F-theory in 4 Dimensions and Phenomenology

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20 years of joy

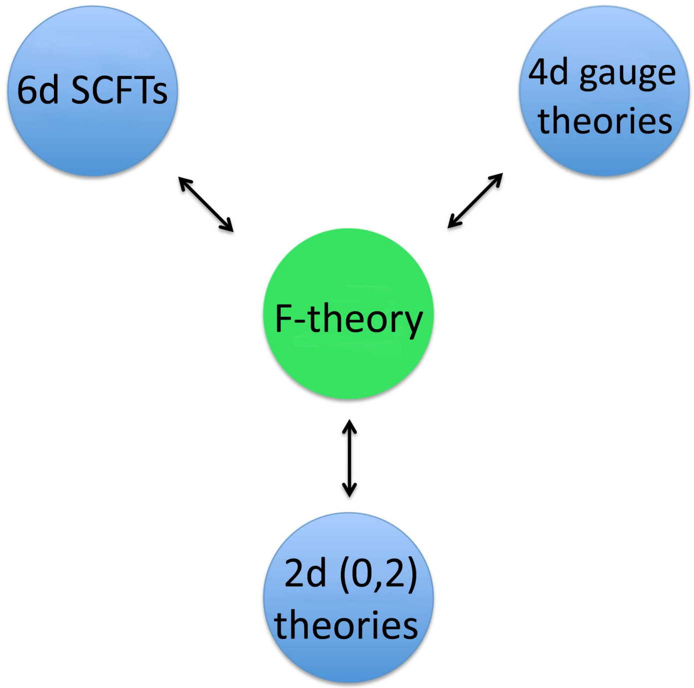
F-theory is perhaps the most general currently controlled framework to think about (non-perturbative) brane configurations in geometric regime

- beyond pert. Type II orientifolds due to [p,q]-branes
- still within (conformal) Calabi-Yau geometry and thus well-controlled

⇒ framework to understand geometric compactifications w/ branes

Geometric engineering of gauge theories in various dimensions - including coupling to gravity, such as

- 6d SCFTs
- 4d gauge theories
- 2d (0,2) theories
  [talk by Schäfer-Nameki]



Themes in F-theory pheno

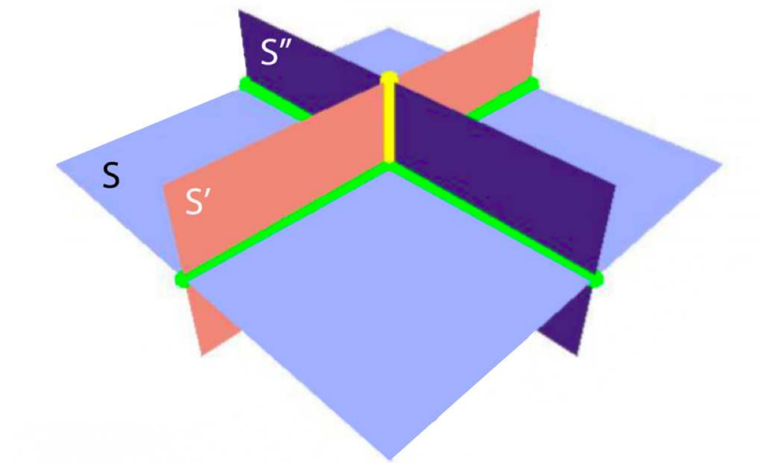
This review will focus on 4d F-theory and applications to phenomenology, in particular application to F-theory GUTs

[Beasley,Heckman,Vafa; Donagi,Wijnholt'08]

Hierarchy of localisation:

- SU(5) $\leftrightarrow$ 4-cycle
- matter $\leftrightarrow$ 2-cycle
- Yukawa $\leftrightarrow$ point

E_6-point $\leftrightarrow$ $\mathbf{10}\,\mathbf{10}\,\mathbf{5}$



Pic: Cordova, 0910.2955

Two key ingredients of $SU(5)$ GUT models in F-theory:

- Hypercharge flux induced GUT breaking
- Extra U(1)/discrete symmetries

The need to understand these has triggered tremendous technical progress in recent years.

Themes in F-theory pheno

1) GUT model building:

$\rightarrow$ mostly within TeV scale SUSY paradigm

$\rightarrow$ some extensions to intermediate scale $SU(5)$ GUTs

- Global models

  $\leftrightarrow$ fully fledged 4-fold and G_4 gauge background

- (Semi)-local models

  $\leftrightarrow$ no known explicit realization, but guiding principle for consistency

- Ultra-local models

  $\leftrightarrow$ focus on open patch containing (some) interaction points

2) Non-GUT model building: direct $SU(3) \times SU(2) \times U(1)_Y$

Progress in 4d F-theory

Global models:

- fiber structure in codimension one, two, three
- explicit resolutions
- U(1) symmetries and discrete symmetries (Mordell-Weil $\&$ TS group)
- G_4 fluxes: construction, massless matter spectrum

Semi-local models:

- U(1) charges from fibre structures or from E_8 decompositions
- Anomalies as constraint on consistency, especially of hypercharge flux

Ultra-local models:

- Explicit and detailed computation of Yukawa couplings in local (T-brane) backgrounds
- Fitting of parameters to phenomenologically viable couplings

Outline

I) Developments in F-theory compactifications

1. Codimension one and two
2. Perturbative and non-perturbative couplings
3. Massless and massive U(1) symmetries
4. G_4 fluxes and massless spectra

II) F-theory phenomenology

1. Hypercharge flux, 3-2 splitting, proton decay
2. Challenges for hypercharge flux
3. Semi-global model scans
4. non-SUSY GUTs, direct MSSM

I) Developments in F-theory compactifications

The magic of F-theory

F-theory epitomises the geometrisation of physics

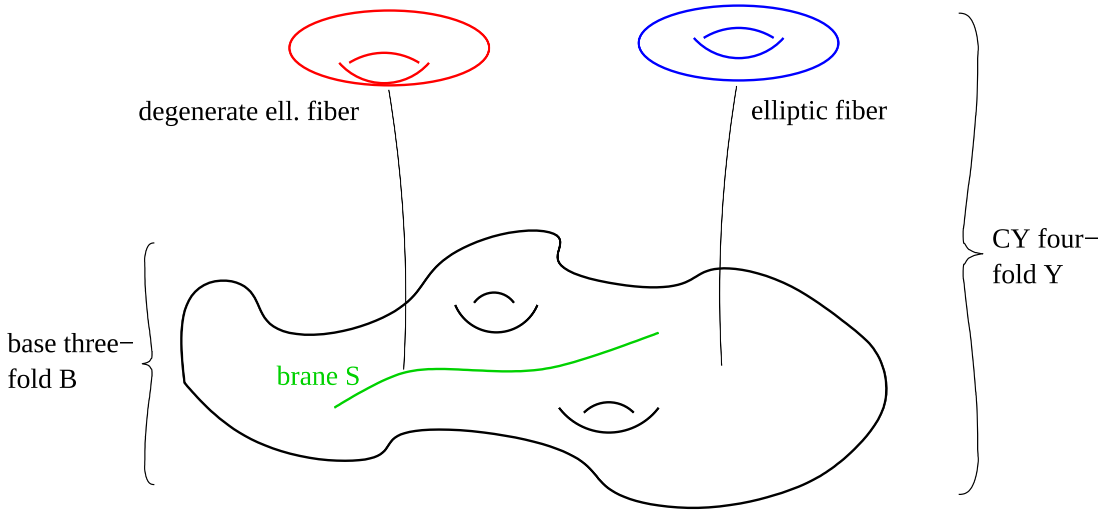
[Vafa][Morrison,Vafa]'96

IIB language:

7-branes wrap 4-cycle $S \in X_6/\sigma$

F-theory language:

S $=$ locus of fiber degeneration



IIB picture		**F-theory picture**
compactification space		base of fibration
varying axio-dilaton $\tau(z)$	$\Longleftrightarrow$	complex structure of fibre
7-branes		codim.-one singular fibres
D(-1) corrections		e.g. $\tau(z)$ [Billo et al.'11-'13]

But not all physics is geometrised...

F-theory via M-theory

F-theory approachable via **duality with M-theory** [Vafa'96] [Witten'96]

- M-theory on elliptic 4-fold $\rightarrow \mathcal{N} = 2$ theory in $\mathbb{R}^{1,2}$
- F-theory limit = suitable limit of vanishing fibre volume $v_{T^2} \rightarrow 0$

Effective action by dimensional reduction of **11D sugra coupled to M2/M5-branes** in this very subtle F-theory limit see talk by T Grimm

$$\begin{array}{ccc} \text{M-theory on } Y_{n+1} & \xrightarrow{\mathrm{Vol}(\mathbb{E}_\tau)\to 0} & \text{F-theory on } Y_{n+1} \\ \downarrow & & \downarrow \\ \text{Effective action in } \mathbb{R}^{1,8-2n} & \xrightarrow{R_A \sim \frac{1}{R_B} \to 0} & \text{Effective action in } \mathbb{R}^{1,9-2n} \end{array}$$

M-theory		F-theory
M2-branes on $\mathbb{R}^{1,2}$	$\Longleftrightarrow$	D3-branes on $\mathbb{R}^{1,3}$
vertical M5-brane instantons		D3-brane instantons
G_4-flux '1 leg along sing. fibres'		gauge fluxes
G_4-flux '1 leg along smooth fibres'		bulk fluxes

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A lot of recent progress in exploring 6D and 4D effective action

[Grimm'10][Grimm,Kerstan,Palti,TW'11][Bonetti,Grimm,(Hohenegger)'11,'12 &13],

including α'-corrections and warping:

