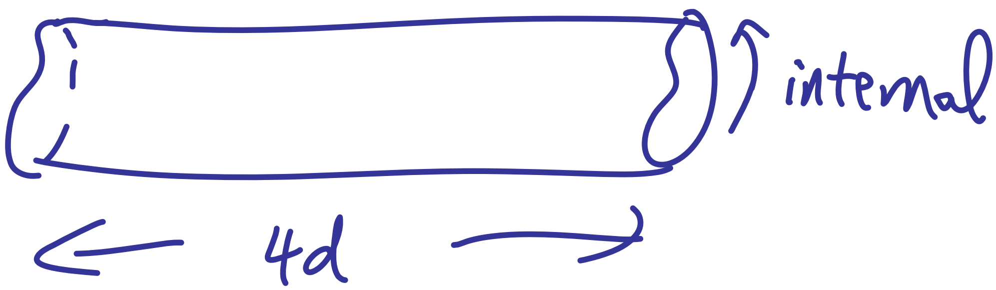
Toward 4d quantum gravity
in string theory

with Joe Polchinski

We'd like a non-perturbative formulation of 4d physics:



with a hierarchy of energy scales

$$M_{\left\{\substack{KK \\ \text{string} \\ D}\right.} \gg \frac{1}{L_{4d}}$$

Outline

- The problem & previous attempts
- Our strategy
- Consistency conditions (cf singularities)
- Candidate Examples

$$\mathrm{AdS}_{\substack{5\\4}} \times \mathrm{Small}_{\substack{5\\6}} \Leftrightarrow \mathrm{CFT}_{\substack{4\\3}}$$

stabilized compactifications $\Leftrightarrow$ IR limit of concrete brane systems (w/ SUSY)

- Generalizations toward dS

AdS/CFT (and previously, BFSS matrix theory) formulate some backgrounds non-perturbatively, but did not (yet) get down to 4d :

BFSS : 11d $\leftrightarrow$ N D0-brane Q.M.

4d (max susy) $\leftrightarrow$ $\underbrace{\text{D7-branes}}$ on T^7

↳ codim 2 → log potential → $N \leq 24$

$\underline{\text{AdS/CFT:}}$ ① $\underbrace{AdS_2 \times S^2}_{\text{want } L_{AdS} \to \infty} \times \underbrace{CY}_{\text{small } \checkmark}$

↳ IR divergences in AdS_2

AdS/CFT ②

$$\underbrace{AdS_4 \times \begin{cases} S^7 & (M) \\ S^7/\Gamma & \\ \mathbb{CP}^3 & (IIA) \end{cases}}_{L_{internal} \sim L_{AdS}}$$

No hierarchy of scales in Freund-Rubin compactifications.

Basic reason: In 11/10 d Einstein equations $\underbrace{R_{MN} - \frac{1}{2} R G_{MN}}_{\text{Internal + 4d}} = 8\pi G \underbrace{T_{MN}}_{\text{flux}}$

all three contributions are of the same order in the solution

For future reference, let us reproduce this in the language of the 4d effective potential energy:

$$S = \int \frac{d^{10}x \sqrt{G}}{\alpha'^4} \left(\frac{\mathcal{R}}{g_s^2} + F_p^2 + \cdots \right)$$

$\rightarrow$ 4d potential energy

$$\mathcal{U}_4 = \frac{1}{\alpha'^2} \left\{ - \int \frac{d^6x \sqrt{G_6}\, \mathcal{R}^{(6)}}{\alpha'^2\, g_s^2} + \int \frac{d^6x \sqrt{G_6}\, F_p^2}{\alpha'^2} + \cdots \right\}$$

$$\sim \frac{R^6}{\alpha'^2 g_s^2} \left(- \frac{1}{R^2} + \frac{g_s^2 Q^2}{R^{2p}} + \cdots \right)$$

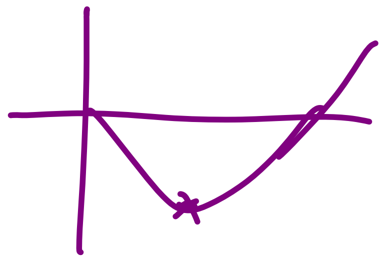
where $R \equiv$ size in string units.

