6d SCFTs and F-theory:
a bottom-up perspective

Yuji Tachikawa

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Isn't it amazing?

Maybe I should try something other than string theory, following his example!

6d SCFTs and F-theory:
a **bottom-up** perspective

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I don't even remember the orders of (f, g, Δ) in Kodaira's table by heart!

I was asked to give a review talk from an outsider point of view,
so it's probably OK.

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I was asked to give a review talk from an outsider point of view,
so it's probably OK.

I guess the idea was like inviting an LQG person to the Strings conference...

I also apologize in advance that my talk will be very subjective,
and will not cite/mention many relevant papers,
although I know I should.

But at least I would like to illustrate ...

The Unreasonable Effectiveness of F-theory in the study of 6d SCFTs

Q. Why 6d SCFT?

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A. When [Heckman-Morrison-Vafa 1312.5746] came out, my student Hiroyuki Shimizu got interested, so as an adviser I needed to study it and come up with a project he could work on.

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A. 6d is the **maximal** dimension where **superconformal groups are available**. So, something interesting might be going on.

Sketch of the proof that **6d is the maximal dimension**:

1. Superconformal algebras are **simple**.
2. Fermionic parts of the superconformal algebras are **spinors**.
3. **Simple** superalgebras are classified.
4. Fermionic parts of almost all of them are in the **fundamental**.
5. Need an accidental isomorphism **spinors** $\simeq$ **fundamental**.
6. The maximal case is therefore $\mathfrak{so}(8)$, or $\mathfrak{so}(6,2)$ for our purpose.

6d $\mathcal{N}=(n,0)$ SCFT corresponds to $\mathfrak{osp}(6,2|2n)$.

Open question

Show that $\mathcal{N}=(n>2,0)$ SCFTs don't exist.

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Show that $\mathcal{N}=(n>2,0)$ SCFTs don't exist.

cf. 5d $\mathcal{N}=1$ SCFT corresponds to $F(4)$, whose bosonic part is $\mathfrak{so}(7) \oplus \mathfrak{su}(2)$ and the fermionic part is **spinor** $\otimes$ **doublet**.

There simply is no 5d $\mathcal{N}>1$ superconformal algebra,
so there's no corresponding open question.

Q. How do you study 6d SCFTs?

- Conformal bootstrap.
- Analysis of the Lagrangian on the tensor branch.
- Analysis of the Lagrangian of the S^1 compactification.
- Brane constructions in M, (massive) type IIA, or type I
- F-theory!

M-theory constructions and F-theory

$\downarrow$

structure on the tensor branch and F-theory

$\downarrow$

massive type IIA constructions and F-theory

M-theory constructions and F-theory

↓

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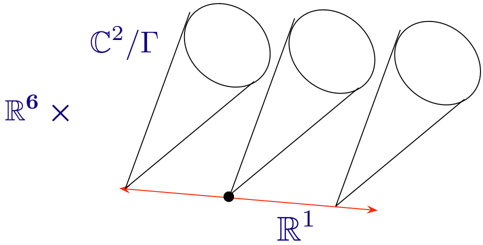
↓

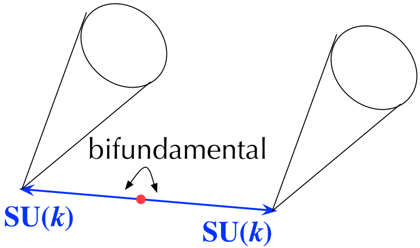
massive type IIA constructions and F-theory

A large class of 6d $\mathcal{N}=(1,0)$ SCFTs can be obtained by putting M5-branes on the ALE singularities:



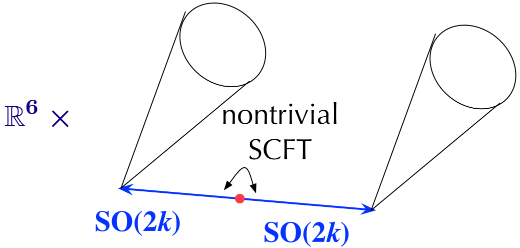
When $\Gamma = \mathbb{Z}_k$, we have $\mathbf{SU}(k)$ gauge fields at the singularity, and an M5 just gives a bifundamental of $\mathbf{SU}(k) \times \mathbf{SU}(k)$:

$\mathbb{R}^6 \times$



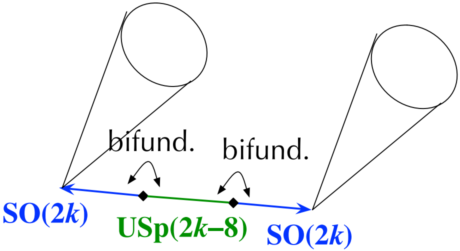
But surprising things happen when Γ is of type D_k or E_k.
[del Zotto-Heckman-Tomasiello-Vafa, 1407.6359]

For example, take Γ of type D_k and put 1 M5:



The M5 becomes two fractional M5s:

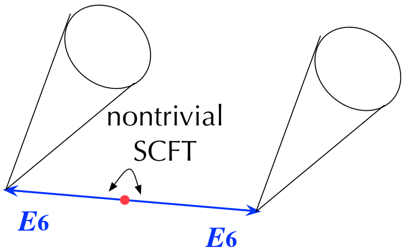
$\mathbb{R}^6 \times$



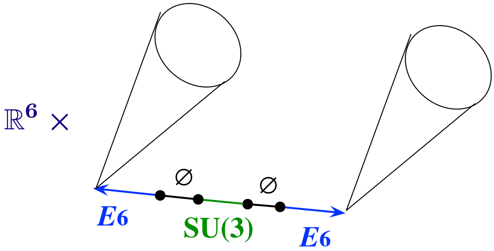
Somehow the middle region the gauge group is $\mathbf{USp(2}k\mathbf{-8)}$,
and each half-M5 gives a bifundamental.