[Hayashi,Garcia-Etxebarria,Savelli,Shiu'12];[Grimm,Savelli,Weissenbacher][Grimm,Pugh]'13;

[Martucci14];[Grimm,Pugh,Weissenbacher14/15];[Minasian,Savelli,Pugh'15]

Non-abelian gauge symmetry

Singularity type in co-dim. $1_{\mathbb{C}} \leftrightarrow$ gauge algebra $\mathfrak{g}$ on 7-brane

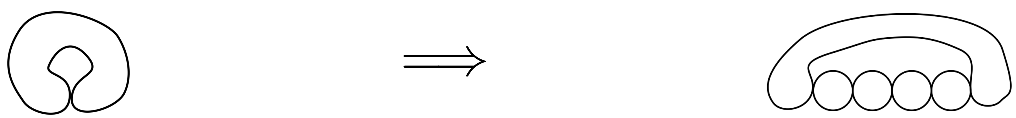
Strategies to study F-theory on singular fibration:

1) Resolve singularity = moving in **Coulomb branch of 3d M-theory**

2) Deform singularity = Higgsing of singularity [Grassi,Halverson,Shaneson'13/14]

Consider resolutions: (provided classical Columb branch unobstructed)

- resolve singular point in fibre by tree of $\mathbb{P}^1_i$ $\qquad i = 1, \ldots, \mathrm{rk}(\mathfrak{g})$



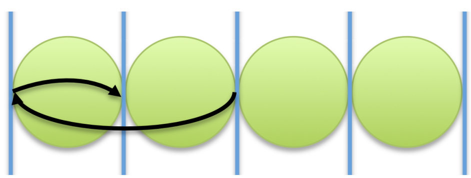
- **Group theory of $\mathfrak{g}$** $\iff$ **extended Dynkin diagram**

  $\mathbb{P}^1_i \leftrightarrow$ simple roots

- Each node of Dynkin diagram

  $\leftrightarrow$ stretched open strings

  $\equiv$ $\mathfrak{g}$-gauge bosons



Absence of Coulomb branch

Crepant resolutions are not available when the classical Coulomb branch in M-theory is obstructed.

Possible reasons:

1) Obstructing gauge data, e.g. discrete C_3-backgrounds see talk by D.Morrison

2) T-brane data ($=$ non-abelian scalar VEVs) see talks by R. Valandro and R.Savelli

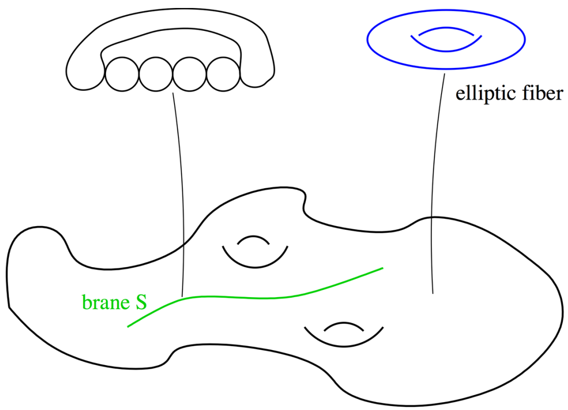
3) Geometric Stückelberg-type mechanisms or other Higgsings of U(1)s

At least 2) and 3) might play a crucial role in F-theory GUTs - more later.

Algebra versus group

$\mathfrak{g}$-gauge bosons:

- non-Cartan part from M2-branes along chains of $\mathbb{P}^1_i \leftrightarrow$ simple roots [Witten'96]
- Cartan part from $C_3 = A_i \wedge [E_i]$



- fibre types on K3 classified by Kodaira in 1-1 with ADE up to a few low rank outliers (II, III, IV) [Kodaira'63][Néron,64]
- on CY_3 extra monodromies along discriminant imply foldings of diagrams and yield all simple gauge algebras [Tate] [Bershadsky et al.'96]
- on CY_4 no further novelties along codimension-one

$\Rightarrow$ Non-abelian gauge algebra is local data

$\Rightarrow$ Topological property of gauge group G is global data:

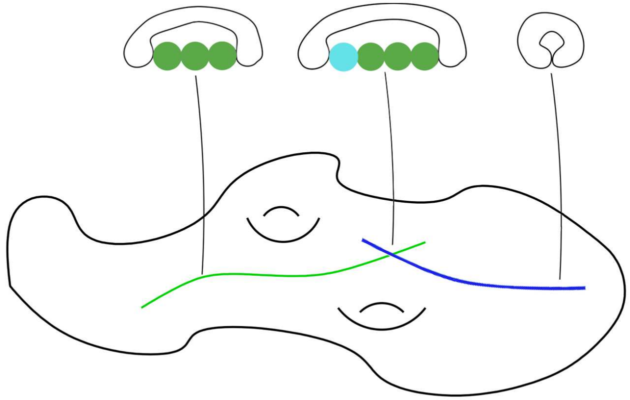
$$\pi_1(G) = \mathrm{MW}_{\mathrm{Tor}}$$ [Aspinwall,Morrison'98][Morrison,Mayrhofer,Till,TW'14]

Codimension two and matter

Enhancement in codimension 2

extra massless states from wrapped M2-branes

[Katz,Vafa'96][Witten'96]

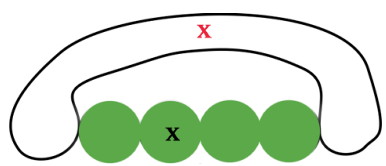


Assume for simplicity existence of a zero-section σ_0

- σ_0 generates KK U(1) in reduction 4d $\rightarrow$ 3d
  ($C_3 = A_0 \wedge \sigma_0 + \ldots \rightarrow A_0$: KK-U(1))
- Massless state in 4d:
  KK zero mode ψ_0 plus tower of KK states ψ_n of KK charge $q_{KK} = n$

$\psi_0 \quad \leftrightarrow \quad$ holomorphic curves C in fibre w/ $q_{KK} = C \cdot \sigma_0 = 0$

$\psi_n \quad \leftrightarrow \quad C + n\mathfrak{F} \qquad \mathfrak{F}:$ full fibre



Codimension two and matter

Mori cone of eff. curves increases in codim 2: [Intriligator,Morrison,Seiberg'97]

Simplest example: Fundamental of $\mathfrak{su}(\mathfrak{n})$

$$\underbrace{F_i}_{\text{simple root}} \to \underbrace{C_+}_{\mathbf{N}-\text{weight}} + \underbrace{C_-}_{\bar{\mathbf{N}}-\text{weight}} \qquad \lambda_i(\mathbf{N}) = \epsilon(C_\pm)\, C_\pm \cdot E_i, \quad \epsilon(C_\pm) = \pm 1$$

'Box Graphs' [Hayashi,Lawrie,Morrison,Schäfer-Nameki'14] [Esole,Yau'14]

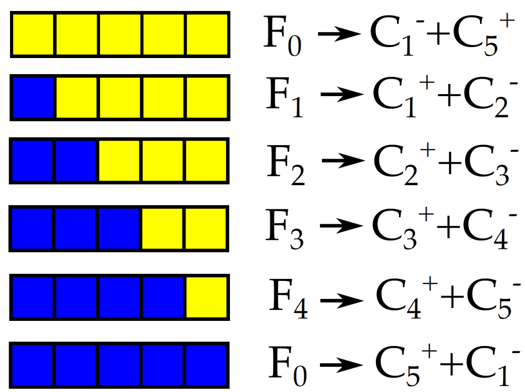
- Find dim($\mathbf{R}$) new effective curves by adding 'original' $\mathbb{P}^1_i$ (roots)

  $C_+ + \sum_i k_i\, F_i \to \mathbf{N}$ $\qquad \epsilon = 1$ (yellow)

  $C_- + \sum_j l_j F_j \to \bar{\mathbf{N}}$ $\qquad \epsilon =$- 1 (blue)

- allowing for positive/negative wrappings of M2-branes recreates full weight lattice of $\mathbf{R} + \bar{\mathbf{R}}$

- specific signs $\leftrightarrow$ phases of classical Coulomb branch of 3D field theory [DeBoer,Hori,Oz][Aharony et al.'97], [Grimm,Hayashi'11][Hayashi,Lawrie,Nameki'13]



- group theoretical classification of all possible enhanced fiber types

Codimension two

Fibers in codimen.-two assuming [Hayashi,Lawrie,Morrison,Schäfer-Nameki'14]

1) rank-one enhancement $\mathfrak{g} \to \mathfrak{h}$

2) smooth non-abelian discriminant component:

- affine Dynkin diagram of $\mathfrak{h}$ if embedding via $\mathfrak{h} \to \mathfrak{g} \oplus \mathfrak{u}(1)$
- monodromy reduced Dynkin diagram of $\mathfrak{h}$ if embedding via $\mathfrak{h} \to \mathfrak{g} \oplus \mathfrak{s}u(2)$ cf [Morrison,Taylor'12]

Further results include:

- Systematic description of 6D matter points for smooth 7-brane curves [Grassi,Morrison,11]
- Inclusion of self-intersecting 7-branes in 6D $\to$ higher tensor reps. [Morrison,Taylor'12] [Cvetič,Klevers,Piragua,Taylor'15] see talk by D. Klevers
- Alternative description of matter via multi-pronged strings made visible through deformations [Grassi,Halverson,Shaneson'13/4]