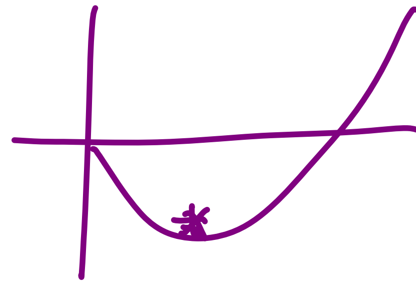
$$\mathcal{U}_4 \sim M_p^4 \left(\frac{g_s^2}{R^6} \right) \left(- \underbrace{\frac{1}{R^2}}_{\mathcal{U}_{\mathcal{R}}} + \underbrace{g_s^2 \frac{Q^2}{R^{2p}}}_{\mathcal{U}_{F_p}} + \cdots \right)$$

in Einstein frame

e.g. IIA on $\mathbb{CP}^3$ w/ F_6 & F_2 :

$$\frac{U_4}{M_p^4} \sim -\frac{g_s^2}{R^8} + \frac{g_s^4 Q_2^2}{R^{10}} + \frac{g_s^4 Q_6^2}{R^{18}}$$





Equivalently

$$U_4 \sim M_p^2\Big(\underbrace{-\frac{1}{R^2\alpha'} + \frac{g_s^2 Q^2}{R^{2p}\alpha'}}_{\text{"}\Lambda\text{"} \sim \frac{1}{R_{AdS}^2\alpha'}} + \cdots\Big)$$

i.e. $\Lambda_{min} \sim \Lambda_{gR}$ in Freund-Rubin

- n.b. CFT_3 dual to IIA$/\mathbb{CP}^3$ is $\left\{\begin{array}{l}\cdot \text{ IR limit of D2, D6, KK} \\ \cdot \text{ strongly coupled CS}\end{array}\right.$

  or M thy $/ S^7/\Gamma$

  Schwarz, BL, ABJM, et seq.

In fact there is a large class of 3d CFTs obtained via RG flow from gauge theory with flavors

Appelquist/Heinz HET
Sachdev.. CMT

$$\frac{1}{g^2} = \underbrace{\frac{1}{g_0^2}}_{\text{classical}} + \underbrace{\frac{\Delta N}{E}}_{\text{1-loop screening}} + \cdots$$

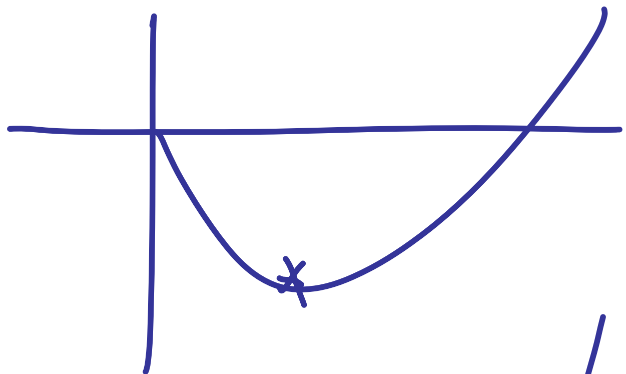
(In 3d $N=4$, $\Delta N = N_f - 2N_c$ and this is exact)

$\rightarrow$ dimensionless coupling has fixed point

$$\frac{1}{\bar{g}^2} = \frac{E}{g^2}\Big|_{E\to 0} = \Delta N \underbrace{+ \cdots}_{\substack{\text{controlled at} \\ \text{large } N_f \text{ independently} \\ \text{of SUSY.}}}$$

e.g. D2-D6, D2-D6-orbifold ; Hanany-Witten ... examples : still Freund-Rubin (or stringy)

Similarly, compactification on S^5 w/ F_5 $\rightarrow$ $AdS_5 \times S^5$



fixed line

$\Updownarrow$

$N=4$ SYM

$L_{AdS} \sim L_{S^5}$

In general, the negative term(s) in the potential are key

e.g. $\left\{ \begin{array}{l} \text{positive curvature} \\ \text{O-planes} \\ -3|W|^2 \end{array} \right.$

Coming from the other direction, we can constuct

$$(A)dS_4 \times X_{small}$$

in an apparently large number of ways B^2S DRS BP MSS GKP+KKLT...

suggesting a rich set of dual CFT_3 s.

- Not a priori realized as near-horizon limit of brane system
- <u>Can</u> read off interesting properties:

$$N_{d.o.f.} \sim L^2_{(A)dS} M_p^2 \le N^b$$

b ← betti #

N ← flux #

ES
BvdB
AAB

Plan: Start from known, Freund-Rubin dual pair:

$$\underbrace{AdS \times S}_{\text{add ingredients which cancel or nearly cancel } U_{\mathcal{R}}} \leftrightarrow \underbrace{QFT}_{}$$