Similarly, when Γ is of type E_6, a full M5-brane fractionates ...

$\mathbb{R}^6 \times$



Similarly, when Γ is of type E_6, a full M5-brane fractionates ...



into 4 fractional M5s, and the gauge groups occur in the sequence

$$E_6, \quad \varnothing, \quad \mathbf{SU(3)}, \quad \varnothing, \quad E_6.$$

In general, we have

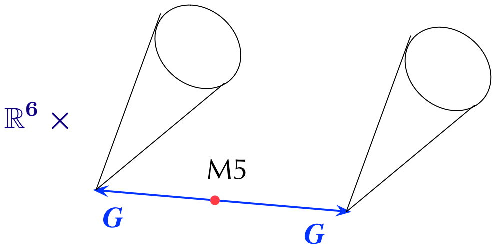
$$\begin{aligned}
\boldsymbol{A} &\quad : \quad \text{doesn't fractionate.} \\
\boldsymbol{D_k} &\quad : \quad \mathbf{SO}(2k), \mathbf{USp}(2k-8), \mathbf{SO}(2k) \\
\boldsymbol{E_6} &\quad : \quad E_6, \varnothing, \mathbf{SU}(3), \varnothing, E_6 \\
\boldsymbol{E_7} &\quad : \quad E_7, \varnothing, \mathbf{SU}(2), \mathbf{SO}(7), \mathbf{SU}(2), \varnothing, E_7 \\
\boldsymbol{E_8} &\quad : \quad E_8, \varnothing, \varnothing, \mathbf{SU}(2), G_2, \varnothing, F_4, \varnothing, G_2, \mathbf{SU}(2), \varnothing, \varnothing, E_8.
\end{aligned}$$

My natural reaction was this:

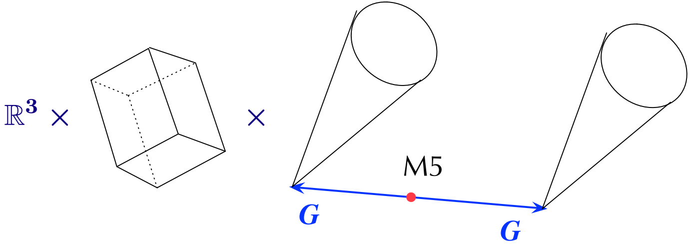
What the hell are these sequences of groups?

M-theoretically, you can go as follows
[Ohmori-Shimizu-YT-Yonekura, 1503.06217, Sec. 3.1]:

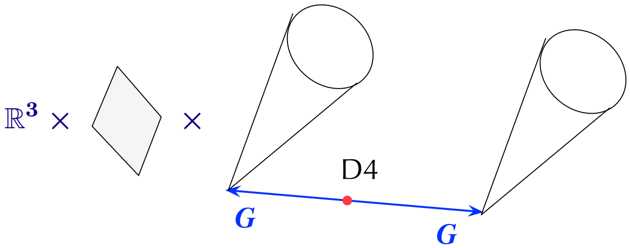
To study the **tensor branch** of this system,



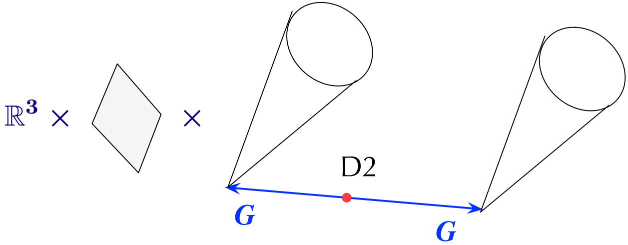
We can instead study the **Coulomb branch** of its T^3 compactification:



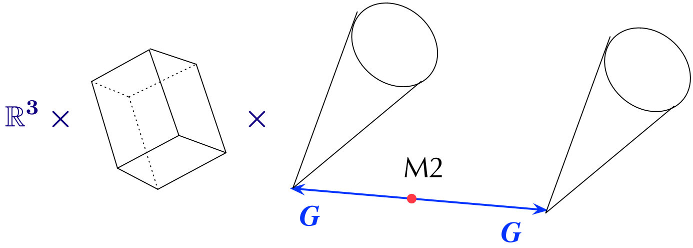
Reduce it to IIA:



Take the double T-dual:

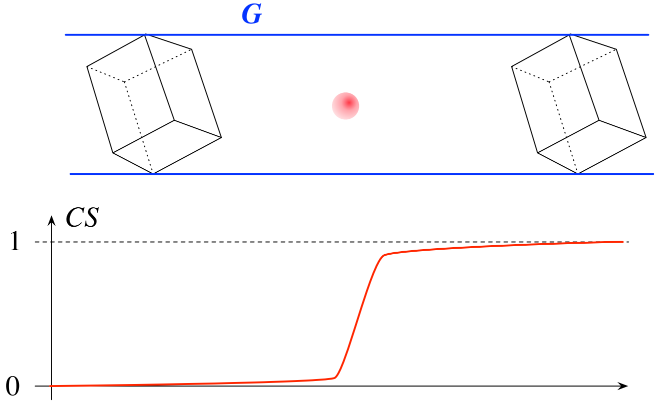


Lift it back to M-theory:



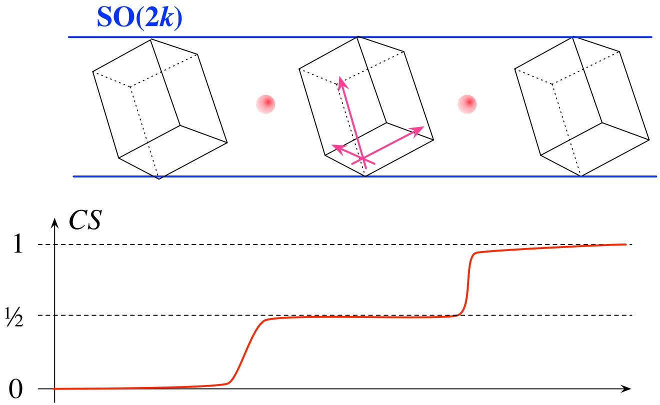
We're now interested in its **Higgs branch**,
since we've effectively taken the 3d mirror.

