Heterotic string solitons and degenerations of K3 surface

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work in progress with A.P.Braun (Oxford)
Feb.23 (tue) '16, F-theory@20 Caltech

* away until Aug. '16

- Duality Het ⟺ IIA @6D

T^4 Narain K3

$$\mathrm{Isom}(\mathrm{II}_{4,20}) \backslash O(4,20;\mathbb{R}) / O(4)\times O(20)$$

- 6D eff. theories w/ (1,1) SUSY
fibred adiabatically over $\mathbb{P}^1$ ⟶ 4D N=2 SUSY.

$$\mathrm{Het} / \text{"}T^2\times\text{"}\mathrm{K3} \Longleftrightarrow \mathrm{IIA} / \text{K3-fib.}CY_3 = M$$

Kachru Vafa ’95
Klemm Lerche Mayr ‘95
…….

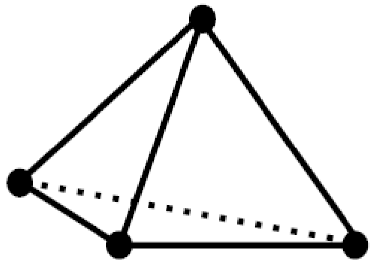
- fibre adiabatically over $\mathbb{P}^1$
  - first step: specify a lattice polarization of K3 (IIA).

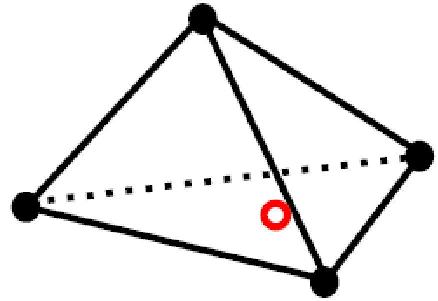
	$[U \oplus \Lambda_S] \otimes \mathbb{C}$	$\oplus \quad \Lambda_T \otimes \mathbb{C}$	$\subset \mathrm{II}_{4,20} \otimes \mathbb{C}$
$(k^8 + ik^9)$	"fixed" over $\mathbb{P}^1$		$(B+iJ)[K3]$
$(k^6 + ik^7)$		vary over $\mathbb{P}^1$	$\Omega(\mathrm{K3})$

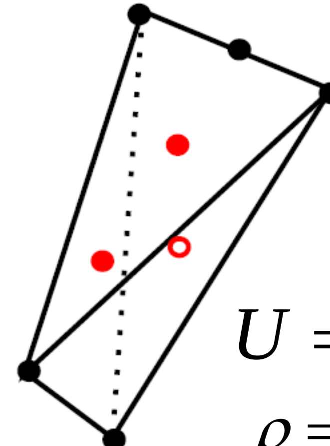
  - second: two aspects to study
    - further discrete choices in fibration.
    - degeneration of fibre. not adiabatic.

- Multiple choices of lattice-pol. K3 fibration

$\tilde{\Delta}_{K3} =$

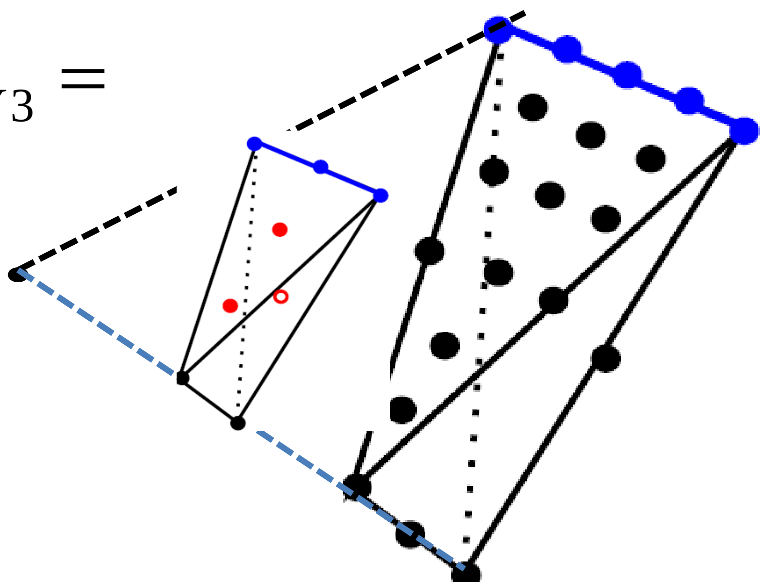






$\Lambda_S =$ $\langle +4 \rangle \quad \rho = 1$ $\quad \langle +2 \rangle \quad \rho = 1$ $\quad U = \mathrm{II}_{1,1} \quad \rho = 2.$

$\tilde{\Delta}_{CY3} =$



Choose any one from

$$2\tilde{\Delta}_{K3} \cap \mathbb{Z}^{\oplus 3}$$

For $h^{1,1}(M) = \rho + 1,$
blue points only.

