_f)$ theories.

Application to $SO(2N_f)$

- Can study CB_A/HB_B only for minimal T-branes.
  - Choose m along root associated to an abelian node.





Self-mirror theory

$m W_- \leftrightarrow m X_1 \tilde{X}^2$, $M^i_j = \tilde{x}^i x_j$

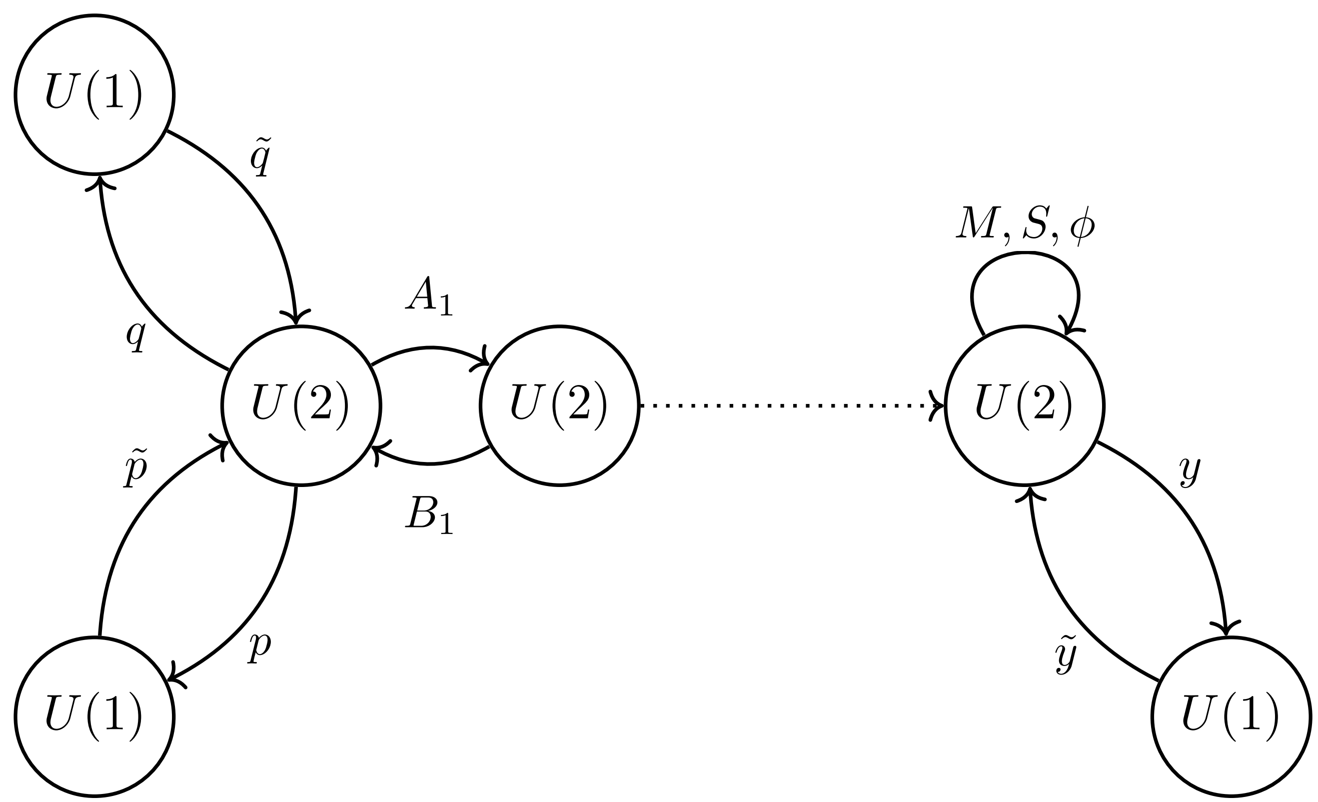
- $W_{B_{loc}} \Longrightarrow W_{A_{loc}} = \mathrm{Tr}(M(\Psi - \phi \mathbb{1}_2)) + M^1_1 \tilde{X}^1 X_1 + M^2_2 \tilde{X}^2 X_2 + m X_1 \tilde{X}^2$

- Integrate out: $W_{A^{eff}_{loc}} = \mathrm{Tr}(M(\Psi - \phi \mathbb{1}_2)) - \dfrac{M^1_1 M^2_2}{m} X_2 \tilde{X}^1$ SQED, N_f=1.

- Mirror back: $W_{B^{eff}_{loc}} = \mathrm{Tr}(M(\Psi - \phi \mathbb{1}_2)) - \dfrac{S}{m} \mathrm{det} M$ only matter!

The geometry of CB_A/HB_B

- Net effect → Mutilated quiver + 3 new chiral fields.
  1. One adjoint of neighbouring U(2): M (HB_B field).
  2. Two singlets: S , ϕ (CB_B fields).



- F-terms: $M^2 = 0$.

➡ We computed HB_B and found still the D_{N_f} singularity.

Conclusions

- Exceptional flavor symmetries:
  - Theory A is non-lagrangian. [Minahan-Nemeschanski `96]
  - ✓ Can apply local mirror sym. on abelian nodes of theory B.
    - ➡ Minimal nilpotent orbits do not change HB_B geometry.
- Need non-abelian mirror symmetry:
  - Check if also non-minimal orbits preserve singularity of HB_B.
  - Check if also for E_n-theories *mM=0* on the residual CB_B.
- Study more deeply the CB_B after deformation (instantons):
  - How to deduce the unbroken flavor symmetry ?
- Probe higher-dimensional F-theory backgrounds (CY_3, CY_4).