An M2 can dissolve into the G gauge field as an instanton on $T^3 \times \mathbb{R}$:

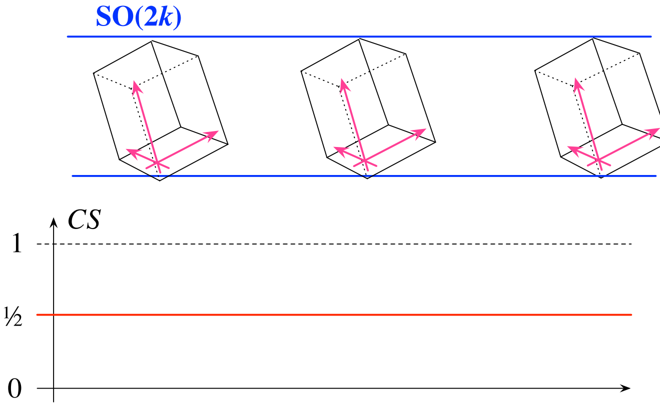


The plot below shows the evolution of the Chern-Simons invariant on T^3 at each slice.

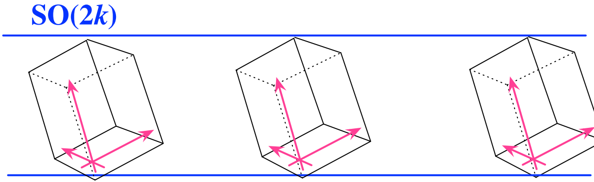
When $G = \mathbf{SO}(2k)$, the instanton can fractionate:



In an extreme situation, we have this:



The bundle is flat but nontrivial.



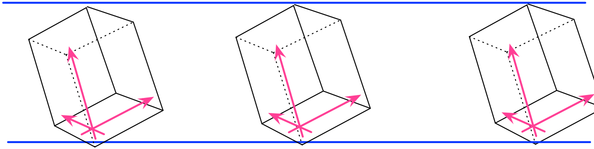
Three holonomies are known to be given by

$$\mathbf{diag}(+,+,+,-,-,-,-,+^{2k-7})$$
$$\mathbf{diag}(+,-,-,+,+,-,-,+^{2k-7})$$
$$\mathbf{diag}(-,+,-,+,-,+,-,+^{2k-7})$$

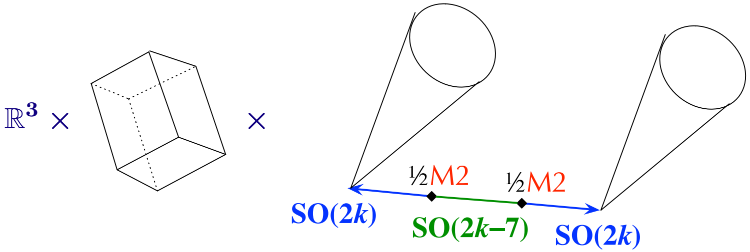
Originally noticed by [Witten, hep-th/9712028].

So the unbroken gauge group is

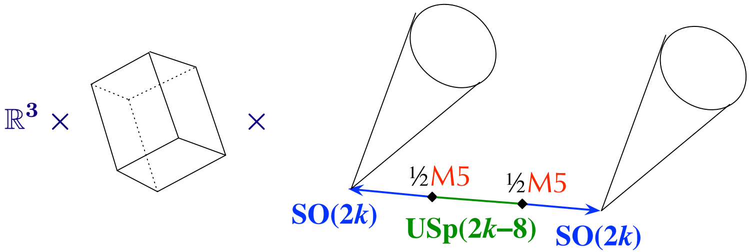
$\mathbf{SO(2}\boldsymbol{k}\mathbf{)} \rightarrow \mathbf{SO(2}\boldsymbol{k}\mathbf{-7)}$



So we have



Going back the duality chain, we have

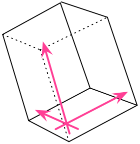


since we need to take 4d S-duality / 3d mirror symmetry:

$$\mathbf{SO(2}\boldsymbol{k}\mathbf{-7)} \leftrightarrow \mathbf{USp(2}\boldsymbol{k}\mathbf{-8)}$$

The analysis can be carried out in a similar manner for any G, using the results in [Borel,Friedman,Morgan math.GR/9907007].

What needs to be done is the classification of flat G bundles on T^3

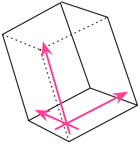


and the computation of their Chern-Simons invariants.

Example: $G = E_7$.

