Introduction to Standard Model

Notation: $\psi = \frac{1}{2}(1 + \gamma_5)\psi$. Nek-transform differently under the Lorentz Group. $\psi_\frac{1}{2}$ is $(\frac{1}{2}, 0)$ and $\psi_0$ is $(0, \frac{1}{2})$. Because we use different representations of the Lorentz group, they can have different quantum numbers. Gauge group of the Standard model is $SU(3) \times SU(2) \times U(1)$.

Field $\quad SU(3) \quad SU(2) \quad U(1) \quad$ Lorentz

| $Q^i_c$ | (3) | 2 | $\frac{1}{6}$ | $(\frac{1}{2}, 0)$ |
| $U^i_L$ | (3) | 1 | $\frac{2}{3}$ | $(0, \frac{1}{2})$ |
| $d^i_R$ | (3) | 1 | $-\frac{1}{3}$ | $(0, \frac{1}{2})$ |
| $l^i_L$ | (1) | 2 | $-\frac{1}{2}$ | $(\frac{1}{2}, 0)$ |
| $H$ | (1) | 2 | $\frac{1}{2}$ | $(0, 0)$ |
| $e^i_R$ | (1) | 1 | $-1$ | $(0, \frac{1}{2})$ |

Gauge group is spontaneously broken by vacuum expectation value of the Higgs doublet:

$$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$$

$$2\lambda H^+ (\partial^\mu H^\dagger H^\mu) - V(H)$$
\[ \sqrt{V(H)} = \frac{1}{4} \left( H^+ H - 0^2 \right)^2 \]

Higgs scalar potential minimized by: \( H^+ H = 0^2 \).

In our SU(2)\times U(1) "global symmetry" it will be vacuume expectation values:

\[ H = \begin{pmatrix} 0 \\ \sqrt{v/2} \end{pmatrix} \]

where \( v \) is real and positive. Expand the 2D \( V \) in the 2D complex

\[ H(x) = \begin{pmatrix} h^+(x) \\ \sqrt{v/2} + h^0(x) \end{pmatrix} \]

\[ \rightarrow \text{complex} \]

\[ H^+ H = |h^+|^2 + \frac{v^2}{2} + \sqrt{2} \sqrt{v^2} \Re h^0 + |h^0|^2 \]

\[ \sqrt{V(H)} = \frac{1}{4} \left( |h^+|^2 + \sqrt{2} \sqrt{v^2} \Re h^0 + |h^0|^2 \right)^2 \]

So only \( \sqrt{v} h^0 \) has a mass

\[ \text{Mass} h^0 = \sqrt{\frac{v^2}{2}} \]

Components \( h^+, \sqrt{v} h^0 \) are massless Goldstino

Because associated with the symmetry breaking: \( SU(2) \times SU(1) \times U(1) \) transform like \( \sqrt{v} \) in the 2D. But local

transformations do not change. In \( h^0, H^+ \)

from \( H \). This is called unitary gauge.

When dual degrees of freedom goes. We will

still 3 gauge boson get mass. Massless
gaug boson on left plots will just 2 degrees
of pseudon (2 polarization) but messenger
are due to 3 degrees of freedom \( J=1, m=1,0,-1 \).
So the degrees of freedom go into additional
defects of magnetic modes. This phenomena
is called the Higgs Mechanism.

Generality of such action on doublets

\[ T^a = \frac{Q^a}{2}, \quad a = 1,2,3 \quad T - T^7 T^6 - \frac{T^8}{2} \]

U(1) generator is called hypercharge \( Y \). Adler \&
Higgs double \( Y = \frac{1}{2} \).

\[ \bar{Y} H = \frac{1}{2} H \]

A U(1) subgroup of gauge group leaves Higgs
vacuum expectation value invariant.

\[ Q = T^3 + Y = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \]

\[ Q < H > = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0 \]

\[ e^{-Q} < H > = < H > \]

The general \( Q = T^3 + Y \) is called electric
doublet. How do the gauge bosons transform with
a doublet \( U(1) \)?

\[ dW^a T^a = i \bar{\alpha} Q W^a T^a = i \bar{\alpha} \left[ T^3, W^a T^a \right] \]

Now \( W^3 \) has zero charge. But what about \( W^1,2 \).
Define \( T^+ = T^1 + iT^2 \)

\[
T^+ = \frac{1}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} + \frac{1}{2} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}
\]

\( T^- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \)

\[
W^1 T^1 + W^2 T^2 = \frac{1}{2} \begin{pmatrix} 0 & W^1 - (W^2) \\ W^1 + iW^2 & 0 \end{pmatrix}
\]

\[
= \frac{1}{2} (W^1 - iW^2) T^+ + \frac{1}{2} (W^1 + iW^2) T^-
\]

