1. Show that in Dirac theory the total angular momentum operator is

\[ \mathcal{J} = \int d^3x \psi^\dagger(x) \left\{ \vec{\sigma} \times (-i\vec{\nabla}) + \frac{1}{2} \begin{pmatrix} \vec{\sigma} & 0 \\ 0 & \vec{\sigma} \end{pmatrix} \right\} \psi(x) \]

(See Peskin and Schroeder section 3.5.)

2. Show that the Feynman propagator

\[ S_F(x-y)_{\alpha\beta} = \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip(x-y)} i(p^\mu + m)_{\alpha\beta}}{p^2 - m^2 + i\epsilon} \]

is given by

\[ S_F(x-y)_{\alpha\beta} = \langle 0| T (\psi_\alpha(x) \bar{\psi}_\beta(y)) |0\rangle \]

3. Deduce \( S^{-1}O(x)S \) for \( S = P, C, T \) for the operators
   a) \( O(x) = \bar{\psi} \sigma_{\mu\nu} \psi \)
   b) \( O(x) = \bar{\psi} \sigma_{\mu\nu} \gamma^5 \psi \)

4. The pion \( \pi \) is a pseudoscalar particle: under parity, it transforms as

\[ P |\pi(\vec{p})\rangle = - |\pi(-\vec{p})\rangle \]

where \( P \) is the parity operator and \( |\pi(\vec{p})\rangle \) is a one-particle pion state with 3-momentum \( \vec{p} \), normalized relativistically. Suppose \( V_\mu(x) \) is a 4-vector and \( A_\mu(x) \) is an axial vector—that is, under parity

\[ PV_0(x)P = +V_0(x_P), \quad PV_j(x)P = -V_j(x_P) \]
\[ PA_0(x)P = -A_0(x_P), \quad PA_j(x)P = +A_j(x_P) \]

Argue that

\[ \langle 0| A_\mu(x) |\pi(\vec{p})\rangle = f e^{-ip^\mu x_\mu} p_\mu \]

for some constant \( f \). What can you say about \( \langle 0| V_\mu(x) |\pi(\vec{p})\rangle \)?