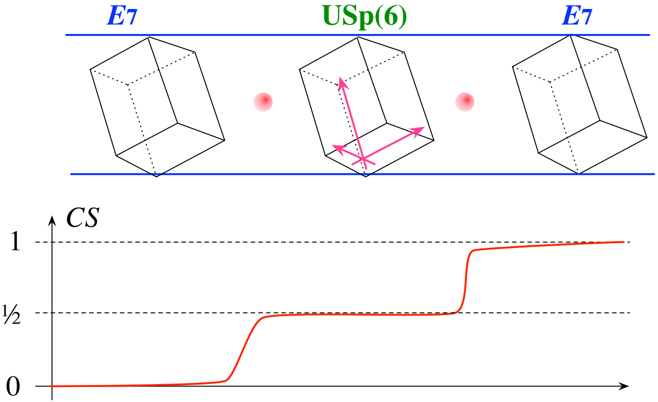
You can construct a bundle with $CS = 1/2$ as follows. Take

$$\begin{array}{l} \mathbf{diag}(+,+,+,-,-,-,-) \\ \mathbf{diag}(+,-,-,+,+,-,-) \\ \mathbf{diag}(-,+,-,+,-,+,-) \end{array}$$



in $\mathbf{SO(7)}$. In fact they are in G_2.

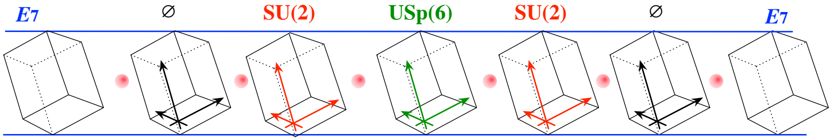
E_7 has a maximal subgroup $G_2 \times \mathbf{USp(6)}$. Therefore

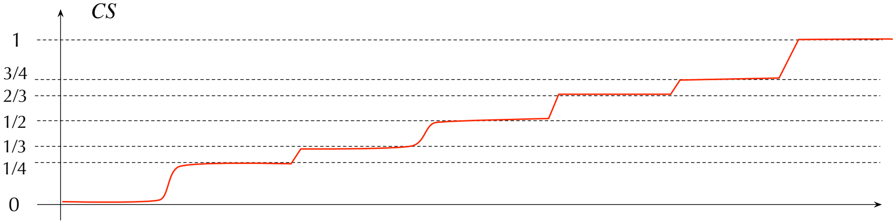


You can fractionate further, since the allowed CS invariants are

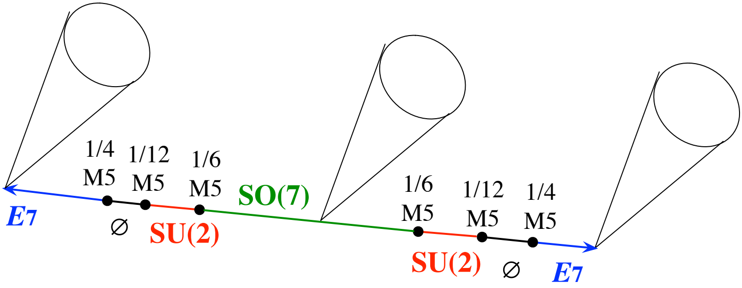
$$CS = 0, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}.$$

We have





In the original duality frame we have

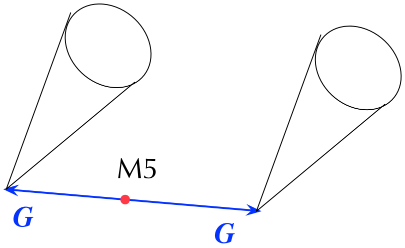


Note that the M5 charges are **not equally distributed**.

In F-theory, the analysis is done as follows [Aspinwall-Morrison, hep-th/9705104]:

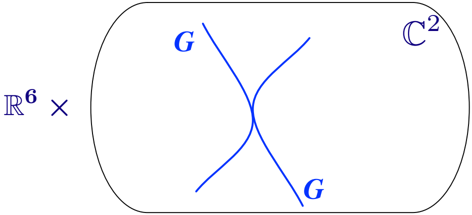
Recall that the M-theory configuration

$\mathbb{R}^6 \times$



is dual to ...

This F-theory configuration:



where two F-theory 7-branes intersect transversally at a point.

I know I don't have to review the following in this workshop, but anyway.

Let's say we put the elliptic fibration to the Weierstrass form

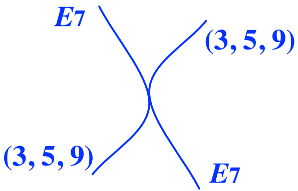
$$y^2 = x^3 + fx + g$$

where f, g are functions on the base.

Let $\Delta = 4f^3 + 27g^2$ be its discriminant.

	g	G	$\mathbf{ord}(f)$	$\mathbf{ord}(g)$	$\mathbf{ord}(\Delta)$
I_k	$\begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$	$\mathbf{SU}(k)$	0	0	k
II	$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$	$\varnothing$	≥ 1	1	2
III	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$\mathbf{SU}(2)$	1	≥ 2	3
IV	$\begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}$	$\mathbf{SU}(3)$	≥ 2	2	4
I_k^*	$\begin{pmatrix} -1 & -k \\ 0 & -1 \end{pmatrix}$	$\mathbf{SO}(2k+8)$	2	3	$k+6$
IV^*	$\begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$	E_6	≥ 3	4	8
III^*	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	E_7	3	≥ 5	9
II^*	$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$	E_8	≥ 4	5	10

So, suppose two E_7 7-branes intersect.



Here $(3, 5, 9)$ means that (f, g, Δ) vanish to these orders there.

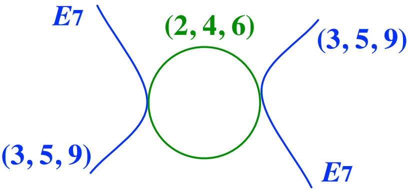
At the intersection,

$$(3, 5, 9) + (3, 5, 9) = (6, 10, 18) \geq (4, 6, 12).$$

A smooth elliptic fibration can't exceed $(4, 6, 12)$.