(at least brane construction)

add ingredients which cancel or nearly cancel $U_{\mathcal{R}}$ $\leftrightarrow$ additional field content, couplings of QFT

stabilize the moduli $\to$ AdS_4 $\leftrightarrow$ CFT_3

7-branes contribute to $\mathcal{U}$ naively as

$$\mathcal{U}_7 \sim M_p^4 \left(\frac{g_s^2}{R^6}\right)^2 \cdot \left(\underbrace{\tau_7}_{\text{tension in string units}} \cdot R^4 \right)$$

compare to curvature energy

$$\mathcal{U}_{\mathcal{R}} \sim M_p^4 \left(\frac{g_s^2}{R^6}\right)^2 \cdot R^6 \cdot \left(-\frac{1}{R^2 g_s^2} \right)$$

$\Rightarrow$ for $\tau_7 \sim \frac{1}{g_s^2}$, i.e. (p,q) 7Bs,

cf Aharony Fayazzuddin Maldacena

they compete.

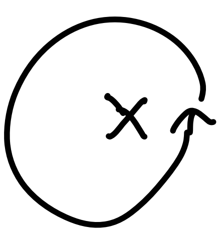
Of course 7Bs, being codimension 2, have large IR back reaction ...

The interplay between curvature & 7-brane energy is accurately captured using the techniques of F-theory:

Vafa '96

$$\begin{array}{ccc} T^2 & \rightarrow & X \\ & & \downarrow \\ & & B \end{array}$$

$$\tau_{T^2} = C_0 + \frac{i}{g_s} \quad \text{in IIB}$$



D7-brane: $\tau(u) \sim \frac{1}{2\pi i} \log u$

$\tau \to \tau + 1$ monodromy

(p,q) 7-brane $\tau \to \dfrac{(1+pq)\tau - p^2}{q^2\tau + (1-pq)}$

Plan: Start from Freund-Rubin dual pair:

AdS x S $\leftrightarrow$ QFT (brane construction)

add 7-branes which cancel or nearly cancel U_{qR} $\leftrightarrow$ additional flavors* couplings of QFT

stabilize the moduli $\rightarrow$ AdS4 $\leftrightarrow$ CFT_3

* in general, both electric & magnetic cf Douglas/Shenker, Argyres/Douglas, Argyres-Plesser-Seiberg-Witten

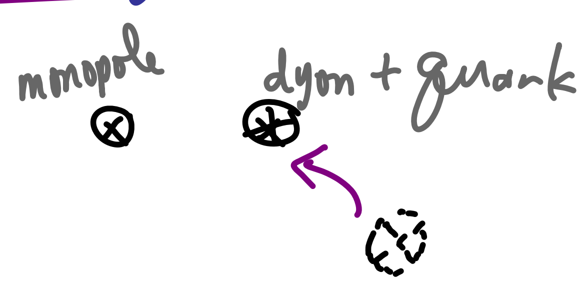
D3, D7 and Electric/Magnetic Matter

- 4d $\mathcal{N}=2$ $SU(2)$ SYM w/ N_f hypermultiplets

(Coulomb branch)

Seiberg-Witten solution

AD/APSW: can <u>change</u> mass matrix M such that mutually nonlocal matter is light.



- In brane constructions (Sen, Banks Douglas Seiberg ...)

  $u \leftrightarrow$ D3 position

  $\otimes \leftrightarrow$ 7B position $\qquad \tau_{YM} \leftrightarrow \tau_{IIB}$

