1. Using the path integral formulation write the expression for
\[ \langle \Omega | T \{ \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4) \} | \Omega \rangle, \]
in $\lambda \phi^4$ theory working to order $\lambda^2$. Keep your answer as integrals over space-time, don’t try to do the integration. Show that the disconnected vacuum pieces cancel.

2. Using the path integral formulation derive Feynman rules for the scalar field theory
\[ \mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 - \frac{g}{m} \partial_\mu \phi \partial^\mu \phi \phi - \frac{\lambda}{4!} \phi^4. \]

3. In $\lambda \phi^4$ theory argue that
\[ W[J] = \log Z[J] \]
is the generating functional for connected two and four point functions. In general, $W[J]$ is the generating functional for connected n-point functions. (You don’t need to prove it.)

4. Using the gauge fixed QED action
\[ S = \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 \right] \]
show that the photon propagator
\[ D_{\mu\nu} = \frac{-i}{k^2 + i\epsilon} \left[ \eta_{\mu\nu} - (1 - \xi) \frac{k_\mu k_\nu}{k^2} \right]. \]