Codimension three and couplings

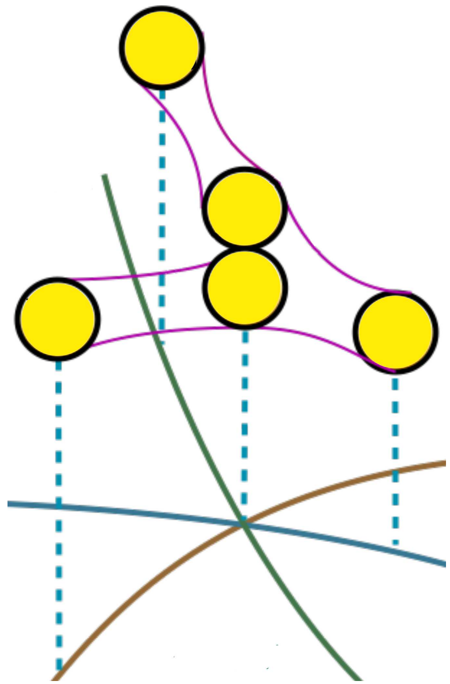
Origin of unsuppressed Yukawa couplings in M/F-theory:

- Incoming M2-brane on fibral curve Σ^1
- Splitting $\Sigma_1 \to \Sigma_2 + \Sigma_3$ in fiber $\mathfrak{f}_p$ over point p

  $-[\Sigma_1] + [\Sigma_2] + [\Sigma_3] = 0 \in H_2(\mathfrak{f}_p, \mathbb{Z})$

- State $\Phi_i \leftrightarrow$ M2 on $[\Sigma^i]$

  Coupling $\tilde{\Phi}_1 \Phi_2 \Phi_3$ from splitting of M2 at point p of fiber 'enhancement'



- First application to GUT models [Beasley,Heckman,Vafa][Donagi,Wijnholt]'08
- Fiber structure in codim.-3

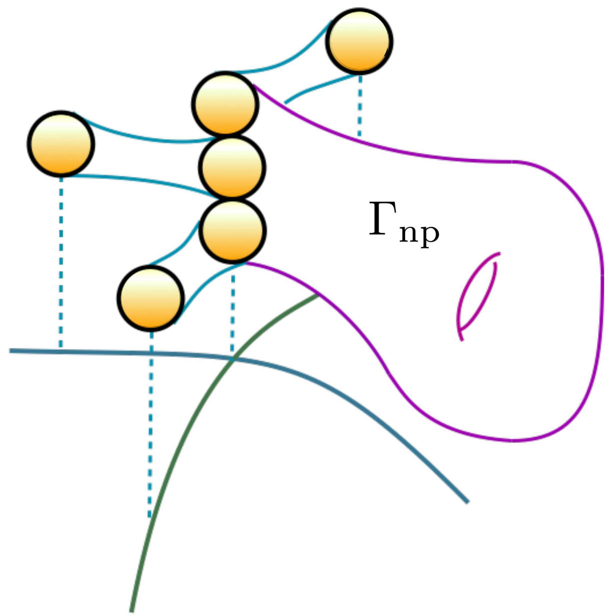
  [Esole,Yau][Marsano,Schäfer-Nameki] [Krause,Mayrhofer,TW']'11

- Quantitative evaluation in local approach see talk by F. Marchesano

  [Font,Ibanez,Marchesano,Regalado'12][Font,Marchesano,Regalado,Zoccarato] [Carta et al]'15

Non-perturbative couplings

$\Sigma_1 + \Sigma_2 + \Sigma_3 = \partial\Gamma_{\rm np}$ [Martucci,TW'15]



- $\Gamma_{\rm np}$ has 2 legs in base and 1 leg in fiber
- $\mathrm{vol}(\Gamma_{\rm np}) \neq 0$ in F-theory limit

Euclidean M2-brane on $x_0 \times \Gamma_{\rm np}$ + timelike M2-branes on $(-\infty, x_0] \times (\Sigma_1 + \Sigma_2 + \Sigma_3)$

F-term coupling requires M2-instanton to form BPS bound state with M5-instanton, i.e. **fluxed M5-instanton** $\Gamma_{\rm np} \to T_3|_{M5}$

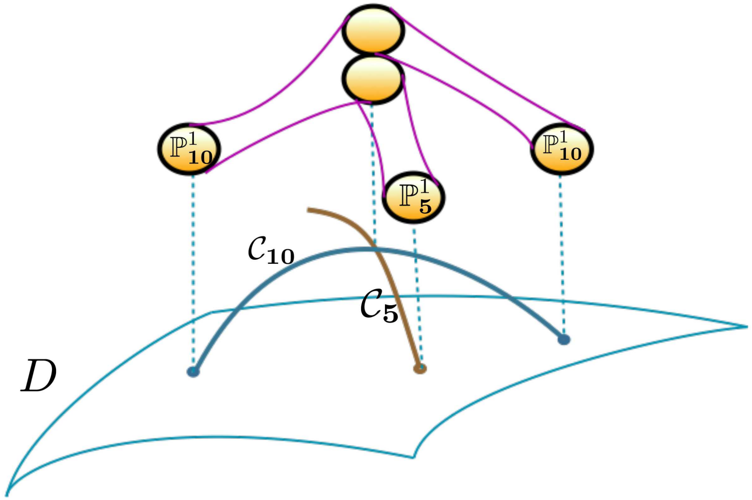
extra suppression by Kähler moduli in F-theory limit

non-perturbative homol. relation	$\longleftrightarrow$	volume suppressed coupling $\prod_i \Phi_i e^{-S_{\rm M5}}$

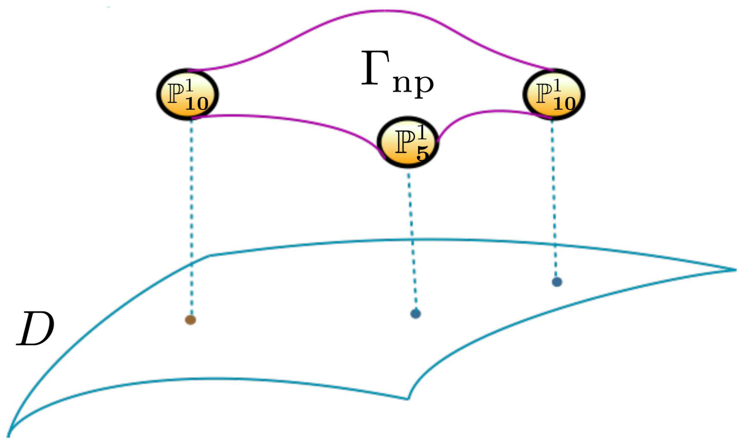
Corrections to Yukawas

Example: $\mathbf{10}\,\mathbf{10}\,\mathbf{5}$ coupling in SU(5) GUT model [Martucci,TW'15]

- Pert. Yukawa requires $[\mathbb{P}^1_{\mathbf{10}} + \mathbb{P}^1_{\mathbf{10}} + \mathbb{P}^1_{\mathbf{5}}]_{\rm p} = 0$ in Y_4
- Pert. coupling is generically of rank one if exists only a single Yukawa point [Cecotti,Cheng,Heckman,Vafa'09]



- On instanton D generically $[\mathbb{P}^1_{\mathbf{10}} + \mathbb{P}^1_{\mathbf{10}} + \mathbb{P}^1_{\mathbf{5}}]_{D,{\rm p}} \neq 0$
- $dT_3 = -\delta^4(\mathbb{P}^1_{\mathbf{10}} + \mathbb{P}^1_{\mathbf{10}} + \mathbb{P}^1_{\mathbf{5}})$
  $\Rightarrow$ correction $\mathcal{O}_{\rm np} = \mathbf{10}\,\mathbf{10}\,\mathbf{5}\,e^{-S}$



Summing up $\mathcal{O}_{\rm p} + \mathcal{O}_{\rm np}$ changes rank

Different to contribution considered in [Marchesano,Martucci'10] that needs to be evaluated

The quest for U(1)

Motivation to study non-Cartan U(1)s:

- desirable for phenomenology as extra **selection rules** (proton decay, flavour structure,...)
- **charged singlets** plays role in phenomenology - e.g. as neutrinos or in SUSY breaking
- precursor to construction of large class of **gauge fluxes**
- **U(1) symmetries and instantons** have rich interplay in Type II and heterotic compactifications
  What's the analogue in F-theory?

General fact from expansion $C_3 = \sum_{i=1} A_i \wedge \mathsf{w}_i$:

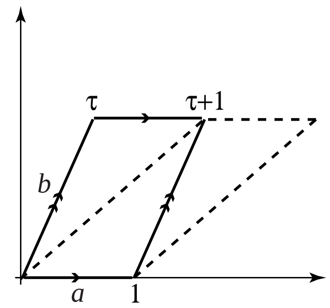
non-Cartan U(1)s $\leftrightarrow$ extra resolution divisors not fibered over base 4-cycle

These correspond to **extra sections of the fibration**. [Morrison,Vafa'96] [Klemm,Mayr,Vafa'96]

Mordell-Weil group

1) Elliptic curve: $\mathcal{E} = \mathbb{C}/\Lambda$

$\leftrightarrow$ addition of points



Rational points:

- have $\mathbb{Q}$-rational coordinates (x, y, z) in Weierstrass model

$$y^2 = x^3 + fxz^4 + gz^6, \qquad [x : y : z] \in \mathbb{P}^2_{2,3,1}$$

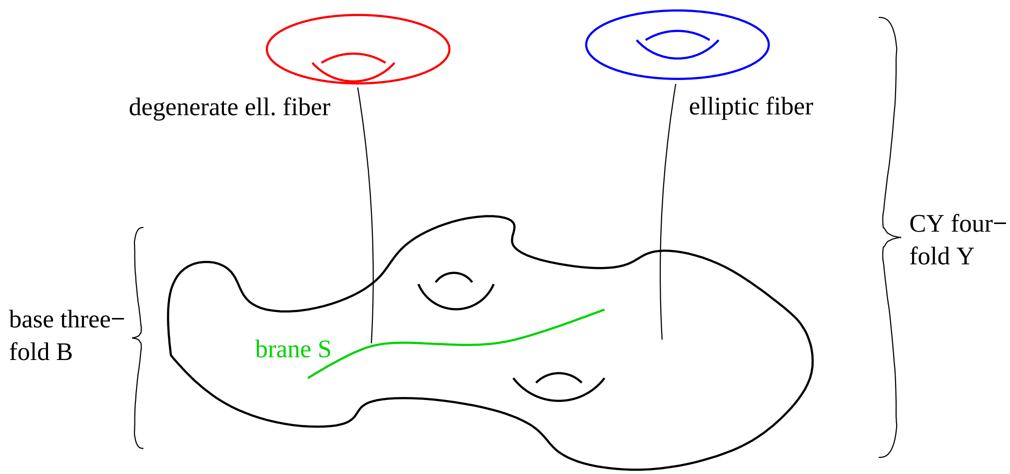
- form an abelian group under addition = **Mordell-Weil group E**

$$E = \mathbb{Z}^r \oplus \mathbb{Z}_{k_1} \oplus \cdots \oplus \mathbb{Z}_{k_n}$$

2) Elliptic fibration: $\pi : Y \to \mathcal{B}$

Rational section σ:

$$\mathcal{B} \ni b \mapsto \sigma(b) = [x(b) : y(b) : z(b)]$$



- $\sigma(b)$ is a K-rational point in fiber
- degenerations in codimension allowed

Mordell-Weil group

Mordell-Weil group $E(K)$= group of rational sections

- zero-element = zero-section $\sigma_0 : b \to [1:1:0]$ in $y^2 = x^3 + fxz^4 + gz^6$
- group law = fiberwise addition

$$E(K) = \underbrace{\mathbb{Z}^r}_{\text{free part}} \oplus \underbrace{\mathbb{Z}_{k_1} \oplus \cdots \oplus \mathbb{Z}_{k_n}}_{\text{torsion part}}$$

Physical significance:

- Free part $\leftrightarrow$ $U(1)$ gauge symmetries

  $\sigma_i \xrightarrow{\text{Shioda}} \mathsf{w}_i = [\sigma_0] - [\sigma_i] - (\text{base class}) + \ldots$

  $C_3 = A_i \wedge \mathsf{w}_i$, A_i: $U(1)_i$ potential

- Torsion part $\leftrightarrow$ Global structure of non-ab. gauge groups $(\pi_1(G))$

  [Aspinwall,Morrison'98],[Mayrhofer,Till,Morrison,TW'14]

Systematic recent study of U(1)s via rational sections:

Anderson,Bizet,Borchmann,Braun,Braun,Choi,Collinucci,Cvetič,Etxebarria,Grassi,Grimm, Hayashi, Keitel,Klevers,Küntzler,Krippendorf,Oehlmann,Kapfer,Klemm,Lawrie,Lopes,Mayrhofer, Mayorga, Morrison, Park,Palti,Piragua,Rühle,S-Nameki,Song,Valandro,Taylor,TW,Wong. . .

Understanding U(1)s

Questions:

1. Which complex structure restrictions lead to extra sections and thus to extra U(1)s?
2. What is the fiber structure in codim 2 and 3, i.e. which charged matter and couplings exist?
3. How does one combine this with non-abelian gauge symmetry?

Different approaches:

1) Construct most generic fibration with at least n extra sections

see talks by M Cvetič and D. Klevers

- valid over any base such that fibration exists: rk(MW)(Y) $\geq n$
- includes full analysis of charged singlet sector

2) Construct full fibrations with concrete bases - includes 'non-generic' base dependent U(1)s

3) Classify possible fibre structures without concrete realization

Towards classifying U(1) charges

[Lawrie,Schäfer-Nameki,Wong]

Classification of possible charges from structure of consistent fibre intersections

Assumptions:

- smooth rational section
- smooth divisors in base

Example for $SU(5) \times U(1)$:

$$U(1) \text{ charges of } \bar{\mathbf{5}} \text{ matter for } \begin{cases} I_5^{(01)} \in \{-15, -10, -5, 0, +5, +10, +15\} \\ I_5^{(0|1)} \in \{-14, -9, -4, +1, +6, +11\} \\ I_5^{(0||1)} \in \{-13, -8, -3, +2, +7, +12\} \end{cases}$$

$$U(1) \text{ charges of } \mathbf{10} \text{ matter for } \begin{cases} I_5^{(01)} \in \{-15, -10, -5, 0, +5, +10, +15\} \\ I_5^{(0|1)} \in \{-12, -7, -2, +3, +8, +13\} \\ I_5^{(0||1)} \in \{-9, -4, +1, +6, +11\}\ . \end{cases}$$

Discrete symmetry in F-theory

Discrete $\mathbb{Z}_k$ gauge symmetry by Higgsing $U(1)$ w/ particle of charge k:

- Higgs $\Phi = \varphi\, e^{ic}$: $A \to A + d\chi, \quad c \to c + k\,\chi$
- After integrating out φ: $S \simeq \int (dc - kA)^2 + \ldots$

Translation into F/M-theory: [Camara,Marchesano,Ibanez][Grimm,Kerstan,Palti,TW]'11

- massive $\mathbb{Z}_k$ gauge field and Stückelberg axion c from expansion

$$C_3 = c \wedge \alpha_3 + A \wedge \mathrm{w}_2, \qquad \mathbf{d}\mathrm{w}_2 = \mathbf{k}\,\alpha_3$$

- $\int_{11D} (dC_3)^2 \simeq \int_{11D} (dc \wedge \alpha_3 - A \wedge d\mathrm{w}_2)^2 + \ldots \Longrightarrow \int (dc - kA)^2 + \ldots$

$\mathbb{Z}_\mathbf{k}$ symmetry in F & M-theory $\leftrightarrow$ $\mathrm{Tor} H^3(Y, \mathbb{Z}) = \mathbb{Z}_k$

Indeed confirmed in F/M-theory in [Mayrhofer,Palti,Till,TW'14]

$k = 1$: [Braun,Collinucci,Valandro'14] [Martucci,TW'15] $\to$ terminal singularity

(Coulomb branch obstructed by Stückelberg)

$k > 1$: Alternative description via smooth fibrations without sections

[Morrison,Taylor][Anderson,Grimm,Etxebarria,Keitel] [Klevers,Mayorga,Oehlmann,Piragua,Reuter]

[Mayrhofer,Palti,Till,TW] [Cvetič,Klevers,Poretschkin][Lin,Till,Mayrhofer,TW]'14/15

G_4-Fluxes

(Gauge) fluxes described by $\mathbf{G_4 \in H^4(Y_4)}$ with **'1 leg along fiber'**

a) $\int_{\hat{Y}_4} G_4 \wedge D_a \wedge D_b = 0$ b) $\int_{\hat{Y}_4} G_4 \wedge D_a \wedge Z = 0$ $\forall D_i \in H^2(B)$, Z: fibre

subject to following constraints:

- Quantisation: $G_4 + \frac{1}{2} c_2(\hat{Y}_4) \in H^4(\hat{Y}_4, \mathbb{Z})$

  [Witten'96] [Collinucci,Savelli '10 &'12]

- D3/M2 tadpole: $N_{M_2} + \frac{1}{2} \int_{\hat{Y}_4} G_4 \wedge G_4 = \frac{1}{24} \chi(\hat{Y}_4)$ [Sethi,Vafa,Witten'96]

- F-term condition: $G_4 \in H^{2,2}(\hat{Y}_4)$ [Gukov,Vafa,Witten'99]

  $\leftrightarrow$ superpotential $W = \int_{\hat{Y}_4} \Omega \wedge G_4$ for $h^{3,1}(\hat{Y}_4)$ compl. structure moduli

- D-term condition: $J \wedge G_4 = 0$

  $\leftrightarrow U(1)_i$ D-term $D_i = -\frac{2}{\mathcal{V}_B} \int_{\hat{Y}_4} J_B \wedge G_4 \wedge \mathrm{w}_i$ from F/M- theory effective action

  [Grimm '10] [Grimm,Kerstan,Palti,TW '11] [Cvetič,Grimm,Klevers'13]

G_4-Fluxes

Construction requires detailed knowledge of geometry of 4-fold Y_4

$$\mathbf{H^4(Y_4) = H^{2,2}_{vert}(Y_4) \oplus H^4_{hor}(Y_4) \oplus H^{2,2}_{rest}(Y_4)}$$

$=$ decomposition orthogonal w.r.t. intersection form

1) $\mathbf{H^{2,2}_{vert}(Y_4)}$ generated by elements of $H^{1,1}(Y_4) \wedge H^{1,1}(Y_4)$:
appears only in D-term, not in F-term

- fluxes associated with massless U(1)s

  [Grimm,TW '10] [Braun,Collinucci,Valandro] [Krause,Mayrhofer,TW'11] [Grimm,Hayashi]'11

  if $C_3 = A \wedge \mathrm{w} \Rightarrow G_4 = F \wedge \mathbf{w}$ $\qquad F \in H^{1,1}(B_3)$

- extra gauge fluxes e.g. 'spectral cover' fluxes [Marsano,Schäfer-Nameki'11]
- **Systematics of** $H^{2,2}_{\text{vert.}}(\hat{Y}_4)$:
  find all independent linear combinations of $H^{1,1} \wedge H^{1,1}$