\( D_1 W^+ = \frac{W^1 - iW^2}{\sqrt{2}} \quad W^- = \frac{W^1 + iW^2}{\sqrt{2}} \)

\[
W^1 T^1 + W^2 T^2 = \frac{1}{\sqrt{2}} W^+ T^+ + \frac{1}{\sqrt{2}} W^- T^-
\]

\[
[\Omega, T^+] = \pm T^+
\]

\[
\Rightarrow \delta W^+ = i\delta x W^+
\]

\[
\delta W^- = -i\delta x W^-
\]

So \( W^+ \) have electromagnetic charge \( \pm 1 \).
The covariant derivative is

\[ D \mu = \partial_\mu + i g A_\mu T^a + i g_2 W^a T^a + i g_3 b_\mu T^a \]

(\text{for } b_\mu \in SU(2))

Adapting \( u \)-veto by fixing \( T^a \) as Brill-Mam Malcev
Adapted doubled \( T^a = 0 \). \( 1 \)

The scalar length for the Higgs doubled scalars uncertain
hence the couplings is made for the gauge boson

\[ Z_{\text{gauge-boson}} = \left( \frac{1}{2} \right) \left( -i g_2 W^a T^a - \frac{1}{2} g_2 b_\mu T^a \right) \]

\[ \left( i g_2 W^a T^a + i g_3 b_\mu T^a \right) \left( \frac{1}{2} \right) \]

\[ \left( \frac{1}{2} \right) \left( \frac{1}{2} \right) \]

\[ \frac{1}{2} W^{a T^a} \]

\[ \frac{1}{2} \]

\[ 1 \quad T^a, \ T^a = \delta^{a b} \]

\[ T^a_{2 2} = \frac{-1}{2} \delta^{a 3} \]

\[ Z_{\text{gauge-boson}} = \frac{1}{2} g_2 \left( W^a W^a + W^2 W^2 + W^3 W^3 \right) \]

\[ + \frac{1}{2} \left( \frac{1}{2} g_2 b_\mu b_\mu - \frac{1}{4} g_4 g_2 W^a b_\mu b_\mu \right) \]
\[ \begin{align*}
\text{Suppose lorentz index} & \\
= & \left( \frac{u^2}{g^2} \right) (W^1 W^1 + W^2 W^2) \\
+ & \frac{u^2}{g} (g_2 W^2 - g_1 B) (g_1 W^3 - g_1 B) \\
\text{So } W^{1,2} \text{ with mass } & \rightarrow \omega^2. \\
M_w = & \left( \frac{\sqrt{g_2}}{2} \right) \\
\text{and} & \\
Z^w = & \frac{g_2 W^3 - g_1 B}{\sqrt{g_1^2 + g_2^2}} \\
\text{with mass} & \\
M_z = & \frac{\sqrt{g_1^2 + g_2^2}}{2} \\
\text{Orthogonal linear combination} & \\
A_w = & \frac{g_1 W^1 + g_1 B}{\sqrt{g_1^2 + g_2^2}} \\
\text{massless as the photon. Conventional is to introduce weak mixing angle} & \\
\sin \theta_w = & \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \quad \cos \theta_w = \frac{g_2}{\sqrt{g_1^2 + g_2^2}}.
\end{align*} \]
\[ Z = \cos \theta_\omega W_\omega - \sin \theta_\omega B \]
\[ A = \sin \theta_\omega W_\omega + \cos \theta_\omega B \]

\[ M_\omega = \frac{M \omega}{\cos \theta_\omega} \]

Can invert

\[ W_\omega = \cos \theta_\omega Z + \sin \theta_\omega A \]
\[ B = -\sin \theta_\omega Z + \cos \theta_\omega A \]

Write covariant derivatives in terms of mass eigenvectors

\[ D_m = \partial_m + i g A_m^A T^A + \frac{ig_2}{\sqrt{2}} \left( W_\omega^+ T^+ + W_\omega^- T^- \right) \]
\[ + ig_2 T^3 \left( \cos \theta_\omega Z_m + \sin \theta_\omega A_m \right) + (\lambda - T^3) g_1 \left( \cos \theta_\omega A_m - \sin \theta_\omega Z_m \right) \]
\[ = \partial_m + i g A_m^A T^A + \frac{ig_2}{\sqrt{2}} \left( W_\omega^+ T^+ + W_\omega^- T^- \right) \]
\[ + i Z_m \left( g_2 \cos \theta_\omega T^3 + g_1 \sin \theta_\omega (T^3 - \lambda) \right) \]
\[ + i A_m \left( g_2 \sin \omega T^3 + (\lambda - T^3) g_1 \cos \omega \right) \]
\[ = \partial_m + i g A_m^A T^A + \frac{ig_2}{\sqrt{2}} \left( W_\omega^+ T^+ + W_\omega^- T^- \right) \]
\[ + i Z_m \left( g_2 \cos \theta_\omega T^3 + g_1 \sin \theta_\omega (T^3 - \lambda) \right) \]
\[ + i A_m \lambda \cos \theta_\omega g_1 \]
\[ E = g_1 \cos \theta w = g_2 \sin \theta w = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} \]

