Ph 205b Problem Set 3

1. Consider the theory of a complex scalar $\phi$ and two Dirac fermions $\psi_1$ and $\psi_2$ with Lagrange density

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}}$$

$$\mathcal{L}_0 = \partial_{\mu} \phi^* \partial^{\mu} \phi - m^2 \phi^* \phi + \sum_{i=1}^{2} \bar{\psi}_i (\not{\partial} - m_i) \psi_i$$

$$\mathcal{L}_{\text{int}} = -g (\phi \bar{\psi}_1 \psi_2 + \text{h.c.}) - \frac{\lambda}{4!} (\phi^* \phi)^2$$

a) Show that this theory has the symmetry $\phi \rightarrow e^{i\alpha} \phi$, $\psi_2 \rightarrow e^{-i\alpha} \psi_2$ for any real $\alpha$. Is this the most general renormalizable Lagrangian with this $U(1)$ global symmetry and this field content?

b) Calculate both the coordinate space and the momentum space Feynman rules for this theory.

c) Using a momentum space cutoff $\Lambda$ as a regulator, calculate $\langle \Omega | T \{\phi(x)^* \phi(y)\} | \Omega \rangle$. Drop terms that vanish as $\Lambda \rightarrow \infty$, and work to order $\lambda$ and order $g^2$.

2. Add to QED the following interaction terms:

$$\mathcal{L}_{\text{int}} = \lambda_1 F_{\mu\nu} F^{\mu\nu} F_{\lambda} + \lambda_2 \bar{\psi} \sigma^{\mu\nu} F_{\mu\nu} \psi.$$ 

a) What are the dimensions of the coupling constants $\lambda_1$ and $\lambda_2$?

b) Using our naive method, what are the momentum space Feynman rules associated with these interactions?

3. Show that

$$\frac{1}{a_1^{m_1} \ldots a_n^{m_n}} = \frac{\Gamma(M)}{\Gamma(m_1) \ldots \Gamma(m_n)} \int_0^1 dx_1 x_1^{m_1-1} \ldots \int_0^1 dx_n x_n^{m_n-1} \frac{\delta(1 - \sum_{i=1}^n x_i)}{x_1 a_1 + \ldots + x_n a_n} M,$$

where $M = \sum_{i=1}^n m_i$. 