Candelas Font '96

- Multiple choices of lattice-pol. K3 fibration

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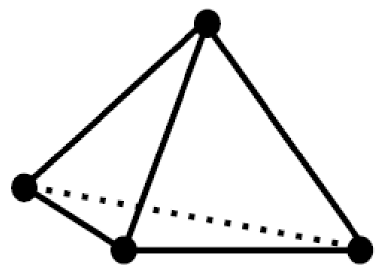

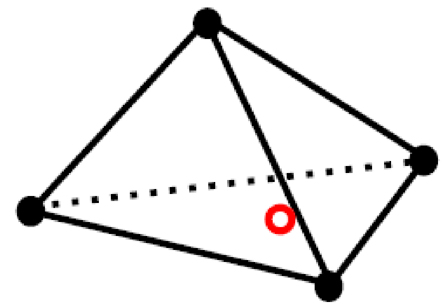

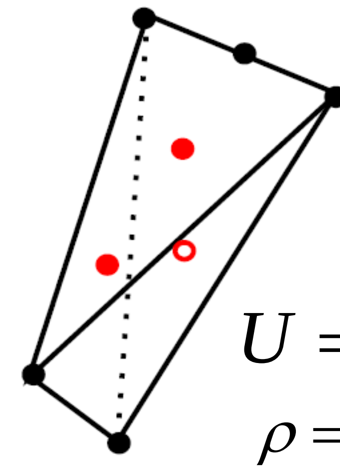

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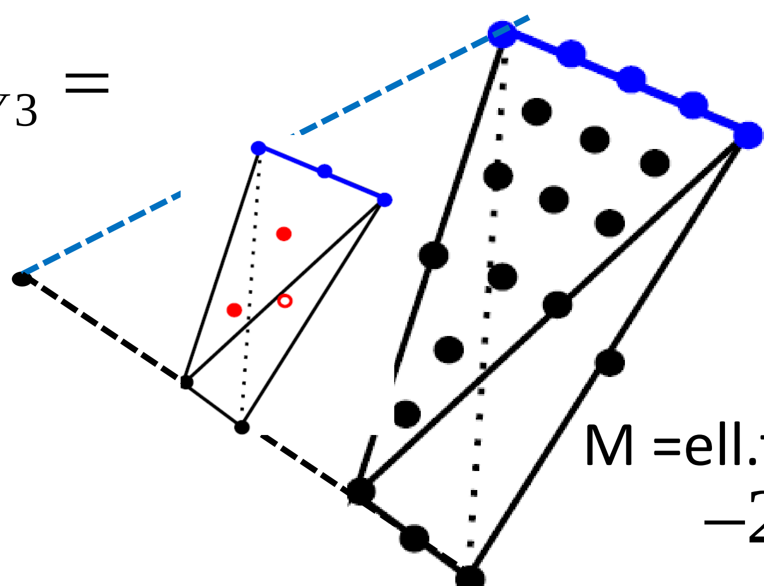

M =ell.fibr. over F_n $-2 \leq n \leq +2.$

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Candelas Font '96

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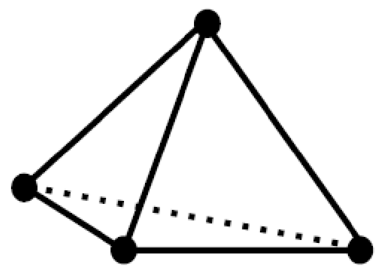