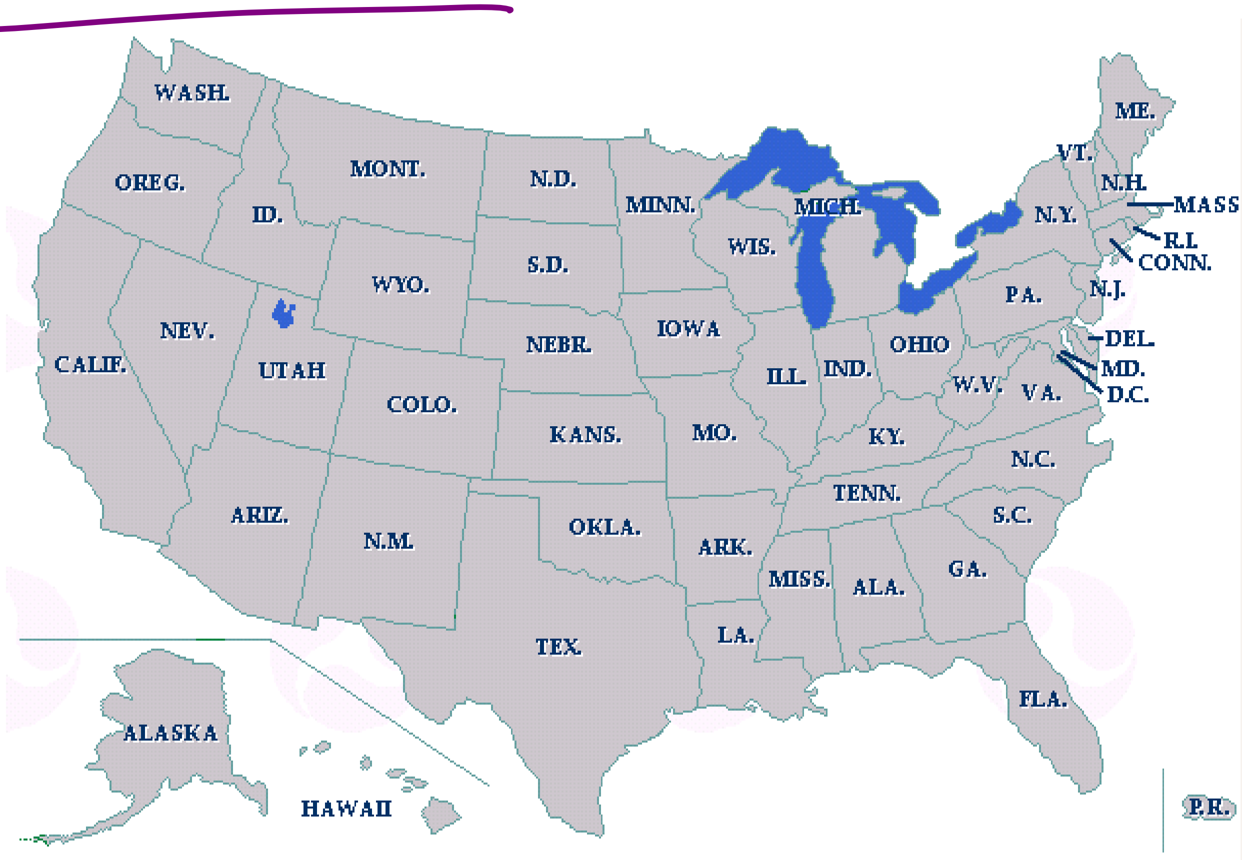
→ Can we unify

"West Coast" & "East Coast" physics?*

Landscape

QFTs from
3-7 systems

Existence proof:



* apologies to rest of world

The T^2 varying over B can be described as

Vafa
Morrison-Vafa
Kachru Intriligator
Morrison Vafa
...

$$y^2 - x^3 - x f(u) z^4 - g(u) z^6 = 0$$

coordinates on B

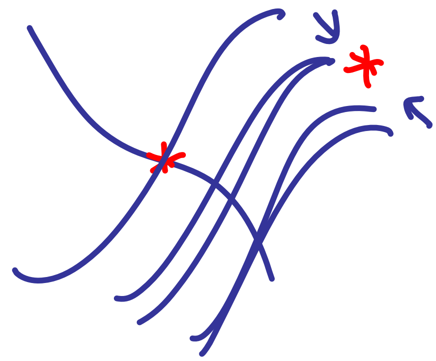
i.e. as a degree 6 hypersurface in $WP^2(2,3,1)$.

For a Kähler base B, one can formulate the T^2 fibration
$$\begin{array}{ccc} T^2 & \rightarrow & X \\ & & \downarrow \\ & & B \end{array}$$
as a hypersurface in $B \times WP^2(2,3,1)$, and as the target space of a (2,2) gauged linear σ-model (GLSM) Witten

7-branes live at the locus

$$\Delta = 27 g^2 + 4 f^3 = 0$$

Singularities

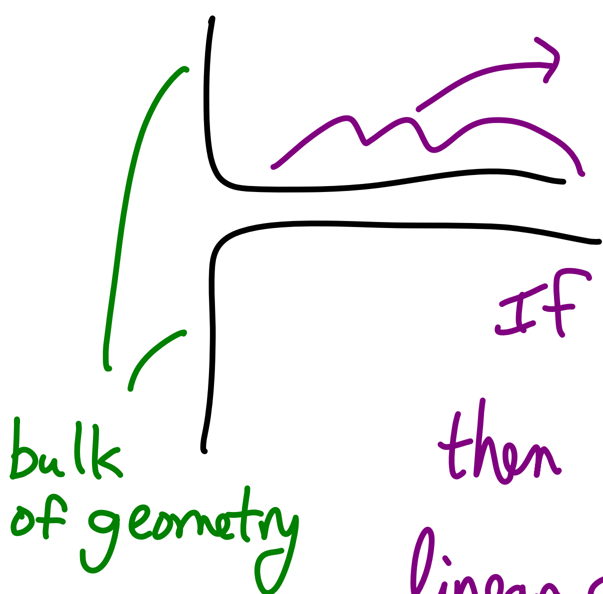


- Some allowed (e.g. enhanced gauge symmetries)
- Some not allowed

A criterion for allowed singularities:

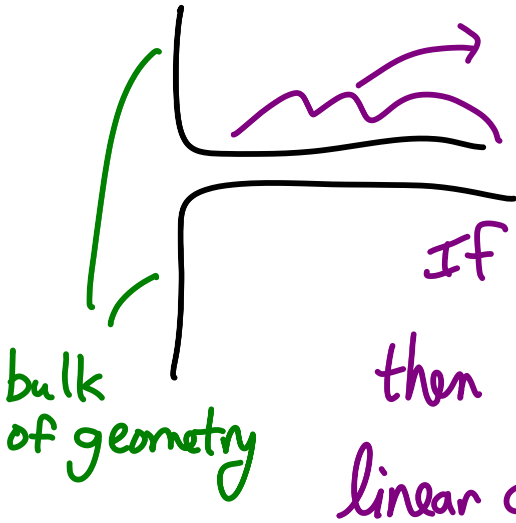
cf W, SW '90s

In the GLSM, singularities arise from extra non-compact branches of scalar field space



→ Compute (using GLSM) $\hat{c}_{\text{throat}}$:

If $\hat{c}_{\text{throat}} \geq \hat{c}_{\text{bulk}}$ then truly singular. (Otherwise linear dilaton → mass gap in throat)



Compute (using GLSM) $\hat{c}_{\text{throat}}$:

If $\hat{c}_{\text{throat}} \geq \hat{c}_{\text{bulk}}$ then truly singular. (Otherwise linear dilaton → mass gap in throat)

This agrees with known cases ...

e.g.

$$\int d^2\theta\, P\left(y^2 - x^3 - x\, \underset{\sim}{f(u)}^{\,u^0} z^4 - \underset{\sim}{g(u)}^{\,u^n} z^6\right)$$

branch with $\underbrace{\langle p\rangle \overset{\text{(D-term)}}{\sim} \langle z\rangle}_{\hat{c}=1}$; $Y = X = 0 = u$

$\hat{c}_Y = 0$ (massive) | $\hat{c}_X = \frac{1}{3}$, $\hat{c}_u = 1 - \frac{2}{n}$

$\Rightarrow$ singular if $n \geq 6$ ✓

... and can be applied widely

Let us start with IIB on

$Y_5 \times S^1$ with $\int_Y F_5 = N_c$

where Y_5 is an S^1 (Hopf) fibration over a Kähler base

(7-branes will be extended along fiber)

$$\begin{array}{ccc} S^1_f & \rightarrow & Y \\ & & \downarrow \\ & & B \end{array}$$

First study 7-branes on B

Examples: $Y = S^5$, $B = \mathbb{CP}^2$ or $\mathbb{WP}^2$

Topologically $S^2 \times S^3$

$Y = T^{1,1}$ $\quad B = \mathbb{CP}^1 \times \mathbb{CP}^1$ Klebanov/Tseytlin/Strassler

$Y = Y^{pq}$ $\quad B = \ldots$ Gauntlett Martelli Sparks Waldram

$Y = L^{abc}$ $\quad B = \ldots$ Cvetic Liu Pope

can have many 3-cycles

base of toric CY_3 cones Hanany...

" " non-toric " Martelli/Sparks/Yau

$$\begin{tikzcd} S^1_f \arrow[r] & Y \arrow[d] \\ & B \end{tikzcd}$$

size R_f

size R

$\cong$ B with "metric flux" $F_{met} = J_B$

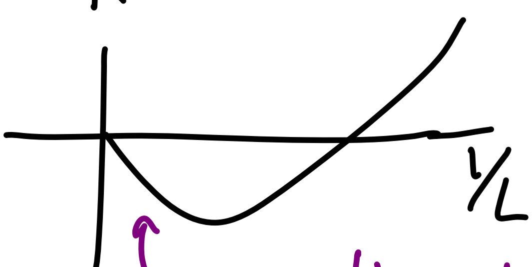
(useful description for small fiber)

Two classes of candidate examples:

① 7-branes fully cancel curvature energy: e.g. 36 7-branes on $\mathbb{CP}^2$

$$U_R + U_7 = 0$$

(F-theory on CY)



negative term from O-planes

$$R_{AdS} \gg R \gg R_f$$

② 7-branes nearly cancel $U_R^{(B)}$

$Y \times S^1_\perp \times AdS_4$

$$\mathcal{U}\Big|_{g_s \sim 1} \sim M_p^4 \left(\frac{1}{R^4 R_6 R_f}\right)\left(\frac{R_f^2}{R^4} - \frac{\varepsilon}{R^2} + \frac{N_c^2}{R^8 R_f^2} + \underbrace{\frac{Q_1^2}{R_\perp^2}}_{\propto \frac{Q_3^2}{R_\perp^2 R^4}}\right)$$

