Gapped Boundary Phases of Topological Insulators via Weak Coupling

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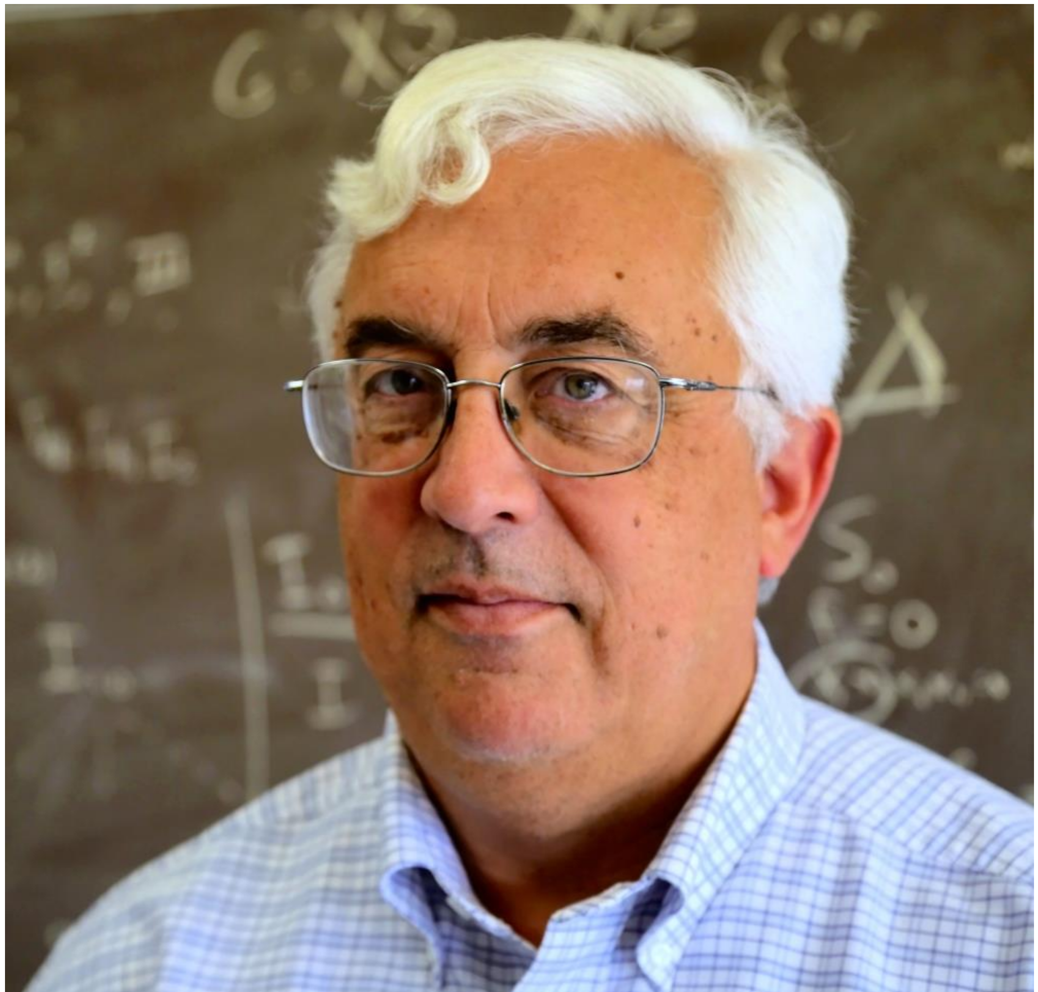
Happy Birthday, Dave



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Nathan Seiberg and Edward Witten, arXiv:1602.04251

Phases of Theories

- Gapless (= massless)
  - Nontrivial fixed point = interacting conformal theory
  - Free theory
- Gapped
  - Trivial bulk theory
    - Trivial boundary
    - Gapless boundary modes
    - Gapped TQFT on the boundary
  - Nontrivial bulk topological quantum theory
    - Same as above

} Bulk is not completely trivial. Symmetry Protected Topological (SPT) phase

Topological Insulators [Kane, Mele; …]

- Insulator
  - Unbroken global $U(1)_A$. The electromagnetic gauge field A can be viewed as a classical background field.
  - Gapped and trivial bulk
- Assume it is time-reversal (T) invariant
- Model: $\frac{1}{8\pi}\int F \wedge F$ (i.e. $\theta = \pi$) inside the material and $\theta = 0$ outside [Qi, Hughes, Zhang; Essin, Moore, Vanderbilt])
- Nontrivial boundary
  - Typically, massless fermions (gapless)
  - Can also lift the fermions and have gapped boundary states. (Examples by [Metlitski, Kane, Fisher; …].)

Topological Insulator: simple example

On the boundary $2+1$-dimensional complex massless fermions.

Parity anomaly:

We would like to preserve $U(1)_A$ and T.

But we can preserve

- either $U(1)_A$ and violate T
- or T and violate $U(1)_A$
- or $U(1)_A$ and T, but the theory is not truly $2+1$-dimensional. It needs a $3+1$-dimensional bulk with $\frac{1}{8\pi}\int F \wedge F$. This is an example of anomaly inflow [Callan, Harvey].