\[ D_m = D_m + ig A_\mu T^\mu + \frac{ig_2}{\sqrt{2}} (W^+ T^3 + W^- T^-) \]
\[ + \frac{1}{\sqrt{2}} \sqrt{g_1^2 + g_2^2} (T^3 - Q \sin \theta w) + i A_\mu Q e \]

SO(3)xSU(2)xU(1) means one force mass lumped to quark & lepton to be zero. However they can couple to the Higgs doublet with different general mass.

\[ Z_{ik} = g_1 \hat{u}^i \hat{v}^j (H^\dagger Q^j) + g_2 \hat{d}^i \hat{u}^j (H^+ Q^j) \]
\[ + g_2 \hat{e}^i \hat{u}^j (H^+ L^j) + \text{h.c.} \]

\[ E = (0, 1), \quad <H> = \left( \frac{v}{\sqrt{2}} \right) \]

\[ \Sigma_{\nu} = -m_\nu \hat{v}^i \hat{v}^j - m_d \hat{d}^i \hat{d}^j - m_e \hat{e}^i \hat{e}^j + \text{h.c.} \]

\[ m_\nu = \frac{v g_2}{\sqrt{2}}, \quad m_d = \frac{v g_3}{\sqrt{2}}, \quad m_e = \frac{-v g_3}{\sqrt{2}} \]

Note that neutrinos are massless. Any matrix \( M \) can be brought and diagonal form by simply applying transformation in left or right.

\[ M = P + D \]

We also diagonal mass matrices by order quark anomaly transformations or left right handed quark & lepton fields. With this leave 3x3 "invonant"
\[ u_t' \cdot \bar{u}_t' = u_t' \cdot \bar{u}_t + d_t' \cdot \bar{d}_t' \]

\[ u_t' \cdot \bar{u}_t', \quad d_t' \cdot \bar{d}_t', \quad \bar{e}_t' \cdot \bar{e}_t' \]

\[ \bar{e}_t' \cdot \bar{e}_t' = \bar{e}_t' \cdot \bar{e}_t + \bar{d}_t' \cdot \bar{d}_t' \]

Define the mass eigenstates

\[ u_L = U(u, L) u' \]

\[ d_L = U(d, L) d' \]

\[ e_L = U(e, L) e' \]

\[ u_R = U(u, R) u' \]

\[ d_R = U(d, R) d' \]

\[ e_R = U(e, R) e' \]

\[ U(u, R) \begin{pmatrix} m & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} U(u, L) \]

\[ U(d, R) \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} U(d, L) \]

\[ U(e, R) \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_e & 0 \\ 0 & 0 & m_e \end{pmatrix} U(e, L) \]
Derivation:

\[
\begin{pmatrix}
    u_L \\
    d_L
\end{pmatrix}
= \begin{pmatrix}
    u(L) \\
    u(d)
\end{pmatrix}
= U \begin{pmatrix}
    U^T \\
    V d L
\end{pmatrix}
\]

\[V = u^T W u \mid u(d, L)\]

Charged W-boson couplings:

\[L = i g_2 \frac{W^+_{ij} u_L Y^i}{\sqrt{2}} \epsilon^{\mu \nu} d_L + \text{h.c.}\]

\[= i g_2 \frac{W^+_{ij} u_L Y^i V_{ij}}{\sqrt{2}} d_L + \text{h.c.}\]

But $Z_3$: A couplings remain diagonal in isospin space.

A 3x3 unitary matrix $V$ specified in general by 9 real parameters. But only four real $\theta_i, \bar{\theta}_i, \bar{\theta}_i$ and $\phi$ are free. $\tan \theta_i$ can be removed $\theta$ but a common phase relation on $U_3$ and $V_3$ does not hold so can remove 5 leaving four parameters. These are $\theta_1, \theta_2, \theta_3$ and $\phi$.

\[V = \begin{pmatrix}
    C_1 & S_1 C_2 & i S_1 S_2 \\
    -S_1 C_2 & C_1 C_3 - S_2 S_3 e^{i \phi} & C_1 S_3 + S_2 S_3 e^{i \phi} \\
    S_1 S_2 & C_1 S_3 + S_2 S_3 e^{i \phi} & C_1 S_3 - S_2 S_3 e^{i \phi}
\end{pmatrix}
\]