So we blow-up the intersection point.

We now get this configuration

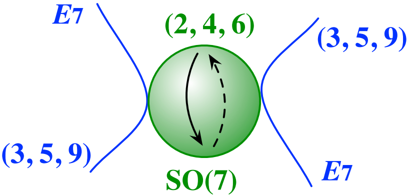


where

$$(2,4,6) = (3,5,9) + (3,5,9) - (4,6,12).$$

Looking up the table, this corresponds to I_0^* with $\mathbf{SO}(8)$.

A more detailed analysis shows that there is
an **outer-automorphism** action of $\mathbf{SO(8)}$ around this S^2 of I_0^* curve



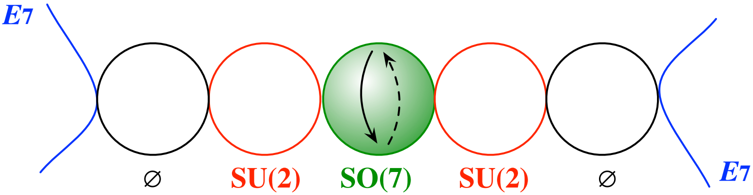
giving $\mathbf{SO(7)}$.

The intersection of $(\mathbf{2, 4, 6})$ and $(\mathbf{3, 5, 9})$ is still singular since

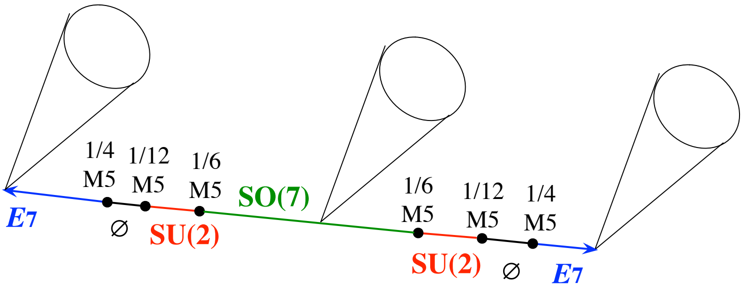
$$(\mathbf{2, 4, 6}) + (\mathbf{3, 5, 9}) \geq (\mathbf{4, 6, 12}).$$

We need to blow up, repeat ...

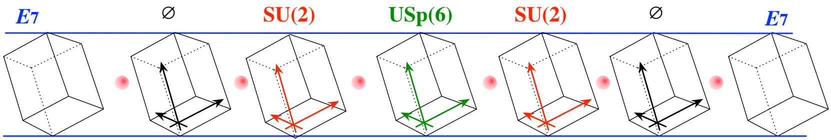
We end up with this final configuration:

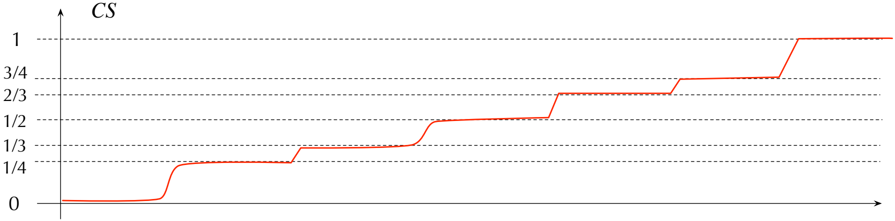


Recall that in M-theory this was

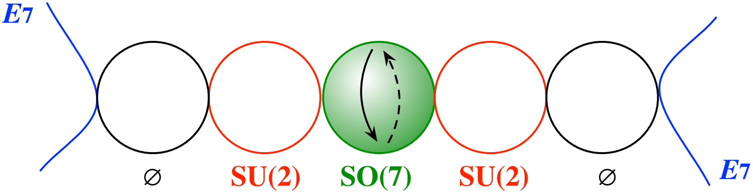


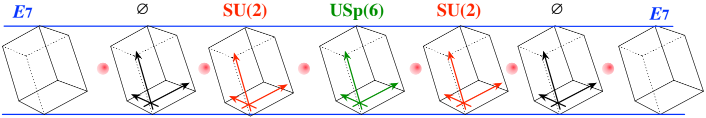
which reflected possible choices of flat E_7 connections on T^3

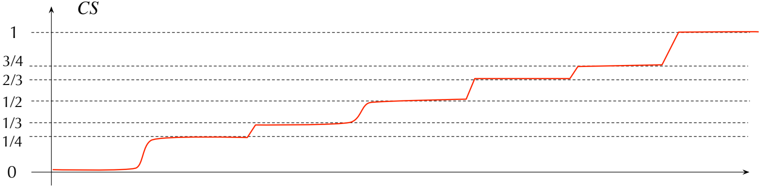




The correspondence works for any $G = A_k$, D_k and $E_{6,7,8}$.







How on earth does the F-theory know the flat connections on T^3?

Note that [Aspinwall-Morrison, hep-th/9705104] appeared before [Borel,Friedman,Morgan math.GR/9907007].

How on earth does the F-theory know the flat connections on $\boldsymbol{T^3}$?

Note that [Aspinwall-Morrison, hep-th/9705104] appeared before [Borel,Friedman,Morgan math.GR/9907007].

F-theory works in mysterious ways.

Open question

The F-theory should know the CS invariants of the flat connections, but how?

M-theory constructions and F-theory

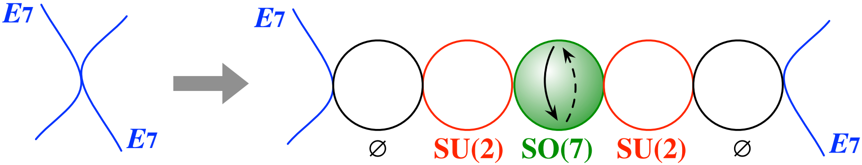
$\downarrow$

structure on the tensor branch and F-theory

$\downarrow$

massive type IIA constructions and F-theory

We're activating scalars in the tensor multiplet.