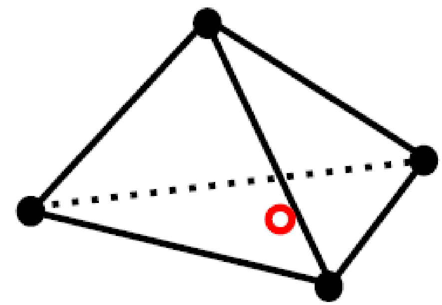

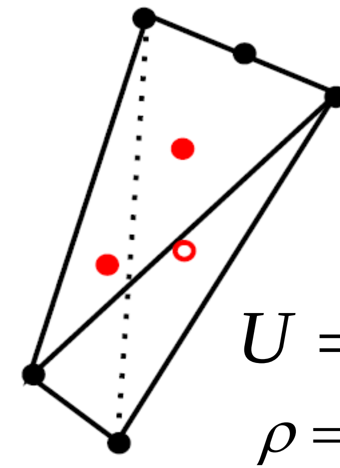

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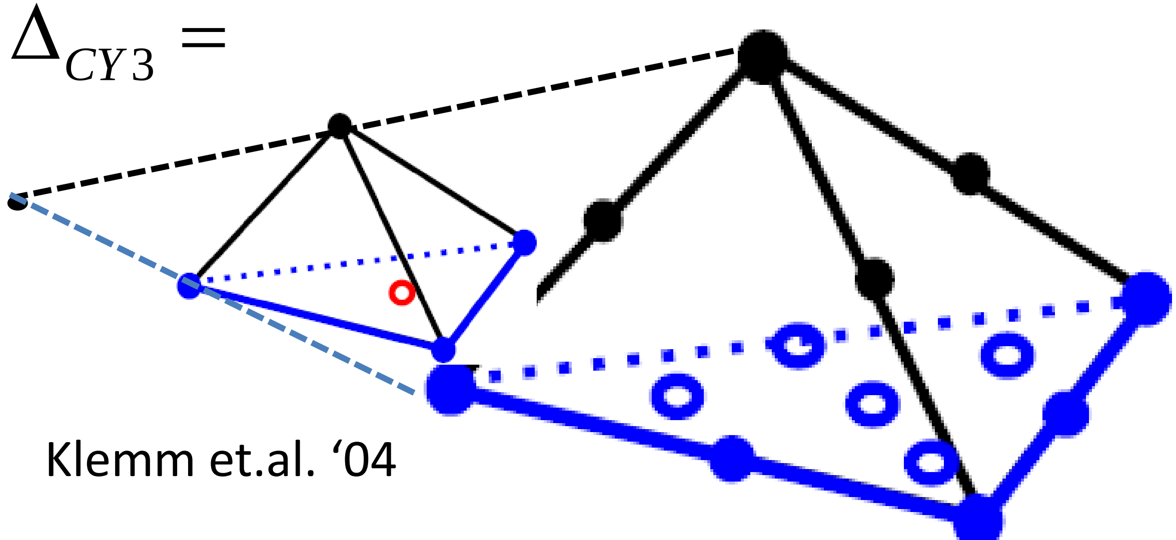

Choose any one from $2\tilde{\Delta}_{K3} \cap \mathbb{Z}^{\oplus 3}$

For $h^{1,1}(M) = \rho + 1$, blue points only.

Klemm et.al. '04

Candelas Font '96

- Multiple choices of lattice-pol. K3 fibration

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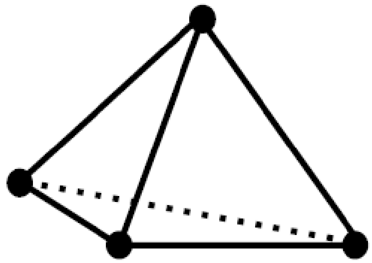