  [Cvetič,Klevers,Grassi,Piragua'13] [Braun,Grimm,Keitel'13] [Bizet,Klemm,Lopez'14]

  [Lin,Mayrhofer,Till,TW'15]

G_4-Fluxes

2) $\mathbf{H^4_{hor}(Y_4)}$ obtained by variation of Hodge structure from $H^{4,0}(Y_4)$
appears only in F-term, not in D-term [Greene,Morrison,Plesser'94]

[Grimm,Ha,Klemm,Klevers'09] [Braun,Collinucci,Valandro '11]

[Intriligator,Jockers,Mayr,Morrison,Plesser'12] [Bizet,Klemm,Lopez'14]

- Poincaré dual to non-vertical 4-cycles algebraic for special complex structure e.g. of form [Braun,Collinucci,Valandro '11]
- applications to 'landscaping' and moduli stabilisation [Bizet,Klemm,Lopez'14] [Braun,Watari'15] [Taylor,Wang'15]

3) $\mathbf{H^{2,2}_{rest}(Y_4)}$: the rest [Braun,Watari'14]
appears neither in D-term, nor in F-term

- non-vertical algebraic cycles available for generic complex structure
- play crucial role in F-theory GUTs

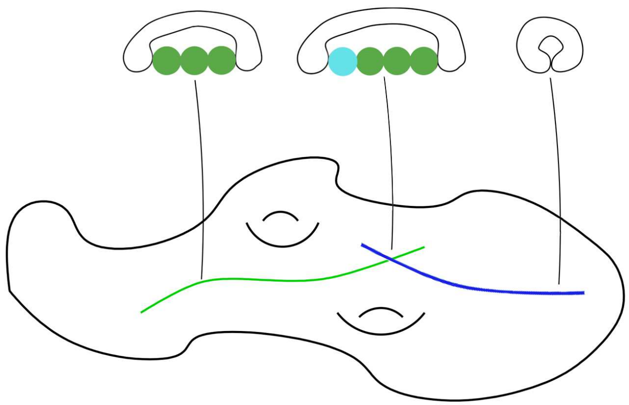
Matter multiplicities in F-theory

$\#$ of charged zero modes $\leftrightarrow$ background gauge field C_3 with $G_4 = dC_3$

- chiral index:

$$\nu_+ - \nu_- = \int_{\mathcal{C}_4} G_4$$

[Donagi,Wijnholt'09], [Braun,Collinucci,Valandro] [Marsano,S-Nameki], [Krause,Mayrhofer,TW], [Grimm,Hayashi]'11 ...



- What is the spectrum of states beyond the chiral index?

$\Longrightarrow$ need C_3 beyond its field strength [Curio,Donagi'98], ...

$$0 \longrightarrow \underbrace{J^2(\hat{Y}_4)}_{\oint C_3 \text{ 'Wilson lines'}} \longrightarrow \underbrace{H^4_D(\hat{Y}_4, \mathbb{Z}(2))}_{\textbf{Deligne cohomology}} \xrightarrow{\hat{c}_2} \underbrace{H^{2,2}_{\mathbb{Z}}(\hat{Y}_4)}_{\text{field strength } G_4} \longrightarrow 0$$

Framework for computation of non-chiral states: [Bies,Mayrhofer,Pehle,TW'14]

T-branes/Gluing data

In Higgs-bundle picture model intersecting 7-branes by varying VEV of scalar field $\Phi \in H^0(S, K_S)$ [Beasley,Heckman,Vafa][Donagi,Wijnholt]'08

$$F^{(0,2)} = 0, \qquad \bar{\partial}_A \Phi = 0, \qquad J \wedge F + i\,[\Phi, \Phi^\dagger] = 0$$

- $\Phi(x) \leftrightarrow$ normal deformations in total space of $K_S \to S$
- location of branes given by spectral cover $\det(s - \Phi) = 0$

T-brane data $\leftrightarrow$ entries in Φ not affecting geometric location

[Cecotti,Cordova,Heckman,Vafa'10]

Equivalently described as gluing morphism [Donagi,Wijnholt'11]

Conceptual interest in 'T-branes/gluing':

Extra data not present in pure geometry as it does involve 'gauge flux'

In particular, not accessible via Coulomb branch of resolution

Phenomenological relevance of 'T-branes/gluing':

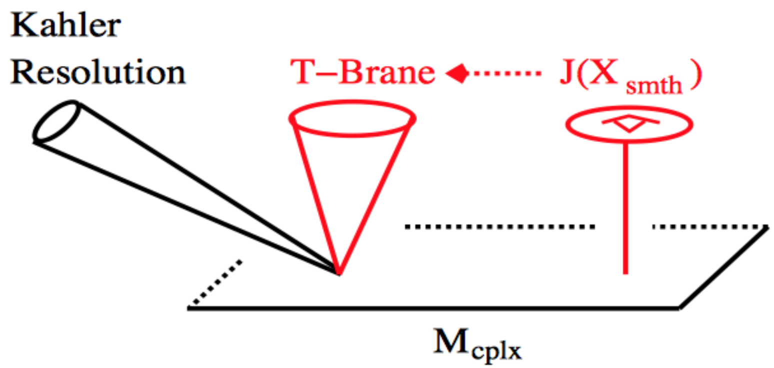
Degrees of freedom affect matter spectrum and couplings

Gluing data/T-branes

How is gluing data captured in compactifications?

1) 6d approach of [Anderson,Heckman,Katz'13]

- Gluing $=$ 3-form moduli $\int C_3$ on X_{smth} in singular limit
- Compensating flux captured in G_4 on resolved space



- Encoded in element in 'singular limit' of Deligne cohomology

$$0 \longrightarrow \underbrace{J^2(X_{smth})}_{\oint C_3 \, '\text{Wilson lines}'} \longrightarrow \underbrace{H^4_D(X_{smth}, \mathbb{Z}(2))}_{\textbf{Deligne cohomology}} \xrightarrow{\hat{c}_2} \underbrace{H^{2,2}_{\mathbb{Z}}(X_{smth})}_{\text{field strength } G_4} \longrightarrow 0$$

2) Approach of [Collinucci,Savelli'14] see talks by Valandro, Savelli

Can be understood via certain matrix factorizations directly in singular limit

II) F-theory phenomenology

F-theory GUT Phenomenology

initiated by [Beasley,Heckman,Vafa; Donagi,Wijnholt'08]

- **GUT breaking** $SU(5) \to SU(3) \times SU(2) \times U(1)_Y$:
  hypercharge flux due to localisation of GUT brane in codimension
- **Doublet-triplet (3-2) splitting** (and μ-problem):
  localisation of $\bar{\mathbf{5}}_m$, $\mathbf{5}_{H^u}$, $\bar{\mathbf{5}}_{H^d}$ on separate curves
- **Proton stability**: U(1) symmetries and localisation
- **Detailed Flavour structure**: locally or via Froggatt-Nielsen/U(1)s

Almost all of these are by now known in global examples - with one crucial exception to be discussed momentarily.

SU(5) GUT breaking

SU(5) field strength on 7-brane $\mathcal{F} = \underbrace{\mathbb{F}}_{4\,\text{large dim.}} + \underbrace{F}_{\text{along } S}$

Decomposition:

$$F = \sum_a T^a_{SU(3)} F_a + \sum_i T^i_{SU(2)} F_i + T^Y F_Y$$

hyperchage generator $T_Y = \text{diag}(-2,-2,-2,3,3) \subset SU(5)$

Vacuum expectation value $\langle F_Y \rangle = \langle dA_Y \rangle \neq 0$ on GUT brane S

- $SU(5) \longrightarrow SU(3) \times SU(2) \times U(1)_Y$

  $$\begin{aligned} &\mathbf{24} \to (\mathbf{8},\mathbf{1})_{0_Y} + (\mathbf{1},\mathbf{3})_{0_Y} + (\mathbf{1},\mathbf{1})_{0_Y} + \cancel{(\mathbf{3},\mathbf{2})_{5_Y}} + \cancel{(\overline{\mathbf{3}},\mathbf{2})_{-5_Y}} \\ &\overline{\mathbf{5}} \to (\overline{\mathbf{3}},\mathbf{1})_{2_Y} + (\mathbf{1},\mathbf{2})_{-3_Y} \\ &\mathbf{10} \to (\mathbf{3},\mathbf{2})_{1_Y} + (\overline{\mathbf{3}},\mathbf{1})_{-4_Y} + (\mathbf{1},\mathbf{1})_{6_Y}\,, \\ &\mathbf{5}_H \to (\mathbf{3},\mathbf{1})_{-2_Y} + (\mathbf{1},\mathbf{2})_{3_Y}\,, \quad \overline{\mathbf{5}}_H \to (\overline{\mathbf{3}},\mathbf{1})_{2_Y} + (\mathbf{1},\mathbf{2})_{-3_Y} \end{aligned}$$

- offers way to project out exotic states $(\mathbf{3},\mathbf{2})_{5_Y} + (\overline{\mathbf{3}},\mathbf{2})_{-5_Y}$ from $\mathbf{24}$ for certain twisted embeddings [BHV'08]
- globally described by $G_4 =' F_Y \wedge \mathrm{w}'_Y$, $\quad \mathrm{w}_Y = \sum_i l_i E_i$