Massless boundary modes are associated with $U(1)_A$ and T. They are robust.

Topological Insulator

Start with $\frac{1}{8\pi}\int F \wedge F$ inside the material, but not outside.

Massless boundary modes are associated with $U(1)_A$ and T.

They are robust – cannot be lifted by small perturbations.

- Can we add a large perturbation and gap the system?
- Something must remain on the boundary to account for the anomaly inflow.
- Can there be a TQFT on the boundary with the same anomaly? (Examples by [Metlitski, Kane, Fisher; …].)
- Not obvious whether a given TQFT has the right anomaly.

Extend the Previous Model

Cast of characters:

- Emergent $U(1)_a$ gauge field on the boundary
- Scalar w of $U(1)_a$ charge 1, which can Higgs it to be trivial.
- Massless fermion χ with $U(1)_A \times U(1)_a$ charges $(1, 2s)$
  - For integer s no additional anomaly associated with a (actually, s has to be even for more subtle reasons).

In the phase with $\langle w \rangle \neq 0$ the low-energy spectrum consists of a massless fermion with $U(1)_A$ charge one.

So this system contains the previous system – same anomaly. (Even the same gravitational anomalies, which we do not discuss today.)

Extend the Previous Model

Add:

- Scalar Φ with $U(1)_A \times U(1)_a$ charges $(2, 4s)$ such that we can have a T-invariant coupling $\chi\chi\Phi^* +$ c.c. .

In a phase with $\langle w \rangle = 0$, but $\langle \Phi \rangle \neq 0$ the theory is gapped:

- Higgs $U(1)_a \to \boldsymbol{Z}_{4s}$. No massless gauge field.
- χ acquires a mass from $\chi\chi\Phi^*$
- Unbroken T and global $U(1)_A$ symmetry (linear combination of the original global $U(1)_A$ and gauge $U(1)_a$)

- Our system has the right anomaly to be a boundary state.
- It has a gapped boundary phase with a TQFT.
- Everything can be analyzed explicitly.

The Massive Spectrum

- w quanta are $U(1)_A$ neutral bosons transforming with "charge" 1 under $\boldsymbol{Z}_{4s}$.
- χ quanta are $U(1)_A$ neutral fermions transforming with "charge" $2s$ under $\boldsymbol{Z}_{4s}$.
- Interesting spectrum of vortices from $U(1)_a \rightarrow \boldsymbol{Z}_{4s}$:
  - The elementary vortex (vorticity $v = \pm 1$) has a single χ zero mode. It exhibits non-Abelian statistics.
  - More generally, all odd v vortices have non-Abelian statistics.
  - Even v vortices have Abelian statistics.

The Low Energy TQFT

First, we describe the $\boldsymbol{Z}_{4s}$ gauge theory as a $U(1)_a \times U(1)_c$ Chern-Simons theory [Maldacena, Moore, NS] $\frac{1}{2\pi} c\, d(4sa + 2A)$.

c is dual to the phase of the Higgs field Φ. Its equation of motion constrains a to be a $\boldsymbol{Z}_{4s}$ gauge field. The coupling to A follows from the coupling of Φ.

Second, we integrate out χ to find a Chern-Simons term

$$\frac{1}{8\pi}(2sa + A)d(2sa + A)$$

The term $\frac{1}{8\pi} AdA$ is the only term that is not properly normalized. It reflects the anomaly. It comes form the bulk of the system. (Need to be more careful and use η.)

The Low Energy TQFT

$$\frac{1}{2\pi}\, c\, d(4sa + 2A) + \frac{1}{8\pi}(2sa + A)\, d(2sa + A)$$

But this cannot be the whole story.

The Wilson lines $\exp(i \oint c)$ should represent the vortices. But this misses the fact that they have non-Abelian statistics.

It turns out that we need to add to this free TQFT another non-Abelian sector. It is the 2+1-dimensional TQFT that corresponds to the 1+1-dimensional Ising model. Further, the two sectors are subject to a $\boldsymbol{Z}_2$ quotient. Same TQFT in [Metlitski, Kane, Fisher].

The line operators of this theory represent the world-lines of the quasi-particles we found semi-classically.

Conclusions

- Topological phases of matter are interesting.
  - They exhibit rich phenomena. Some of them have already been encountered by high energy physicists, but most of them have not.
  - Mathematics, quantum field theory, condensed matter physics…
- We have presented a weakly coupled T-invariant theory with a global $U(1)_A$ symmetry. It has two interesting phases:
  - Massless charged fermions. Hence, the right anomaly to be the boundary of a topological insulator.
  - Gapped phase with a TQFT.
  - Explicit, calculable.

Conclusions

- The analysis of this system, despite being weakly coupled, has many interesting subtleties.
- New consistency conditions
- New anomalies
- Many more models
- Topological superconductors ($U(1)_A$ is broken to $\boldsymbol{Z}_2$)
- Many interesting questions

Dave,

Thank you for a wonderful friendship.

Happy Birthday