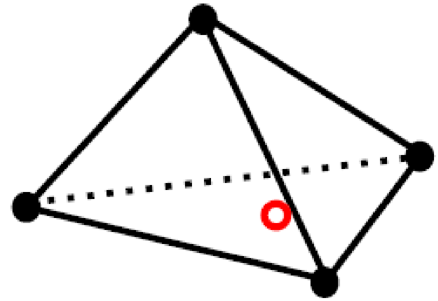

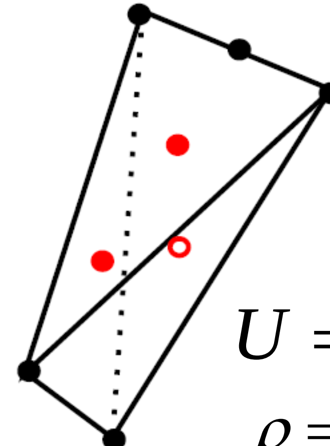

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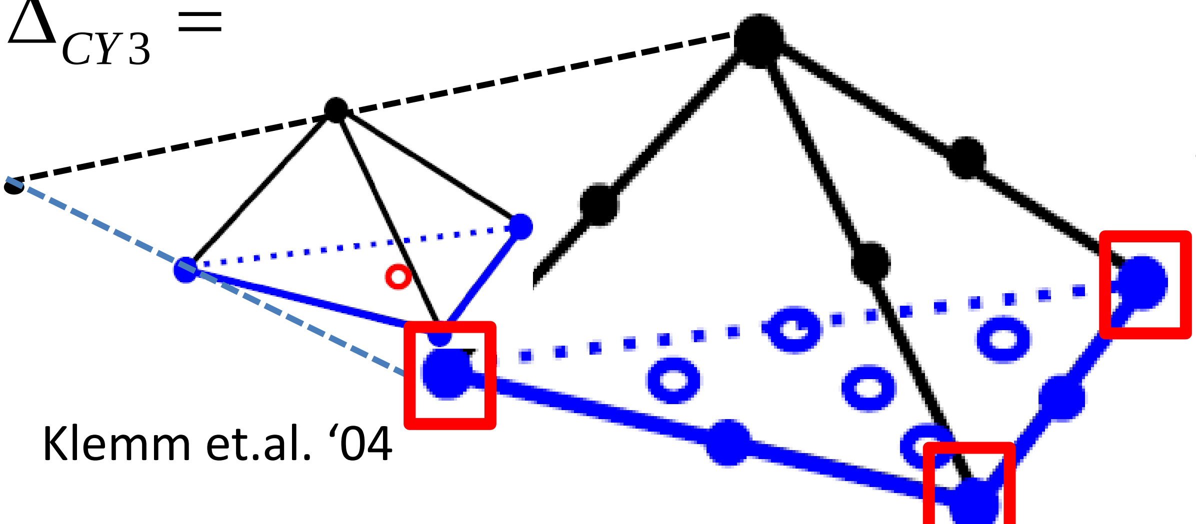


Klemm et.al. '04

Choose any one from

$2\tilde{\Delta}_{K3} \cap \mathbb{Z}^{\oplus 3}$

Type IIA on M

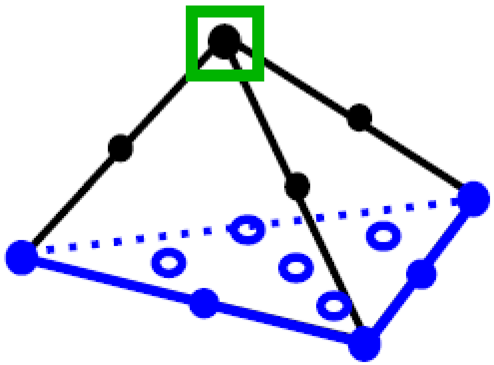
Kachru Vafa '95

Het on "T2 x" K3

instanton 4+10+10

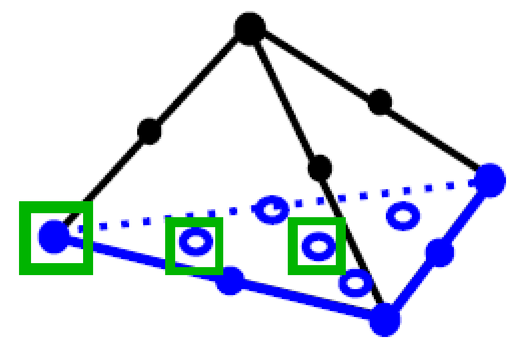
- Type IIA / M = deg-2 K3 fibr. over $\mathbb{P}^1$