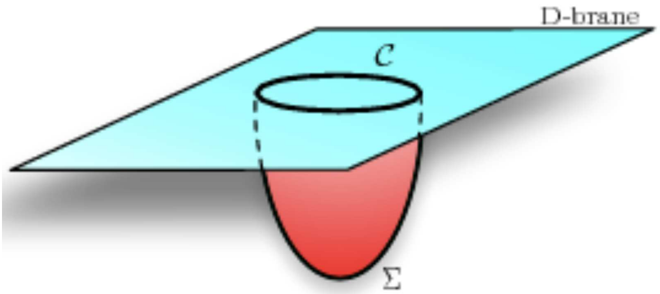
Global constraints

- Naively, GUT breaking by $\langle F_Y \rangle \neq 0$ on GUT brane S is a local effect.
- Challenge: $U(1)_Y$ must remain massless by avoiding Stückelberg mass

Masslessness constraint:

[Buican et al.'06] [BHV'08][DW'08]

$\int_C F_Y \neq 0$ only if curve $C \subset S$ has 'contribution' homologically trivial on B_3



- Stückelberg couplings $\int_{\mathbb{R}^{1,3}} F_Y \wedge c_2^\alpha$ from $\int_{M^{11}} C_3 \wedge G_4^{4D} \wedge G_4^{\text{int.}}$
- c_2^α dual to c_1^α from $C_3 = c_1^\alpha \wedge \mathrm{w}_\alpha$ $\qquad \mathrm{w}_\alpha \in H^{1,1}(B_3)$
- $G_4^{4D} = \mathbf{F}_Y \wedge \mathrm{w}_Y \Rightarrow \int_{\hat{Y}_4} G_4^{\text{int.}} \wedge \mathrm{w}_Y \wedge \mathrm{w}_\alpha \stackrel{!}{=} 0 \qquad \forall\, \mathrm{w}_\alpha \in H^{1,1}(B_3)$

$$\int_S F_Y \wedge \iota^* \mathrm{w}_\alpha \stackrel{!}{=} 0 \qquad \forall \mathrm{w}_\alpha \in H^{1,1}(B_3)$$

- Note: associated $\mathrm{G}_4 \in \mathrm{H}^{2,2}_{\text{rest}}(\mathrm{Y}_4)$ [Braun,Watari'14] [Mayrhofer,Palti,TW'13]

Explicit realizations in compact Calabi-Yau fourfolds are known:

[Marsano,Saulina,S-Nameki][Blumenhagen,Grimm,Jurke,TW]'09

[Chen,Knapp,Kreuzer,Mayrhofer'10]

Proton Decay

Conventional SU(5) GUTs suffer from too large proton decay

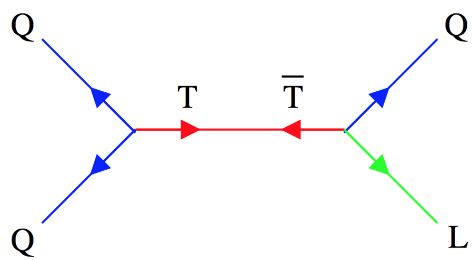
Dimension 4: $W \supset \lambda \, \mathbf{10} \, \overline{\mathbf{5}}_{\mathbf{m}} \, \overline{\mathbf{5}}_{\mathbf{m}}$

- gives rise to R-parity violating $u^c_R \, d^c_R \, d^c_R$, $L \, L \, e^c_R$, $Q \, L \, d^c_R$
- Experimental bound: $\lambda \overset{!}{\leq} 10^{-12}$
- **Solution**: Global $U(1)$ to distinguish $\overline{\mathbf{5}}_{\mathbf{m}}$ and $\overline{\mathbf{5}}_{\mathbf{H}}$ [Watari et al.'09]
  implies: $C_{\mathbf{5_m}}$, $C_{\mathbf{5_H}}$ are different curves [BHV, DW '08]

Dimension 5: focus on effective terms of type $W \supset \frac{c^2}{M_{\mathrm{eff}}} \mathbf{10} \, \mathbf{10} \, \mathbf{10} \, \overline{\mathbf{5}}_{\mathbf{m}}$

- via triplet exchange, e.g.
  $\mathbf{5_H} = (T_u, H_u), \overline{\mathbf{5}}_{\mathbf{H}} = (T_d, H_d)$:
  $QQT_u + QLT_d + M_{KK} T_u T_d \rightarrow \frac{1}{M_{KK}} Q \, Q \, Q \, L$

  $\leftrightarrow$ present if T_u, T_d on same curve C_{5_H}



- **Solution**: $U(1)$ charge must distinguish also $\mathbf{5}_{H_u}$ and $\overline{\mathbf{5}}_{H_d}$
  implies: $C_{\mathbf{5_{H_u}}}$ and $C_{\overline{\mathbf{5}}_{\mathbf{H_d}}}$ are different curves [BHV, DW '08]

The need for U(1) symmetries

Sufficient criterion for stable proton:

- **Extra U(1) selection rules** must distinguish between matter curves
- Example $SU(5) \times U(1)$: $\mathbf{10}_{q_1}$ $(\bar{\mathbf{5}}_{\mathbf{m}})_{q_2}$ $(\mathbf{5}_{\mathbf{H^u}})_{q_3}$ $(\bar{\mathbf{5}}_{\mathbf{H^d}})_{q_4}$

  $\mathbf{10}\,\bar{\mathbf{5}}_{\mathbf{m}}\,\bar{\mathbf{5}}_{\mathbf{H}} : q_1 + q_2 + q_4 \overset{!}{=} 0$ $\quad$ $\mathbf{10}\,\mathbf{10}\,\mathbf{5}_{\mathbf{H}} : 2q_2 + q_3 \overset{!}{=} 0$

  $\cancel{\mathbf{10}\,\bar{\mathbf{5}}_{\mathbf{m}}\,\bar{\mathbf{5}}_{\mathbf{m}}} : q_1 + 2q_2 \overset{!}{\neq} 0$ $\quad$ $\cancel{\mathbf{10}\,\mathbf{10}\,\mathbf{10}\,\bar{\mathbf{5}}_{\mathbf{m}}} : 3q_1 + q_2 \overset{!}{\neq} 0$
- $q_{\mathbf{5}_{H_u}} \neq -q_{\bar{\mathbf{5}}_{H_d}}$ dubbed 'Peccei-Quinn' U(1)

Further benefits: [BHV '08], ...

✓ automatically forbids leading order μ term $\mu H_u H_d$

✓ window to addressing doublet-triplet splitting via hypercharge flux

Studied first semi-locally in spectral covers [Marsano,Saulina,S.-Nameki '09-12]

Geometry globally realized in minimal form in [Mayrhofer,Palti,TW]'12

SU(5) × U(1) × U(1)

[Borchmann,Mayrhofer,Palti,TW] [Cvetič,Klevers,Grassi,Piragua]'13

Explicit description of Y_4 as hypersurface in $\mathbb{P}^2[3]$:

5 inequivalent toric $SU(5) \times U(1) \times U(1)$ realisations

Example:

$$0 = b_{0,2} w^2 s_0^2 \mathsf{v}^2 \mathsf{u} + c_{2,1} w s_0 \mathsf{w}\mathsf{v}^2 + d_{0,2} w^2 \mathsf{v} s_0^2 s_1 \mathsf{u}^2 + + b_1 s_0 s_1 \mathsf{w}\mathsf{v}\mathsf{u} + c_1 \mathsf{w}^2 \mathsf{v} s_1 + d_{2,2} w^2 s_0^2 s_1^2 \mathsf{u}^3 + d_1 s_0 s_1^2 \mathsf{w}\mathsf{u}^2 + b_2 s_1^2 \mathsf{w}^2 \mathsf{u}$$

Curve on $\{w = 0\}$	Matter representation	
$\{b_1 = 0\}$	$\mathbf{10}_{-1,2}$	$\mathbf{10}$
$\{b_{0,2} = 0\}$	$\mathbf{5}_{-3,1}$	–
$\{c_{2,1} = 0\}$	$\mathbf{5}_{2,-4}$	$\mathbf{5}_{H^u}$
$\{c_1 = 0\}$	$\mathbf{5}_{2,6}$	$\mathbf{5}_m$
$\{b_1 b_2 - d_1 c_1 = 0\}$	$\mathbf{5}_{-3,-4}$	$\mathbf{5}_{H^d}$
$\{d_{2,2} b_1^2 + d_1(b_{0,2} d_1 - d_{0,2} b_1 = 0\}$	$\mathbf{5}_{2,1}$	–

In addition: 6 charged singlet curves, including $\mathbf{1}_{5,10} \equiv N_R^c$

Hypercharge - Models (I)

Two types of approaches:

- All 3 families on one $\mathbf{10}$ and $\bar{\mathbf{5}}_m$ curve - no further U(1)s required ultra-local versions studied by

  [Font,Ibanez,Marchesano,Regalado'12],[Font,Marchesano,Regalado,Zoccarato'],

  [Carta,Marchesano,Zoccarato]'15
- Extra U(1)s can be present to distinguish individual family curves

  [Dudas,Palti'09/10] [Krippendorf,S-Nameki,Wong'15], ...

