Elliptic Calabi-Yau fourfolds and 4D F-theory vacua

Dave Day
F-theory at 20 conference
Burke Institute, Caltech

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Washington (Wati) Taylor, MIT

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Written in collaboration with various subsets of:
L. Anderson, A. Grassi, T. Grimm, J. Halverson, S. Johnson, G. Martini, D. Morrison, J. Shaneson, Y. Wang

Dave and I began working together in the summer of 2009 in Aspen

We had a common interest in understanding 6D supergravity and F-theory models and explaining the connection between these

Since then we have written roughly one paper a year. Gone from 6D to 4D, and explored lots of fascinating physics and math; it has been an ongoing adventure!

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Warm-up: Elliptic Calabi-Yau threefolds/6D models

Recent work: Elliptic Calabi-Yau fourfolds/4D models

Philosophy: need the big picture to figure out how our world fits in

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Summary: 6D F-theory and elliptic Calabi-Yau threefolds

Using tools from algebraic geometry and physics intuition, we have a systematic approach to constructing elliptic Calabi-Yau threefolds and understanding 6D F-theory models

Classifying elliptic CY threefolds

Elliptic CY3 $\pi : X_3 \to B_2$
Weierstrass model $y^2 = x^3 + fx + g$,
$f \in \Gamma(\mathcal{O}(-4K_B)), g \in \Gamma(\mathcal{O}(-6K_B))$

- Basic idea: classify bases B, then tune Weierstrass for each base
  Focus on Weierstrass models on smooth bases (*e.g.* not SCFT)
- Minimal models + work of Grassi:
  $B = \mathbb{P}^2, \mathbb{F}_m$ or blowup thereof (or Enriques)
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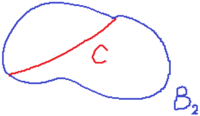
Geometry of non-Higgsable groups

The base B_2 is a complex surface.

Contains homology classes of complex curves C_i



For $C \cong \mathbb{P}^1 \cong S^2$, local geometry encoded by *normal bundle* $\mathcal{O}(m)$

$C \cdot C = m$; *e.g.*, $N_C \cong \mathcal{O}(2) \cong TC$: deformation has 2 zeros, $C \cdot C = +2$

If $N_C \cong \mathcal{O}(-n), n > 0$, C is *rigid* (no deformations)

For $\mathcal{O}(-n), n > 2$, base space is so curved that singularities must pile up to preserve Calabi-Yau structure on total space $\Rightarrow$ non-Higgsable gauge group

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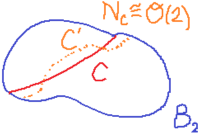

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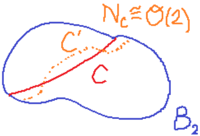

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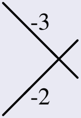
Classification of 6D "Non-Higgsable Clusters" (NHC's) [Morrison/WT]

Clusters of curves imposing generic nontrivial codimension one singularities:

$-m$

$(m = 3, 4, 5, 6, 7, 8, 12)$

$\mathfrak{su}(3), \mathfrak{so}(8), \mathfrak{f}_4$
$\mathfrak{e}_6, \mathfrak{e}_7, \mathfrak{e}_8$



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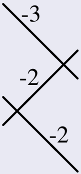

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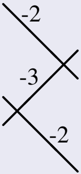


$\mathfrak{su}(2) \oplus \mathfrak{so}(7) \oplus \mathfrak{su}(2)$

- Any other combination including -3 or below $\Rightarrow$ (4, 6) at point/curve

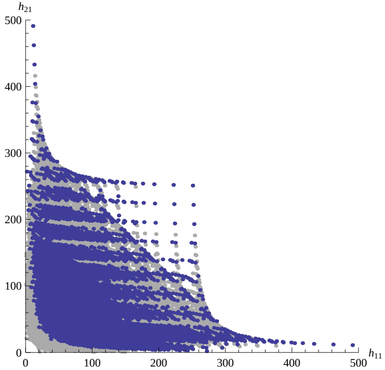
NHC's a useful tool in classifying bases B_2 for EF CY3's

– Also useful in classifying 6D SCFT's, LST's (cf. Heckman, Rudelius talks)

Classifying bases I: toric B_2

Start with $\mathbb{P}^2$, $\mathbb{F}_m$, blow up torically

Generic EF Hodge #'s [Morrison/WT, WT]