$g_s \sim 1$ enforce with e.g. E_n 7-branes

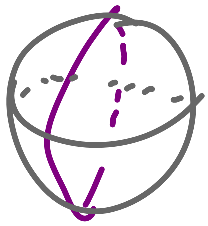
↳ stable minimum with $R_f \ll R \ll R_{AdS}$

Related (simpler) AdS_5 model:

$$\mathcal{U}\Big|_{g_s \sim 1} \sim \frac{M_5^5}{(R^5 R_f)^{2/3}}\left(\frac{R_f^2}{R^4} - \frac{\Sigma}{R^2} + \frac{N_c^2}{R^8 R_f^2}\right)$$

* Are 7-brane moduli tachyonic?

On S^5



allowed tachyon for $R \sim R_{AdS}$
but what about $R \ll R_{AdS}$?

Note that 7-brane moduli

$$y^2 - x^3 - x\, f(u)\, z^4 - g(u)\, z^6 = 0$$

are flat directions in the CY
and nearly flat for $U_R \propto \frac{\varepsilon}{R^2}$

To get started, consider

$$Y = S^5 \quad \text{(topologically)}$$

Start from the $\mathbb{CP}^2$ model:

(2,2) chiral multiplets u_1, u_2, u_3

$U(1)$ Gauge symmetry

$$(u_1, u_2, u_3) \simeq e^{2\pi i \alpha}(u_1, u_2, u_3)$$

$$\Rightarrow D^2 = \left(|u_1|^2 + |u_2|^2 + |u_3|^2 - R^2\right)^2$$

$\hookrightarrow D = 0$ alone gives S^5

$\rightarrow \alpha$ parameterizes S^1_{fiber}

$$\begin{array}{ccc} S^1_f & \rightarrow & S^5 \\ & & \downarrow \\ & & \mathbb{CP}^2 \end{array} \qquad ds^2_{S^5} = d\sigma^2_{\mathbb{CP}^2} + R_f^2 (d\alpha + A)^2$$

$$dA = J$$

Gibbons, Pope

To add the 7-branes, want a T^2 fibration over $B = \mathbb{CP}^2$

Gauged Linear σ-model becomes

	u_1	u_2	u_3	x	y	z	P
T^2 { $U(1)$	0	0	0	2	3	1	-6
×							
$\mathbb{CP}^2$ { $U(1)$	1	1	1	q_x	$\frac{3}{2}q_x$	0	$-3q_x$

$$S_W = \int d^2\sigma\, d^2\theta\; P\left(y^2 - x^3 - f(u)\,x z^4 - g(u) z^6\right)$$

Now, $\sum_{\text{fields } I} q_I = 0$ is the Calabi-Yau condition (ensuring anomaly-free $U(1)\times U(1)$ R-symmetries appropriate to $(2,2)$ SCFT) Witten

The running of R^2 in

$$D^2 = \left(|u_1|^2 + |u_2|^2 + |u_3|^2 + q_x |x|^2 + \frac{3}{2} q_x |y|^2 - |p|^2 - R^2\right)^2$$

is $\mu \frac{\partial R^2}{\partial \mu} \sim \sum_I q_I$

Now $\sum q_I = 3 - \frac{1}{2} q_x$

and the degree of $G = y^2 - x^3 - f x z^4 - g z^6$

is $\deg_G = \deg_g = 3 q_x = 18 - 6 \sum q_I$

<u>Fully</u> canceling curvature energy means $\sum q_I = 0 \Rightarrow \deg_g = 18$

$\Rightarrow \deg \Delta = 36 \Rightarrow$ 36 7-branes

(This agrees with naive result from $U_7 \sim \cdots (\varphi_7 \times \mathrm{vol})\ r$)

- The 7Bs are extended along U(1) fiber

As next step, generalize to cases where we do <u>not</u> fully cancel the curvature energy. Consider T^2 fibration over $B = \underline{\underline{W}}\mathbb{P}^2$

Gauged Linear σ-model

	u_1	u_2	u_3	x	y	z	P
$T^2 \{ U(1)$	0	0	0	2	3	1	-6
$\times$							
$W\mathbb{P}^2 \{ U(1)$	$\underline{\underline{w_1}}$	$\underline{\underline{w_2}}$	$\underline{\underline{w_3}}$	0	0	$w_0 - \Sigma w$	0

$$S_W = \int d^2\sigma\, d^2\theta \; P\left(y^2 - x^3 - f(u)\, x z^4 - g(u)\, z^6\right)$$

Again $\beta_{R^2, \text{full}} \sim \sum q = w_0$ in the full system including the 7Bs.