  Flavour structure via Froggatt-Nielsen mechanism (singlet VEVs)

However:

Two generic problems remain - both indirectly related to our as yet incomplete understanding of 'massive U(1)s' and their realization in F-theory

Hypercharge - Models (II)

Prior to $\not{SU}(5)$: GUT universal gauge flux F

$$\mathbf{10} \leftrightarrow \quad \text{bundle on } C_{10} \text{ .w/curvature } F_{10}: \quad \chi_{\mathbf{10}} = \int_{C_{10}} F_{10}$$
$$\mathbf{5} \leftrightarrow \quad \text{bundle on } C_5 \text{ w/curvature } F_5: \quad \chi_{\mathbf{5}} = \int_{C_5} F_5$$

After $\not{SU}(5)$:

- $\mathbf{10} \to (\mathbf{3},\mathbf{2})_{1_Y} + (\overline{\mathbf{3}},\mathbf{1})_{-4_Y} + (\mathbf{1},\mathbf{1})_{6_Y}$

$$\chi_{(\mathbf{3},\mathbf{2})_1} = \int_{C_{10}} (F_{10} + F_Y), \qquad \chi_{(\overline{\mathbf{3}},\mathbf{1})_{-4}} = \int_{C_{10}} (F_{10} - 4F_Y),$$
$$\chi_{(\mathbf{1},\mathbf{1})_6} = \int_{C_{10}} (F_{10} + 6F_Y)$$

- $\mathbf{5} \to (\mathbf{3},\mathbf{1})_{-2_Y} + (\mathbf{1},\mathbf{2})_{3_Y}$

$$\chi_{(\mathbf{3},\mathbf{1})_{-2}} = \int_{C_{\mathbf{5}}} F_5 - 2F_Y, \qquad \chi_{(\mathbf{1},\mathbf{2})_3} = \int_{C_{\mathbf{5}}} F_5 + 3F_Y$$

Curve	MSSM	Chirality	Curve	MSSM	Chirality
$\mathbf{10}_a$	$(\mathbf{3},\mathbf{2})_1$	M_a	$\mathbf{5}_i$	$(\mathbf{3},1)_{-2}$	M_i
	$(\overline{\mathbf{3}},\mathbf{1})_{-4}$	$M_a - N_a$		$(1,\mathbf{2})_3$	$M_i + N_i$
	$(\mathbf{1},\mathbf{2})_6$	$M_a + N_a$			

Hypercharge - Models (III)

Curve	MSSM	Chirality	Curve	MSSM	Chirality
$\mathbf{10}_a$	$(\mathbf{3},\mathbf{2})_1$	M_a	$\mathbf{5}_i$	$(\mathbf{3},1)_{-2}$	M_i
	$(\bar{\mathbf{3}},\mathbf{1})_{-4}$	$M_a - N_a$		$(1,\mathbf{2})_3$	$M_i + N_i$
	$(\mathbf{1},\mathbf{2})_6$	$M_a + N_a$			

Minimal would-be scenario: Complete matter multiplets from curves

$$1 \times C_{\mathbf{10}}, \qquad 1\times C_{\bar{\mathbf{5}}_{\mathbf{m}}}, \qquad 1 \times C_{\bar{\mathbf{5}}_{\mathbf{H_d}}}, \qquad 1 \times C_{\mathbf{5}_{\mathbf{H_u}}}$$

- No exotics: $M_{\mathbf{10}} \stackrel{!}{=} 3, \quad N_{\mathbf{10}} \stackrel{!}{=} 0 \qquad M_{\bar{\mathbf{5}}_{\mathbf{m}}} \stackrel{!}{=} 3, \quad N_{\bar{\mathbf{5}}_{\mathbf{m}}} \stackrel{!}{=} 0$
- Doublet-triplet splitting:
  $M_{\mathbf{5}_{H_u}} \stackrel{!}{=} 0 \stackrel{!}{=} M_{\mathbf{5}_{H_d}}, \qquad N_{\mathbf{5}_{H_u}} \stackrel{!}{=} 1 \stackrel{!}{=} -N_{\mathbf{5}_{H_d}}$

forbidden by $U(1)_Y$ anomalies:

$$\sum_i q_i N_i + \sum_a q_a N_a = 0, \quad \sum_i N_i = \sum_a N_a = 0, \quad \sum_i q_i^\alpha q_i^\beta N_i + 3 q_a^\alpha q_a^\beta N_a = 0$$

excludes just $N_{\mathbf{5}_{H^u}} = 1 = -N_{\mathbf{5}_{H^d}}$ unless $q_{\mathbf{5}_{H^u}} = q_{\mathbf{5}_{H^d}}$

Hypercharge anomalies

[Marsano],[Dudas,Palti]'11, [Palti,12]

No couplings $F_Y \wedge c_\alpha^2$ $\Longrightarrow$ no Green-Schwarz terms can cancel specific hypercharge anomalies

$\Longrightarrow$ No proper hypercharge anomalies must occur in field theory!

In presence of extra U(1) symmetry:

1. $\mathcal{A}_{U(1)_Y^2-U(1)} \stackrel{!}{\propto} \mathcal{A}_{\mathbf{SU(5)^2}-U(1)}$
2. $\mathcal{A}_{U(1)_Y^2-U(1)_Y} \stackrel{!}{\propto} \mathcal{A}_{\mathbf{SU(5)^2-SU(5)}}$
3. $\mathcal{A}_{U(1)_Y-U(1)_A-U(1)_B} \stackrel{!}{=} 0$

For models with $q_{H_u} \neq -q_{H_d}$ 1. - 3. leave two options:

- Either allow for vector-like exotics
- or for incomplete GUT multiplets from different matter curves

A challenge

In all explicit F-theory models known so far*:
matter curve C = GUT divisor $S \cap$ divisor $b = 0$ on B_3

- restriction $F_Y|_C \stackrel{!}{=} 0$ for massless $U(1)_Y$ boson
- clashes with chiral doublet triplet splitting on split Higgs curves
  $\chi_{(\mathbf{3},1)_{-2Y}} = \int_{C_{\mathbf{5_H}}}(F - 2F_Y) = 0, \qquad \chi_{(1,\mathbf{2})_{3Y}} = \int_{C_{\mathbf{5_H}}}(F + 3F_Y) = \pm 1$

$\Rightarrow$ **need new class of fibrations?**

Investigation in better-understood Type IIB models (no E6-coupling!) shows: [Mayrhofer,Palti,TW'13]

- Well-defined Type IIB models with massless hypercharge **and** $F_Y|_{C_H} \neq 0$ exist thanks to orientifold odd components of Higgs curve
- All anomalies are automatically cancelled
- Can even relax anomaly constraints if extra U(1) is geometrically massive!

A challenge

Current global models realize doublet triplet-splitting with $q_{H_u} = -q_{H_d}$

- non-split $\mathbf{5}_H$ curve and arrange for line bundle cohomologies appropriately
  not done yet, but in principle possible
- or: Split $C_H \to C_{H_u} + C_{H_d}$ without changing the charges
  [Braun,Collinucci,Valandro'14]

Drawback:

- no U(1) available to suppress dim 5 proton decay
- ok with bounds with intermediate SUSY [Ibanez,Marchesano et al.'13]

Semi-global model building

Study phenomenology in absence of full compact models

Need criterion for possible $U(1)$ charges and consistent flux configurations

1) Spectral cover models [Marsano,Saulina,S-Nameki'09-'12], ...

- correctly describes charged matter and SU(5) sector, but not completely reliable for U(1) symmetries
- Mixed abelian hypercharge anomalies not automatically cancelled [Palti'12] $\rightarrow$ consistency less clear

2) Classify all possible Higgsing chains $E_8 \rightarrow SU(5) \times U(1)^5$

impose absence of hypercharge anomalies

[Dudas,Palti'09-'10] [Baume,Palti'15] [Palti'12,'16]

3) U(1) charges from consistent smooth fibers

[Lawrie,S-Nameki,Wong] [Krippendorf,S-Nameki,Wong]'15

- This is a proper subset of all possible configurations.
- Contains all possible Higgsing chains $E_8 \rightarrow SU(5) \times U(1)^5$ of 2)
- In addition impose absence of $U(1)_Y$ anomalies on spectrum

An exhaustive scan

over all such SU(5) GUT models with up to 2 U(1)s within class 3)

Constraints: [Krippendorf,S-Nameki,Wong'15]

1. Absence of anomalies and of exotics $\rightarrow$ allow for incomplete GUT multiplets from curves
2. Absence of dim 4 and 5 proton decay operators by U(1)s (Superpotential and Kahler potential) and of tree-level μ-term
3. Rank 1 top Yukawa coupling, rank 1 or rank 0 down Yukawa coupling
4. Generation of subleading Yukawas by suitable singlet VEVs without re-generating terms in 2) (Froggatt-Nielsen)
5. Phenomenologically viable flavour hierarchy patterns

Best results: see talk by J.Wong

1) New geometrically realized $SU(5) \times U(1) \times U(1)$ model satisfying 1) - 4) based on [Cvetič,Klevers,Piragua,Taylor'15]

2) Charge configuration for models with 1) - 5) without known geometric realization

Complete GUT multiplets

Necessarily includes vectorlike exotics

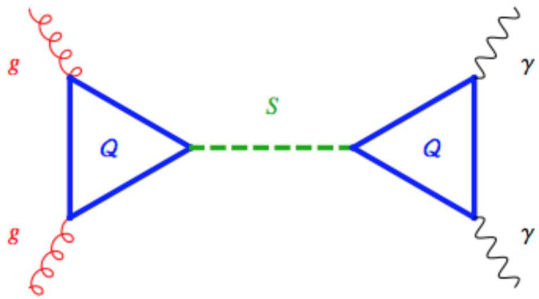
Strategy prior to Dec 2015:

- Decouple these by $\mathcal{O}(1)$ singlet VEVs $\langle S\rangle\Phi_1\Phi_2$
- **Problem:** Danger of re-generating too big unwanted couplings such as proton decay, μ-terms etc.