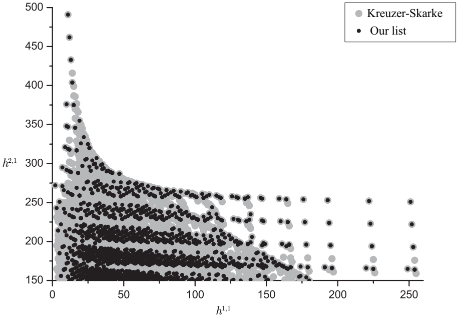
- 61,539 toric bases (some not strictly toric: -9, -10, -11 curves)
- Reproduces large subset of Kreuzer-Skarke database of CY3 Hodge #'s
  Boundary of "shield" from generic elliptic fibrations over blowups of $\mathbb{F}_{12}$.

Beyond toric: approach allows construction of general (non-toric) bases

– Computed all 162, 404 "semi-toric" bases w/ 1 $\mathbb{C}^*$-structure [Martini/WT]

Generally: Keep track of cone of effective divisors as combinatorial data

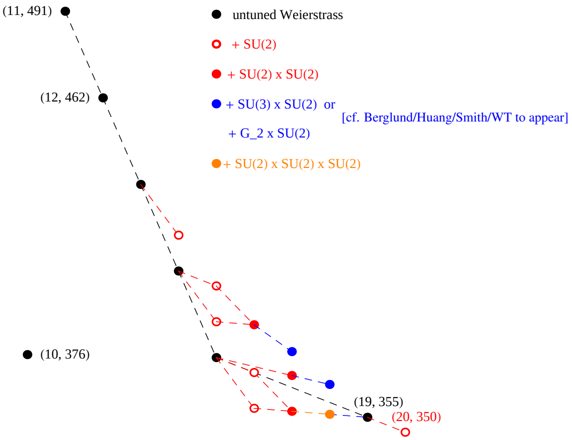
- All bases for EF CY threefolds w/ $h^{2,1}(X) \geq 150$ [WT/Wang]



Technical issues at large $h^{1,1}(X)$, small $h^{2,1}(X)$:
Infinite generators for cone, Multiply intersecting -1 curves

Upshot: modest expansion of possibilities beyond toric, semi-toric

EFCY3's w/ $h^{2,1} \geq 350$, $\mathbb{F}_m$+ tuning → full WM classification [Johnson/WT]



- Matches KS; non-toric + toric at (19, 355); new non-toric below 350
- Empirical data on Calabi-Yau's suggests: "most" (known) CY's are elliptic, particularly at large Hodge numbers (cf. [Gray/Haupt/Lukas])

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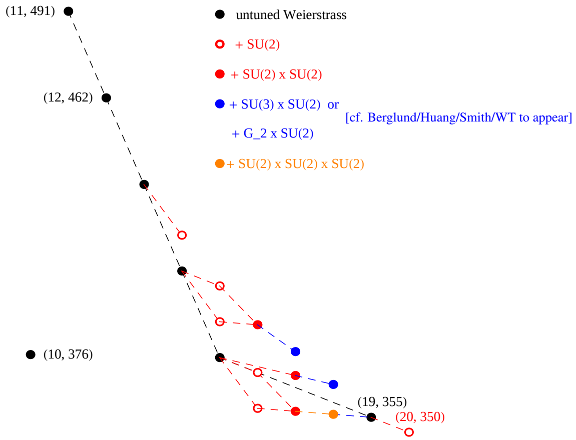

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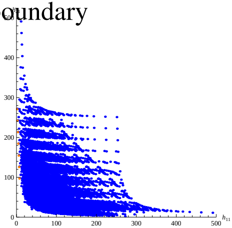
Elliptic Calabi-Yau threefolds: upshot

- Systematic approach to construction
- Complete control at large $h^{2,1}(X)$ (*e.g.*, proof $h^{2,1} \leq 491$)
- Toric bases give good representative global picture, capture boundary
- Finite number of bases, minimal $\mathbb{P}^2, \mathbb{F}_m$ on left boundary



- "Most" bases B_2 have non-Higgsable G_{NA}
  (all but weak Fano = gdP)

Some outstanding issues:

- Difficult regime: large $h^{1,1}(X)$, small $h^{2,1}(X)$ (cf. Park talk)
- Classifying matter/codim. 2 tuning + transitions
  (cf. Anderson, Klevers, Morrison, Raghuram talks)
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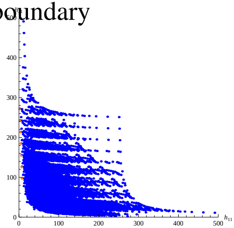