Again $\beta_{R^2, \text{full}} \sim \sum q = W_0$ in the full system including the 7Bs.

For WP^2 alone,

$$\beta_{R^2, WP^2} = W_1 + W_2 + W_3$$

$\Rightarrow$ If $W_0 << W_1 + W_2 + W_3$ then we almost cancel the curvature.

$$\rightarrow \mathcal{U}_{R, \text{full}} \sim M_p^4 \left(\frac{g_s^2}{Vol}\right)^2 \cdot Vol \cdot \left(-\frac{\varepsilon}{R^2}\right)$$

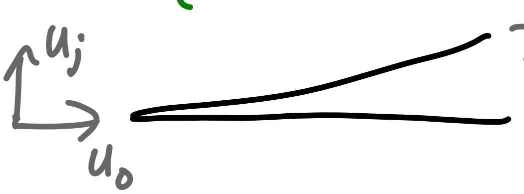
with $\boxed{\varepsilon \sim \frac{W_0}{W_1 + W_2 + W_3}}$

(using the NLSM result $\beta \sim R_{MN}$)

We can describe the full <u>brane system</u> (whose low-energy limit is the QFT) as F-theory on the following noncompact $\underline{\underline{CY_4}}$:

u_0	u_1	u_2	u_3	x	y	z	p
0	0	0	0	2	3	1	-6
$-w_0$	$\underline{\underline{w_1}}$	$\underline{\underline{w_2}}$	$\underline{\underline{w_3}}$	0	0	$w_0 - \Sigma w$	0

· $\Sigma w = 0$ overall; <u>cross-section</u> geometry as above



} <u>small</u> if $w_0 \ll w_j$: $w_0 |u_0|^2 \sim w_j |u_j|^2$

· This + N_c D3-branes at tip $(u_0 = u_j = 0)$ is the brane construction.

Altogether preserves 4 supercharges.

* What about the singularities of $WP^2(w_1, w_2, w_3)$?

$$(u_1, u_2, u_3) \simeq (\lambda^{w_1} u_1, \lambda^{w_2} u_2, \lambda^{w_3} u_3)$$

$\rightarrow$ for $\lambda = e^{\frac{2\pi i}{w_1}} \in U(1)$

$(u_1, 0, 0)$ is a (generally non-SUSY) $\mathbb{C}^2/\mathbb{Z}_{w_3}$ orbifold fixed point.

By itself*, this has twisted tachyons which condense, smoothing it out

Adams Polchinski ES ... .. Morrison ..

* Don't Panic! • Must include $\underbrace{\text{7-branes}}_{\text{can restore SUSY}}$

• and S^1_{fiber} : Removes $U(1)$ projection entirely

* full brane construction is supersymmetric

Concrete Examples

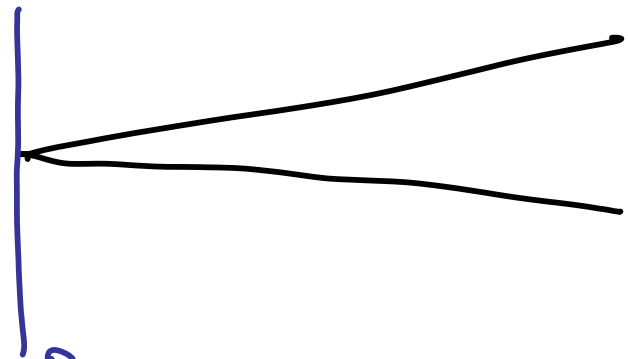
GLSM $U(1)^3$

$$D^2 = \Big(-2|\Phi_0|^2 + (w+1)|\Phi_1|^2 + w(|\Phi_2|^2 + |\Phi_3|^2) - (3w-1)|z|^2 - r_1\Big)^2$$

$$+ \Big(2|\eta|^2 - |z|^2 + \tfrac{1}{3}|\Phi_1|^2 - \tfrac{2}{3}(|\Phi_2|^2 + |\Phi_3|^2) - r_2\Big)^2$$

$$+ \Big(2|X|^2 + 3|y|^2 + |z|^2 - 6|p|^2\Big)^2$$

$$\int d^2\theta\, W = \int d^2\theta\, P z^6 \sum_{a,I} \eta\, \Phi_0^{9-\frac{a}{2}}\, \Phi_1^{3+\frac{a}{2}}\, \Phi_2^{a}\, \Phi_3^{I}\,{}^{18-a-I}$$



$p \sim z \sim \eta \sim \Phi_1 \neq 0$

$\Phi_0 = \Phi_2 = \Phi_3 = 0$

$\hat{c} = 4$ (marginal)

This branch is lifted for $r_2 < 0$

physics depends holomorphically on

$$t_2 = r_2 + i\theta_2$$

$\Rightarrow \quad \theta_2 \neq 0$ desingularizes the tip.