On generic points on the tensor branch, we just have

- tensor multiplets
- vector multiplets
- hypermultiplets

so one can apply a more traditional field-theoretical analysis.

Caveat: we are assuming that all nontrivial SCFTs have tensor branch.

Open question

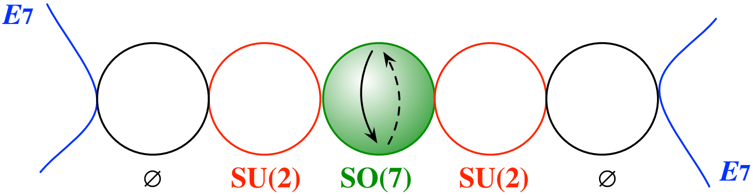
Show that if a 6d SCFT does not have any tensor branch,
it is a theory of free hypermultiplets.

Open question

Show that if a 4d $\mathcal{N}=2$ SCFT does not have any Coulomb branch,
it is a theory of free hypermultiplets.

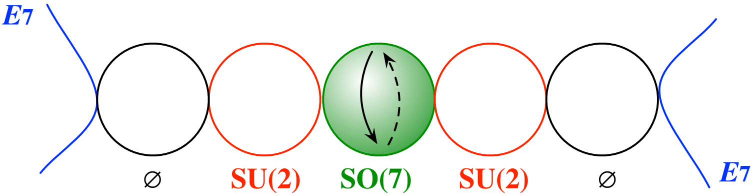
Comments:

- If you believe every 6d SCFT comes from F-theoretic singularities, then the answer is yes:
  3-dim CY singularities can always be resolved.
- Bootstrap?



In F-theory, each $\mathbb{P}^1$ is associated to

- a **tensor** multiplet
- a **string** coming from D3 wrapped there,
  whose tension is given by the scalar in the tensor multiplet
- possibly a **gauge multiplet**, whose inverse coupling squared is also given by the scalar in the tensor multiplet
- When $\exists$ gauge multiplet, the **string** is the **instanton-string**.



In F-theory, each intersection of two $\mathbb{P}^1$ supporting G_1 and G_2 gives a particular hypermultiplet charged under $G_1 \times G_2$.

So you can try to work **bottom-up**: what is the possible structure of

- tensors H_i, $i = 1, \ldots, n_T$
- vectors for G_a, $a = 1, \ldots, n_V$
- hypers charged under $\prod G_a$

on generic points of the tensor branch of a 6d SCFT?

Strong constraints come from the **anomaly cancellation**.

An important role is played by the **Dirac pairing** $\langle \cdot, \cdot \rangle$
on the charge lattice $\Lambda \simeq \mathbb{Z}^{n_T}$ of the strings.

Note that in 6d, the pairing is **symmetric**.

SCFT requires it to be **positive definite**.

Before getting further, I should say the analysis that follows would be **rather defective**.

- There's **no guarantee** that a given anomaly-free combination comes from a 6d SCFT that really exists.
- The analysis does not tell us anything about the **models where there are no vectors.**

I will come back to the first point later.

On the second point:

There should be a way to understand better the **models** that do **not have vector multiplets on the tensor branch**.

So far, known examples are

- **ADE** $\mathcal{N}=(2,0)$ theories, and
- **E-string** theory and its higher-rank analogues, which are $\mathcal{N}=(1,0)$.

Why does $\mathcal{N}=(2,0)$ theories classified by **ADE**?

Where does the E_8 **symmetry** come from, for $\mathcal{N}=(1,0)$ examples?

Why does $\mathcal{N}=(2,0)$ theories classified by ADE?

There is a nice argument [Henningson, hep-th/0405056] showing that the **anomaly cancellation on the string worldsheet** requires that the charge lattice of any $\mathcal{N}=(2,0)$ theory should be a **simply-laced root lattice**.

Essential points of his idea are as follows.

- Take the string of charge vector $\boldsymbol{q} \in \boldsymbol{\Lambda}$.
- The string breaks half the SUSY.
  Nambu-Goldstone modes become worldsheet fields.
  They are chiral, and therefore anomalous.
- It also couples to the self-dual fields in the bulk.
  This gives the anomaly inflow proportional to $\langle \boldsymbol{q}, \boldsymbol{q} \rangle$.
- The cancellation requires $\langle \boldsymbol{q}, \boldsymbol{q} \rangle = \mathbf{2}$.
- So $\boldsymbol{\Lambda}$ is an integral lattice generated by elements of $\mathbf{(length)^2 = 2}$.

How about $\mathcal{N}=(1,0)$ examples?

Open question

Give an argument that if a genuine $\mathcal{N}=(1,0)$ SCFT does not have any vector multiplet on generic points on the tensor branch, it is the E-string or its higher-rank analogue.

The first step would be to study the rank-1 case.

Assume that the Dirac pairing of the charge lattice $\mathbf{\Lambda} \simeq \mathbb{Z}$ is minimal.

Then it seems that the cancellation of the worldsheet anomaly requires that $\exists$ a left-moving $c = 8$ modular-invariant sector, showing that it automatically has E_8 symmetry.

Anybody interested in filling in the gaps?

Or is it already given in the literature?