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4D F-theory compactifications

Story parallel in many ways:

– Compactify on Calabi-Yau fourfold, base B_3 = complex threefold

– Empirical data suggest similar structure (though less complete for CY4's)

No proof of finiteness
Mori theory threefold analog of minimal model bases more subtle

All evidence so far: moduli space of CY 4's quite parallel to CY3 story

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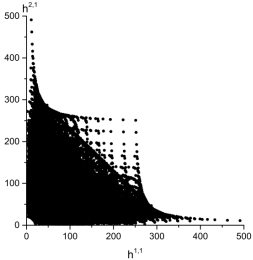

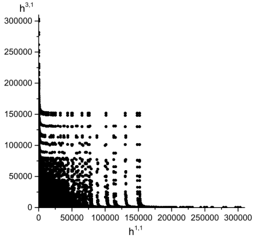

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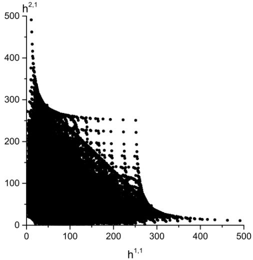

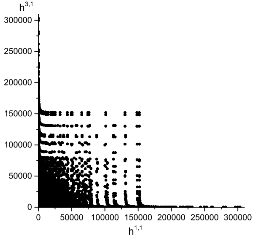

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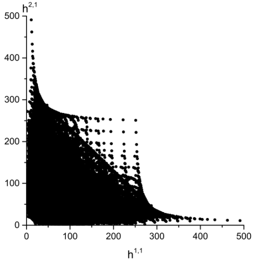

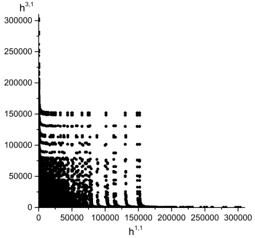

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4D non-Higgsable clusters [Morrison/WT]
(see also: Anderson/WT, Grassi/Halverson/Shaneson/WT, cf. Halverson talk)

At level of geometry/complex structure, similar to 6D but more complicated

Expanding in coordinate z, around divisor (surface) $S = \{z = 0\}$,

$$f = f_0 + f_1 z + f_2 z^2 + \cdots$$

Compute using geometry of *surfaces*: up to leading non-vanishing term,

Single group clusters: $\boxed{SU(2), SU(3), G_2, SO(7), SO(8), F_4, E_6, E_7, E_8}$

(cannot have: non-Higgsable $SU(5), SO(10)$)

the only 2-factor products that can appear are:

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Classification of elliptic Calabi-Yau fourfolds

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No proofs, but finite classification seems manageable.
Rough "physicist's" picture – ignore various subtleties
Focus on classifying bases B_3, apparently finite number

"minimal models" $\sim \mathbb{F}_m$ but more complex – populate LHS of Hodge plot
Roughly, min $B_3 = \{\mathbb{P}^1$ (conic) bundle over B_2, B_2 bundle over $\mathbb{P}^1$, Fano$\}$

Blow up curves, points: $h^{3,1} \downarrow, h^{1,1} \uparrow$; finite # of options on each minimal B_3

w/Halverson: $\mathbb{P}^1$ bundles over toric bases B_2
(w/Anderson: B_2 = gdP, smooth heterotic dual)
Finite # $\mathbb{P}^1$ bundles over fixed B_2 (cf. 2015 talk)

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Max $h^{3,1}$ = 303,148 (cf. Wang talk)

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Possible issue: irrational bases

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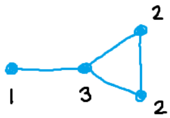
Random walk on a graph: $p_i \propto \nu_i =$ # of neighbors, *e.g.*



Explore connected toric threefold bases from $\mathbb{P}^3$ by blow-up, -down transitions

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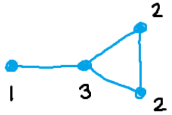

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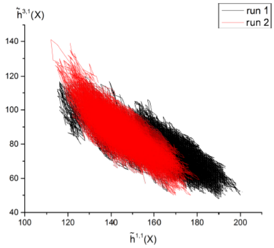