The string coupling

7-branes corresponding to mutually non-local flavors have $g_s \sim 1$

$\otimes$ e.g. $\tau \sim e^{\frac{i\pi}{3}}$ $f(u) = 0$ branch

Dasgupta/Mukhi

In a (near-) SUSY background, the (approximate-) moduli are

- R^2 (GLSM D-term)
- polynomial coefficients in f, g

(other modes of the metric + dilaton have KK-scale masses $\sim \frac{1}{R\sqrt{\alpha'}}$.)

In the LSM, f + g are superpotential couplings $\Rightarrow$ don't run even at $\mathcal{O}(\varepsilon)$

$\Rightarrow$ expect $|m^2| \leq \mathcal{O}(\varepsilon^2)$

Sen Limit

In F theory, $\exists$ limit (Sen)

$$f = -3h^2 + \epsilon\eta \qquad \varepsilon \to 0$$
$$g = -2h^3 + \varepsilon h\eta - \varepsilon^2 \frac{\chi}{12}$$

for which $g_s \to 0$. i.e. all the (p,q) 7-branes boil down to O7-planes + D7-branes

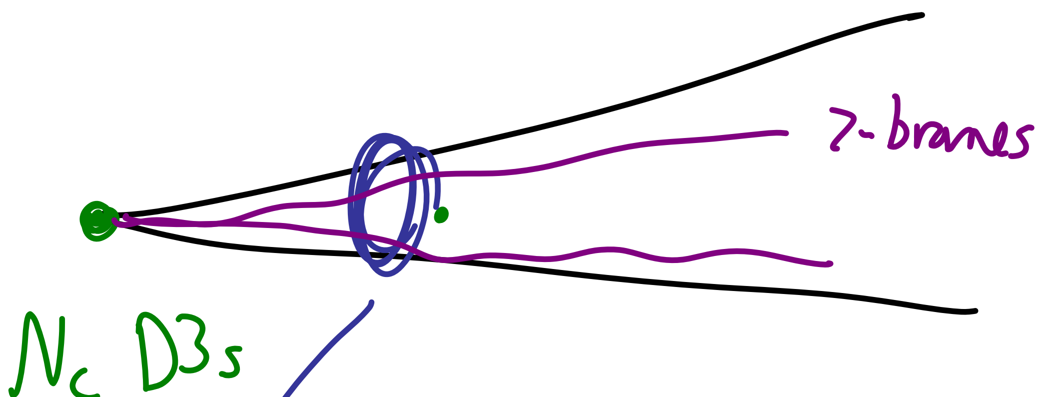
Such examples, if they can also be stabilized (including g_s), would be purely electric on the QFT side.

Number of degrees of freedom:

AdS_5 :

$$N_{d.o.f.}^{(7Bs)} \sim M_5^3 \, R_{AdS}^3 \sim \frac{1}{\varepsilon^3} \cdot N_{d.o.f.}^{(no\ 7Bs)}$$

$$R_f \sim \varepsilon R_{AdS} \ , \quad R^2 \sim \varepsilon R_{AdS}^2$$

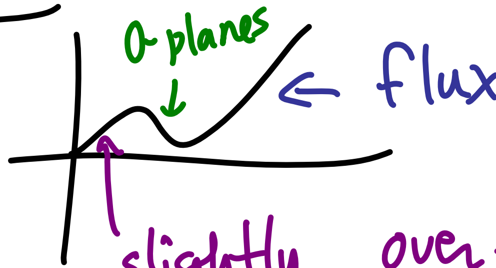


\# of light states

$$\sim \frac{1}{(\varepsilon^{\frac{1}{2}})^4} \cdot \frac{1}{\varepsilon} \sim \frac{1}{\varepsilon^3}$$

Next Moves

- dS_4 :



slightly over-cancel curvature energy

Now no tachyons are allowed.

+ Torroba, Shenker, Xi Dong,

→ dS entropy?; dS holography?

- Lifshitz solutions cf Kachru Liu Mulligan top-down

  part of ongoing work w/ S. Hartnoll, J. Polchinski, D. Tong

  Landscape of dynamical critical exponents

- QFT content & couplings from brane system?