Ph 205b Problem Set 5

1. a) Show that near $p^2 = m^2$,

$$
\int d^4x e^{ip \cdot x} \langle \Omega | T \{ \psi(x) \bar{\psi}(0) \} | \Omega \rangle \rightarrow \frac{iZ(p + m)}{p^2 - m^2 + i\epsilon},
$$

where $\langle \Omega | \psi(0) | p, s \rangle = \sqrt{Z} u^s(p)$. (Imagine that the photon has a small mass.)

b) What is the analogous formula for the time-ordered product of two photon fields?

2. In class, we derived the LSZ formula for $\langle \text{out} | p_1 p_2 | k_1 k_2, i_n \rangle$ in scalar field theory. It related this matrix element to a time-ordered product of four fields. Derive the corresponding formulas for photons and fermions in QED.

3. Consider $2 \rightarrow 2$ scattering:

![Diagram]

The Mandelstam variables $s$, $t$, $u$ are defined by

$$
\begin{align*}
    s &= (p + p')^2 = (k + k')^2 \\
    t &= (k - p)^2 = (k' - p')^2 \\
    u &= (k' - p)^2 = (k - p')^2
\end{align*}
$$

a) Show that

$$
s + t + u = \sum_{i=1}^{4} m_i^2,
$$

where $m_i$ is the mass of particle $i$.

b) In $e^- (p) + e^+ (p') \rightarrow \mu^- (k) + \mu^+ (k')$, show that

$$
\frac{1}{4} \sum_{\text{spins}, s} |M_{fi}|^2 = \frac{8e^4}{s^2} \left[ \left( \frac{t}{2} \right)^2 + \left( \frac{u}{2} \right)^2 \right].
$$

4. Peskin and Schroeder problem 5.2 (Bhaba scattering).