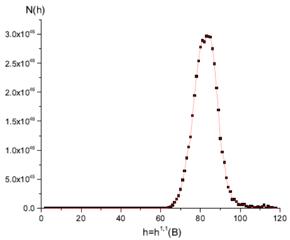

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Structure of elliptic fourfold over "typical" toric base B_3

(note: in connected set, misses most w/ E_8 divisors)

- $h^{1,1}(B) \cong 82 \pm 6$
- # flops ~ 20
- Codimension 1 Kodaira singularity $\Rightarrow G_{\rm NA}$:

$\sim 14\times$ SU(2), $\sim 10 \times G_2$, $\sim 3 \times F_4$, $\sim 2\times$ SU(3), $\sim 1\times$ SO(8)

- Connected products:

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$\sim 10\%$ of NH products are SU(3) $\times$ SU(2)!

- <Biggest cluster>: ~ 16, max found: 37
- Typical base has several codim 2 singularities w/o smooth CY resolution (?)

Structure of elliptic fourfold over “typical” toric base B_3
(note: in connected set, misses most w/ E_8 divisors)

• $h^{1,1}(B) \cong 82 \pm 6$

• # flops ~ 20

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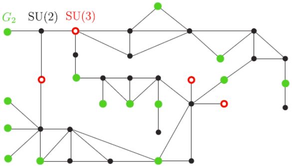

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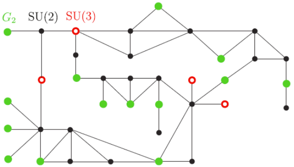

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Irrational F-theory models (w/Morrison, 16xx.xxxxx)

For 6D F-theory/elliptic CY3's, all B_2 rational (birational to $\mathbb{P}^2$)

Not true for 4D/elliptic CY4's!

Clemens-Griffiths ('72): cubic threefold (in $\mathbb{P}^4$) is *not* birational to $\mathbb{P}^3$.

$$B = \{[x_1 : \cdots : x_5] \in \mathbb{P}^4, f_3(x_1, \ldots, x_5) = 0\}$$

$$f_3 = \sum_{i \leq j \leq k} c_{ijk} x_i x_j x_k = c_{111} x_1^3 + c_{112} x_1^2 x_2 + \cdots$$

can build EF CY4's over B

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Elliptic Calabi-Yau fourfolds over the cubic threefold

Need $f \in \Gamma(\mathcal{O}(-4K)), g \in \Gamma(\mathcal{O}(-6K))$

By adjunction $K_B = (K_{\mathbb{P}^4} + B)_B = (-5H + 3H)|_B = -2H|_B$

$\Rightarrow f, g$ from homogeneous degree 8, 12 polynomials on $\mathbb{P}^4$

Fano $\Rightarrow X$ generically smooth (no NHC's)

Hodge numbers:

$h^{1,1}(B) = 1$, w/no generic U(1)'s $\Rightarrow h^{1,1}(X) \cong 2$

count c's $\in \mathcal{O}(3H)$: 35

f's: $0 \to \mathcal{O}_{\mathbb{P}^4}(5H) \to \mathcal{O}_{\mathbb{P}^4}(8H) \to \mathcal{O}_X(8H) \to 0$: $330 - 126 = 204$

g's: $1365 - 495 = 870$

$h^{3,1} \cong 870 + 204 + 35 - 24 = 1085$

Compare Fano $\mathbb{P}^3$, $h^{3,1} = 3878$, Max $h^{3,1} = 303,148$

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Upshot: cubic threefold $B_3 \rightarrow$ simple base for 4D F-theory

– Seems relatively unremarkable despite irrational nature

– Relatively small $h^{3,1}, h^{1,1}$

– No NHC's

– Can blow up points and curves *e.g.* by blowing up in ambient $\mathbb{P}^4$

– Can connect to rational by multiple conifold transitions

– Could do similar analysis on other irrational Fano threefolds

Conclusions

- Getting a birds eye view of the 4D F-theory landscape
- Apparently a finite number (but $> 10^{50}$) of bases B_3
- Finite tunings over each base
- NHC's generic

Next problem: systematics of fluxes, moduli stabilization in typical vacua w/NHC's (cf. Weigand talk)

Several possible scenarios for standard model:

– Typical GUT tuning: seems expensive [Braun/Watari]

– Typical base: NHC's may contribute to nonabelian SM group [Grassi/Halverson/Shaneson/WT]

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In all scenarios: NHC's promising source of dark matter.

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Happy Birthday to F-theory and to Dave!