New strategy in view of LHC 750 excess: [Palti'16]

- Breaking of U(1) around TeV scale to keep singlets and vectorlike pairs light
- Links TeV singlet mass to smallness of μ-term and thus SUSY

Resulting singlet and vectorlike fermions of the right form to fit 750 diphoton excess (in broad brushes)



Other stringy interpretations:

[Heckman] [Cvetič,Halverson,Langacker] [Ibanez,Lozano] [Lüst et al.]'15

Complete GUT multiplets

Unification preserved provided singlets effectively behave like complete multiplets [Marsano,Saulina,S-Nameki'09]

Example: [Palti'16]

GUT Field	$U(1)$ Charge	$U(1)$ Flux	Hyper Flux	MSSM	Exotics
10_E	-2	-1	-1		$(\bar{3},2)_{-\frac{1}{6}} + 2\times(1,1)_{-1}$
10_M	3	4	1	3×10	$(3,2)_{\frac{1}{6}} + 2\times(1,1)_{1}$
5_{H_d}	4	0	-1	$(1,2)_{-\frac{1}{2}}$	
5_M	-1	-4	1	$3\times\bar{5}$	$(\bar{3},1)_{\frac{1}{3}}$
5_{H_u}	-6	1	0	$(1,2)_{\frac{1}{2}}$	$(3,1)_{-\frac{1}{3}}$
S_1	5				$(1,1)_0$
S_2	10				$(1,1)_0$

at 1-loop level of β-function:

$$[(\mathbf{3},\mathbf{2})_{1/6} + (\bar{\mathbf{3}},\mathbf{1})_{1/3} + 2(1,1)_1] \sim [(\mathbf{3},\mathbf{2})_{1/6} + (\bar{\mathbf{3}},\mathbf{1})_{-2/3} + (1,1)_1] = \mathbf{10}$$

Ultra-local Yukawa textures

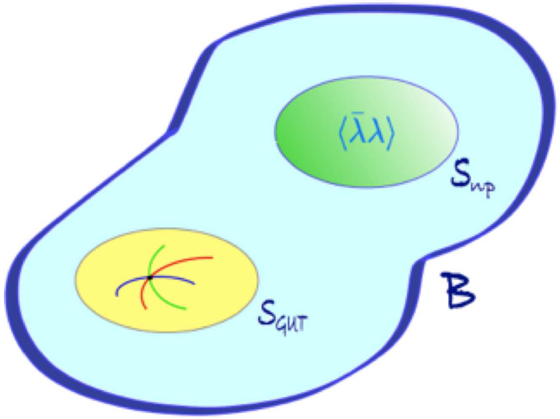
Yukawas $\leftrightarrow$ overlap of matter wavefunction at curve intersection point

Approach: All families from the same curve

- For single Yukawa point, mass matrix of rank 1 [BHV'08],[Cecotti,Cheng,Heckman,Vafa'10]
- Subleading non-pert corrections from D3/M5-instantons

[Marchesano,Martucci'09][Font,Ibanez,Marchesano13]

$M_i \simeq (1, \epsilon, \epsilon^2)$ with $\epsilon \simeq e^{-S_{\rm inst}}$



Font et al.,1307.8089

Computation of holomorphic and physical Yukawa couplings by unfolding of bulk superpotential

$$W = M_*^4 \int_S F \wedge \Phi + \epsilon \frac{\theta_0}{2} \mathrm{Tr}(F \wedge F) + \mathcal{O}(\epsilon^2)$$

E_7 [Carta,Marchesano,Zoccarato] or E_8 patches [Marchesano,Regalado,Zoccarato]

Realistic flavour structures matched to flux/geometric parameters

How to embed into global models?

Gauge coupling unification

To leading order:

$S_{\rm YM} = M_*^4 \int_{\mathbb{R}^{1,3}\times S} F^2 \Rightarrow \alpha_{GUT}^{-1} = M_*^4 {\rm Vol}(S)$

Complication: [Donagi,Wijnholt; Blumenhagen'08]

Non-GUT universal contribution to $\alpha_{\rm GUT}$ from hypercharge flux

Effect still hard to quantify in fully-fletched F-theory

Reliable computation in Type IIB limit [Blumenhagen'08]

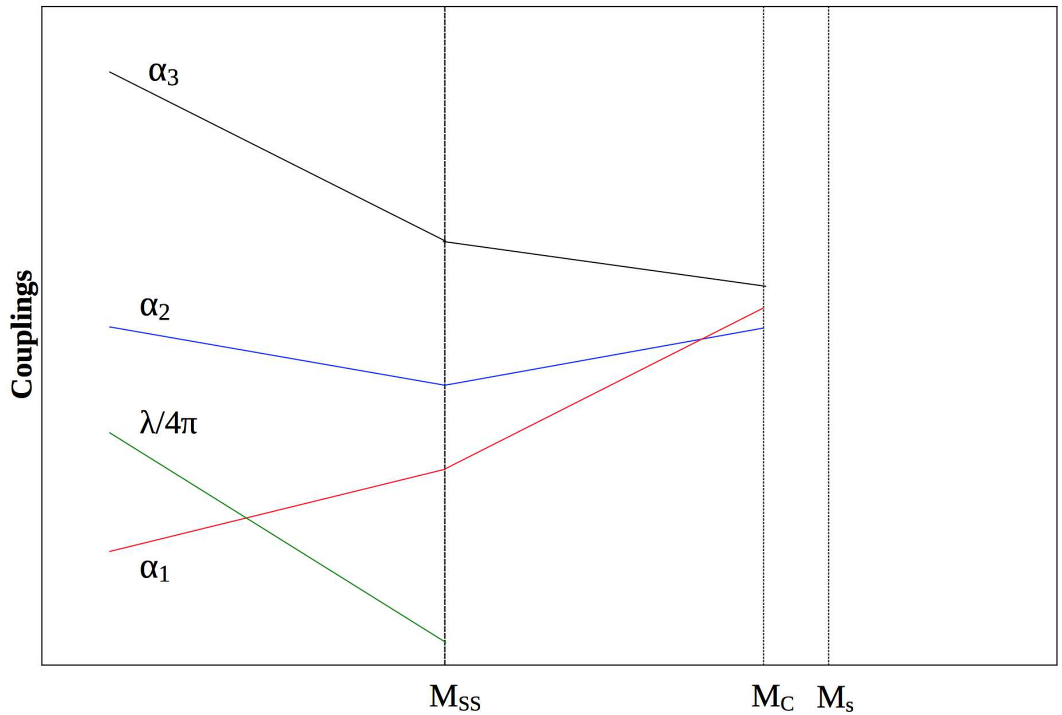
- classical contribution from $S_{CS} \supset \int C_0 {\rm tr}\, F^4$
  $f_{SU(3)} \simeq T - \frac{1}{2}\tau \int_S F_a^2$ $f_{SU(2)} \simeq T - \frac{1}{2}\tau \int_S (F_a^2 + F_Y^2)$
  $f_{U(1)} \simeq T - \frac{1}{2}\tau \int_S (F_a^2 + \frac{3}{5} F_Y^2) \Rightarrow \frac{1}{\alpha_1} = \frac{1}{\alpha_2} + \frac{2}{3}\frac{1}{\alpha_3}$ at $M_{\rm GUT}$
- size depends on quantization of F_Y and other gauge fluxes F_a
- $\delta\alpha_{\rm GUT} \simeq (0-1)\%\, \alpha_{\rm GUT}$ [Blumenhagen'08;Mayrhofer,Palti,TW'13]

Largely open:

- Further contributions from KK-states and D(-1) instantons?
- Precise analogue of specific flux quantization in F-theory?

Intermediate-SUSY GUTs

[Ibanez,Marchesano,Regalado,Valenzuela'12]



- Push $M_{\rm SUSY}$ up to 10^{11} GeV, where quartic Higgs coupling $\lambda = 0$.
- Standard gauge coupling unification is destroyed.
- Effect can be cancelled in principle against hypercharge-flux correction of [Blumenhagen'08]

Scenario:
$M_{\rm SUSY} = 10^{11}$GeV $\qquad M_{\rm GUT} = \ 10^{14}$GeV

Claim: see, however, [Hebecker,Unwin'14]
Dimension 6 proton decay from $X - Y$ boson exchange can be suppressed due to wavefunction distortion via hypercharge flux

Non-GUT model building

Direct approaches to Standard Model might be phenomenologically preferred if TeV scale SUSY were to be excluded

- Classification of toric $SU(3) \times SU(2) \times U(1)_Y \times U(1)$
  - classical realizations of $SU(3)$ and $SU(2)$ and including dim 4/5 proton decay operator analysis [Lin,TW]'14
  - inclusion of fluxes [Lin,TW] to appear
- Spot-on toric $SU(3) \times SU(2) \times U(1)_Y$ with 3 generations via fluxes [Cvetič,Klevers,Oehlmann,Reuter,'14]

  but no extra $U(1)$ to forbid R-parity violation
- Weierstrass model with non-classical Type III and Type IV realizations of $SU(3) \times SU(2)$ [Grassi,Halverson,Shaneson,Taylor'14]

  non-Higgsable SU(3) particularly attractive

Summary

F-theory GUTs exploit 2 key properties of F-theory

1) Localisation on 7-brane $\leftrightarrow$ GUT breaking with hyper flux

2) Exceptional symmetry E_6 $\leftrightarrow$ Yukawa points $\mathbf{10}\,\mathbf{10}\,\mathbf{5_H}$

Fruitful interplay

local model building ideas $\leftrightarrow$ global constraints of geometry

- triggers formal progress, e.g.
  U(1) selection rules $\overset{\checkmark}{\leftrightarrow}$ multi-section fibrations
- distinguishes landscape from swampland, e.g.
  Massless $U(1)_Y \to F_Y$ on trivial cycles $\to$ no new anomalies

Frontiers:

fluxes $\leftrightarrow$ hypercharge anomalies, unification

gluing/recombination $\leftrightarrow$ VEVs in Froggatt-Nielsen

M5-instantons $\leftrightarrow$ Yukawa couplings