Let us come back to the setup:

- tensors H_i, $i = 1, \ldots, n_T$
- vectors for G_a, $a = 1, \ldots, n_V$
- hypers charged under $\prod G_a$

Let us further assume $n_T = n_V$.
Instanton strings are charged under the tensors, so we have

$$dH_a = c_2(G_a).$$

This contributes to the anomaly by

$$I_8^{\text{tensor}} = \frac{1}{2}\Omega^{ab} c_2(G_a) c_2(G_b)$$

where Ω^{ab} is the integral Dirac pairing of the charge lattice.

A side comment:

The IIB F_5 also satisfies $dF_5 = (\text{something})$, and therefore there is

$$I_{12}^{\text{GS}} = \frac{1}{2}(\text{something})^2$$

which is not usually discussed. Is it zero?

If it isn't, it ruins the anomaly cancellation of IIB supergravity.

A side comment:

The IIB $\boldsymbol{F_5}$ also satisfies $\boldsymbol{dF_5 = (\text{something})}$, and therefore there is

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which is not usually discussed. Is it zero?

If it isn't, it ruins the anomaly cancellation of IIB supergravity.

Yes, since $\boldsymbol{(\text{something}) = H_3 \wedge G_3}$.

Coming back to 6d, the total anomaly should vanish:

$$0 = \frac{1}{2}\Omega^{ab}c_2(G_a)c_2(G_b) + \sum_a I_8^{\text{vector}}(G_a) + \sum I_8^{\text{hyper}}$$

First analyzed by Seiberg [hep-th/9609161] for $n_V = n_T = 1$.

For $\mathbf{SU(2)}$ with $2N_f$ half-hypers in the doublet, we have

$$0 = \frac{1}{2}\Omega\, {c_2}^2 - \frac{32 - 2N_f}{24}{c_2}^2$$

We need $32 > 2N_f$, because $\Omega > 0$.

For $\mathbf{SU(2)}$ with N_f half-hypers in the doublet, we have

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We need $32 > 2N_f$, because $\Omega > 0$.

Bershadsky and Vafa in [hep-th/9703167] pointed out that there are more constraints.

- F-theory constructions only gave $N_f = 4$ and $= 10$. Why?
- In 6d, there's a global anomaly associated to $\pi_6(\mathbf{SU(2)}) = \mathbb{Z}_{12}$, and it requires $32 - 2N_f = 0 \mod 12.$
- Also, Ω needs to be an integer, which gives the same condition.

How on earth does the F-theory know the global anomaly associated to the subtle homotopy group $\boldsymbol{\pi_6(SU(2)) = \mathbb{Z}_{12}}$?

F-theory works in mysterious ways.

We can extend the field-theoretical analysis to general $n_T = n_V$, with arbitrary $\prod G_a$ and arbitrary hypers.
[Heckman-Morrison-Rudelius-Vafa 1502.05405][Bhardwaj, 1502.06594]

One example is $n_T = n_V = 2$, $G = \mathfrak{su}(2) \times \mathfrak{so}(8)$, with a half-hyper in $\mathbf{2} \otimes \mathbf{8}_V$ and full spinors in $\mathbf{8}_S \oplus \mathbf{8}_C$.

This is free of both local and global anomalies, with the charge pairing

$$\begin{pmatrix} 2 & -1 \\ -1 & 3 \end{pmatrix}$$

But **this is never realized in F-theory!**
Is there a field-theoretical way to see this?

Yes. [Ohmori-Shimizu-YT-Yonekura, 1508.00915, Appendix A]

This model tries to gauge the $\mathfrak{so}(8)$ flavor symmetry of the 6d model

$\mathfrak{su}(2)$ and 4 doublets on the tensor branch.

The point is that this SCFT only has $\mathfrak{so}(7) \subset \mathfrak{so}(8)$
where 8 doublets transform as a spinor of $\mathfrak{so}(7)$.

So you can't gauge $\mathfrak{so}(8)$.

To see that there is only $\mathfrak{so}(7)$, compactify on T^2 the 6d model

$\mathfrak{su}(2)$ and 4 doublets on the tensor branch.

You get

4d $\mathcal{N}$=2 $\mathfrak{su}(2)_G$ with 4 doublets

+ an additional $\mathfrak{su}(2)_T$ coming from the 6d tensor.

The Weyl group of $\mathfrak{su}(2)_T$ is the S-duality of this
4d $\mathcal{N}$=2 $\mathfrak{su}(2)_G$ with 4 doublets, and maps $\mathbf{8}_V$ to $\mathbf{8}_S$.

So, only the $\mathfrak{so}(7) \subset \mathfrak{so}(8)$ s.t. $\mathbf{8}_C \to \mathbf{7} + \mathbf{1}$ is compatible.

Another way is to compactify on S^1 the 6d model

$\mathfrak{su}(2)$ and 4 doublets on the tensor branch.

You get

5d $\mathcal{N}=1$ $\mathfrak{su}(2)_G$ with 4 doublets

$+$ an additional $\mathfrak{su}(2)_T$ coming from the 6d tensor.

5d $\mathfrak{su}(2)_G$ with 4 doublets has an enhanced $\mathfrak{so}(10)$ flavor symmetry,
and the $\mathfrak{su}(2)_T$ gauges the $\mathfrak{so}(3)$ subgroup,
which comes from the enhanced instanton number symmetry.

The commutant is clearly $\mathfrak{so}(7)$, since $\mathfrak{so}(3) \oplus \mathfrak{so}(7) \subset \mathfrak{so}(10)$.

How does the F-theory know this subtle issue?

F-theory works in mysterious ways.