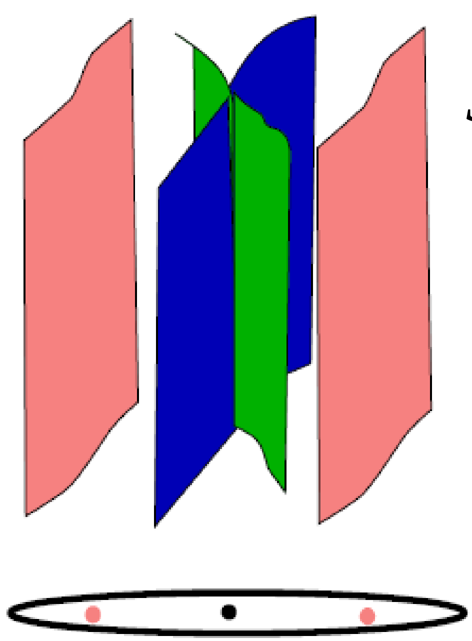
Braun TW '16



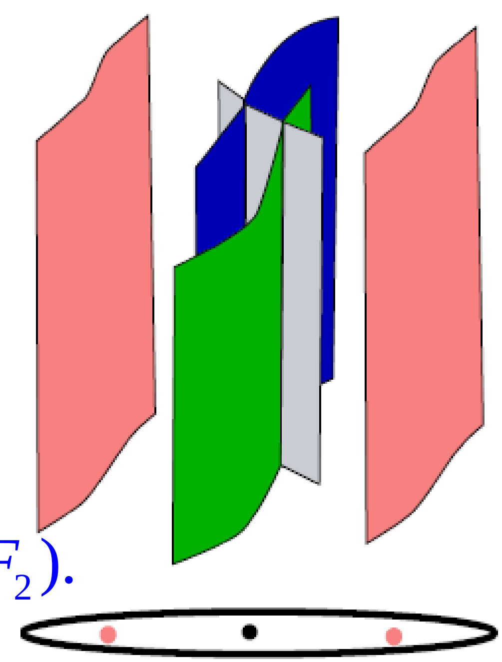
Add point(s) from

$$2\tilde{\Delta}_{K3} \cap \mathbb{Z}^{\oplus 3}$$





$$S_0 = \mathbb{P}^2 \cup \mathrm{Bl}^{18}(\mathbb{P}^2)$$



$$S_0 = dP_7 \cup (T^2 \times \mathbb{P}^1) \cup \mathrm{Bl}^{10}(F_2).$$

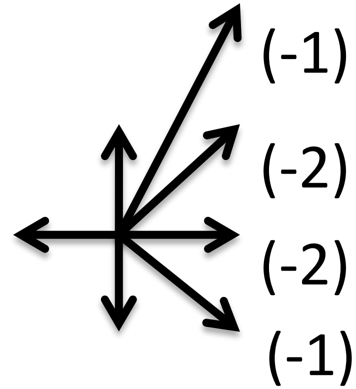
- Generalization

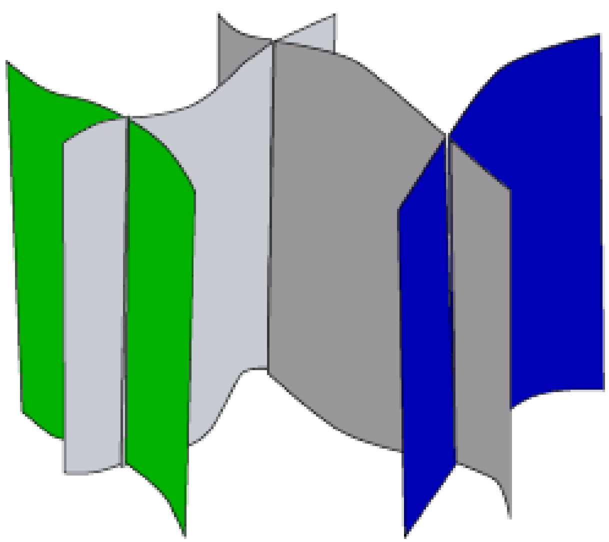
of IIA / $CY_3 = \text{ell.K3 fibr. over } \mathbb{P}^1$

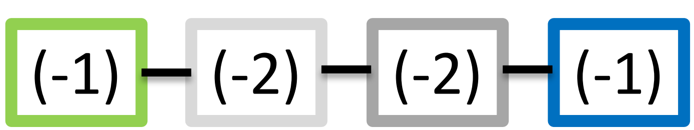
with degeneration

$$= \text{ell.fibr. over } \mathrm{Bl}^k(F_n)$$

$$S_0 = \text{RES} \cup (\mathrm{T}^2 \times \mathbb{ZP}^1)^{k-1} \cup \text{RES}$$







