Ph 205a Problem Set 7

1. Show that in Dirac theory the total angular momentum operator is

$$\vec{J} = \int d^3x \psi^\dagger(x) \left\{ \vec{x} \times (-i\vec{\nabla}) + \frac{1}{2} \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \right\} \psi(x)$$

(See Peskin and Schroeder section 3.5.)

2. Show that the Feynman propagator

$$S_F(x-y)_{\alpha\beta} = \int \frac{d^4p}{(2\pi)^4} e^{-ip\cdot(x-y)} \frac{i(\not{p}+m)_{\alpha\beta}}{p^2 - m^2 + i\epsilon}$$

is given by

$$S_F(x-y)_{\alpha\beta} = \langle 0| T\left(\psi_\alpha(x)\bar{\psi}_\beta(y)\right)|0\rangle$$

3. Deduce $S^{-1}O(x)S$ for $S = P,\ C,\ T$ for the operators
   a) $O(x) = \bar{\psi}\sigma_{\mu\nu}\psi$
   b) $O(x) = \bar{\psi}\sigma_{\mu\nu}\gamma^5\psi$

4. The pion π is a pseudoscalar particle: under parity, it transforms as

$$P\,|\pi(\vec{p})\rangle = -\,|\pi(-\vec{p})\rangle$$

where P is the parity operator and $|\pi(\vec{p})\rangle$ is a one-particle pion state with 3-momentum $\vec{p}$, normalized relativistically. Suppose $V_\mu(x)$ is a 4-vector and $A_\mu(x)$ is an axial vector–that is, under parity

$$PV_0(x)P = +V_0(x_P), \quad PV_j(x)P = -V_j(x_P)$$
$$PA_0(x)P = -A_0(x_P), \quad PA_j(x)P = +A_j(x_P)$$

Argue that

$$\langle 0|A_\mu(x)|\pi(\vec{p})\rangle = f e^{-ip\cdot x} p_\mu$$

for some constant f. What can you say about $\langle 0|V_\mu(x)|\pi(\vec{p})\rangle$?