M-theory constructions and F-theory

$\downarrow$

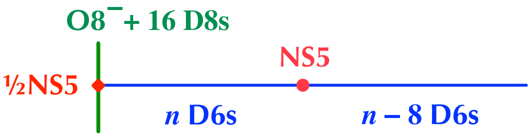
structure on the tensor branch and F-theory

$\downarrow$

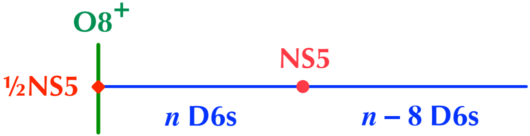
massive type IIA constructions and F-theory

We can also use (massive) IIA to engineer 6d SCFTs.
[Brunner-Karch, hep-th/9712143] [Hanany-Zaffaroni, hep-th/9712145]

Two examples:



gives $\mathfrak{su}(n)$ with an **antisymmetric** and $n + 8$ fundamentals, and



gives $\mathfrak{su}(n)$ with a **symmetric** and $n - 8$ fundamentals.

They are free from anomalies, both local and global.

However, it so happens that the F-theoretic atomic classification [Heckman-Morrison-Rudelius-Vafa, 1502.05405] **includes**

$\mathfrak{su}(n)$ with an **antisymmetric** and $n + 8$ fundamentals

but **does not include**

$\mathfrak{su}(n)$ with a **symmetric** and $n - 8$ fundamentals.

This conundrum was noticed by many people simultaneously, in the US, in Canada, in Japan, and maybe elsewhere too.

I didn't notice it myself, but I learned about it from multiple sources.

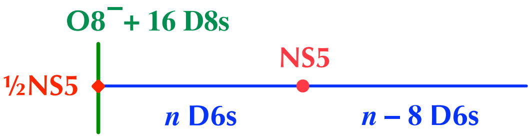
Does the model

$$\mathfrak{su}(n) \text{ with a } \textbf{symmetric} \text{ and } n-8 \text{ fundamentals.}$$

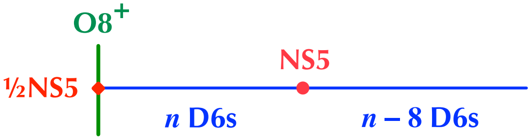
have some secret inconsistency?

No. Classifications in F-theory so far forgot to include **O7+**.
[work in progress, with Bhardwaj, Morrison and Tomasiello]

You can take either

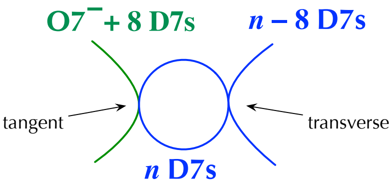


or



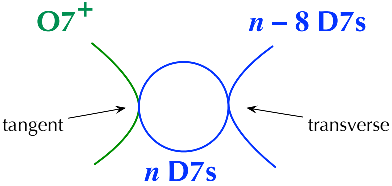
and compactify one transverse direction,
then take the T-dual to go to the IIB.

You get



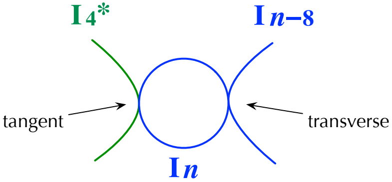
O7⁻ + 8 D7s
n − 8 D7s
tangent
transverse
n D7s

or

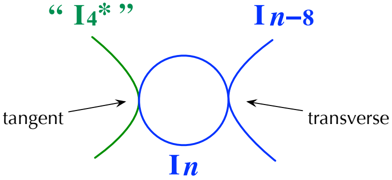


O7⁺
n − 8 D7s
tangent
transverse
n D7s

In the F-theory language, we have



or



What's this ***“I_4^*”***?

The point is that in F-theory, there are **two** distinct types of I_n^* singularities when $n \geq 4$.

- I_n^* with $\mathfrak{so}(2n+8)$ symmetry, deformable to be smooth.
- "I_n^*" with $\mathfrak{usp}(2n-8)$ symmetry, deformable only down to "I_4^*".

The latter is frozen by a mysterious discrete flux [Witten, hep-th/9712028].

You might worry that other Kodaira types might have frozen versions, but that doesn't happen[YT, 1508.06679].

F-theory geometrizes most of the field theory phenomena, but it still needs some additional data. **cf.** T-brane.

Summary

- 6d SCFTs can be studied in many ways.

- Various subtle field-theoretical features are always encoded in F-theory, but in mysterious ways.

  – Properties of $\boldsymbol{ADE}$ Instantons on $\mathbb{R} \times \boldsymbol{T^3}$
  – Global anomaly $\boldsymbol{\pi_6(\mathrm{SU}(2)) = \mathbb{Z}_{12}}$
  – Reduction of flavor symmetry $\boldsymbol{\mathfrak{so}(8) \supset \mathfrak{so}(7)}$ of $\boldsymbol{n_T = 1}$ $\boldsymbol{\mathfrak{su}(2)}$ with $\boldsymbol{4}$ doublets
  – Issues on $\boldsymbol{O^+}$-planes

- There are many open questions.

Open question

Does every 6d SCFT come from F-theory?

Open question

Does every 6d SCFT come from F-theory?

Open question

Does every 4d $\mathcal{N}=2$ SCFT come from string theory?

cf. As far as I know, there is neither a stringy realization of

- 4d $\mathcal{N}=2$ $\mathbf{SU(7)}$ or $\mathbf{SU(8)}$ with **3-index antisymmetric**
- 4d $\mathcal{N}=2$ $\mathbf{Sp(3)}$ or $\mathbf{Sp(4)}$ with **3-index antisymmetric**
- 4d $\mathcal{N}=2$ $\mathbf{SO(13)}$ or $\mathbf{SO(14)}$ with **spinor**

nor the Seiberg-Witten solutions to them. Does F-theory help?

Note that almost exactly the same list of groups were given by Dave on the 1st day of the workshop, as subtle cases in Tate's algorithm. Any relation?