Dual to Het / T2 x K3
with k NS 5-branes

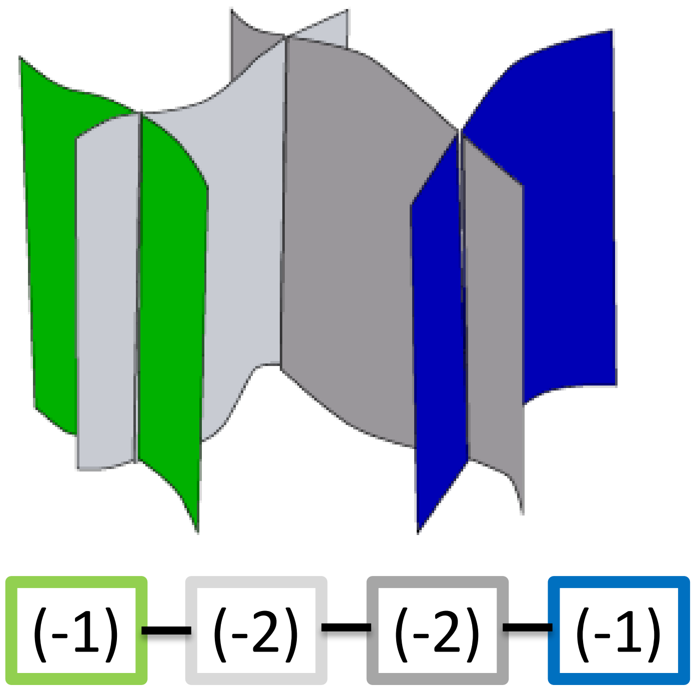
Morrison Vafa '96

- Generalization

IIA / $CY_3 = \Lambda_S$ pol. K3 fibr. over $\mathbb{P}^1$ with degeneration

Type II degeneration of lattice-pol. K3 surface

$$S_0 = \text{RES} \cup (\text{T}^2 \times \mathbb{ZP}^1)^{k-1} \cup \text{RES}$$



generic fibr. S_t degen. to

$$S_0 = V_0 \cup V_1 \ldots V_{k-1} \cup V_k$$

rational surfaces

monodromy

$$T : \Lambda_T(S_t) \to \Lambda_T(S_t)$$

$$T =: \exp[N], \quad N^2 = 0.$$

Kulikov , Persson, Pinkham, Friedman,
Morrison, Looijenga, Saha, Scattone, ...

Type II degeneration of lattice-pol. K3 surface

geometry of central (singular) fibre
subj. to birational modification.

characterization in monodromy

$\mathrm{Im}(N) =: W_1,$ rank 2

$W_1 \subset W_2 := [W_1^{\perp} \subset \Lambda_T]$ rank $20-\rho$

classify rank-2 isotropic sublat. $W_1 \subset \Lambda_T$

↓ mod $\Gamma \subset \mathrm{Isom}(\Lambda_T)$

classify (W_2 / W_1)

generic fibr. S_t degen. to

$$S_0 = V_0 \cup V_1 \ldots V_{k-1} \cup V_k$$

rational surfaces (V_0, V_k)

monodromy

$$T : \Lambda_T(S_t) \to \Lambda_T(S_t)$$

$$T =: \exp[N], \quad N^2 = 0.$$

Braun TW ‘16

- back to examples. (deg-2 K3 fibre)

degen. to $S_0 = dP_7 \cup (T^2 \times \mathbb{P}^1) \cup \mathrm{Bl}^{10}(F_2).$ $\Rightarrow$ $W_2/W_1 = (E_7 + D_{10}); \mathbb{Z}_2,$

degen. to $S_0 = \mathbb{P}^2 \cup \mathrm{Bl}^{18}(\mathbb{P}^2)$ $W_2/W_1 = A_{17}; \mathbb{Z}_3.$

Clemens—Schmid

both fall into 4 classes for deg2 K3 [Scattone]

- Het interpretation: defects in $\mathbb{P}^1$
  - NS 5-brane: $\Lambda_S = U,\ W_2/W_1 = E_8 + E_8,$
  - 1st eg. above: $\Lambda_S = \langle +2 \rangle,\ W_2/W_1 = (E_7 + D_{10}); \mathbb{Z}_2.$

precisely the same N (2 x 2) matrix

analogous to (p,q) 7-branes.
indicates which defects can live together on $\mathbb{P}^1$