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Magnets, boiling kettles and the secret code underlying reality

A strange, unifying mathematical pattern is popping up in all sorts of unexpected places – and it could explain some profound questions about the cosmos

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By **Gabriel Popkin**

SOME people see the future in tea leaves. David Simmons-Duffin is more interested in the boiling water. The jostling of water molecules as they turn from liquid to gas represents a problem that, for theoretical physicists like him, is just too hot to handle.

So what, you might say, as long as we can still make a decent cup of tea. But dive a little deeper into how water boils, and a pattern begins to emerge – the same pattern that crops up in all sorts of places where matter starts to shift shape. Whether it's the collective properties of electrons that make a material magnetic or superconducting, or the complex interactions by which everyday matter acquires mass, a host of currently intractable problems might all follow the same mathematical rules. Cracking this code could help us on the way to everything from more efficient transport and electronics to a new, shinier, quantum theory of gravity.

Simmons-Duffin, who works at the Institute for Advanced Studies in Princeton, New Jersey, and his band of fellow researchers don't claim to have cracked this code yet. But they have made more headway in a few years than people did in the generation before – using a key in a problem that first surfaced almost a century ago.

Physicists like simplicity. Their discipline is all about keenly observing the world and drawing out unifying mathematical rules that govern its workings. Take orbiting planets. First Johannes Kepler meticulously sifted through the available data to establish three mathematical rules that governed their motions. Then Isaac Newton showed that those three rules were just facets of one simple equation: his universal law of gravitation.

The challenge of change

Sadly, most things in the world are messier. Even heavenly motions become too complex to calculate from first principles when many bodies are involved. When it comes to atoms in a gas, or electrons in a solid, all thought of tracing their individual motions goes out the window. The quantities of information involved are too vast and the behaviour too complex to make accurate predictions.

Physicists call such problems strongly coupled, and have invented a number of ways to get to grips with them: rules of thumb and sneaky approximations that allow them to characterise what's going on. One particular focus of interest for such models is what happens when a strongly coupled system changes state – when it undergoes a "phase transition".

Boiling water is a textbook example of a phase transition. As the temperature rises, the liquid molecules start jostling about more vigorously, allowing the most energetic ones to escape as gas. We're all used to this happening at 100 °C under atmospheric pressure. If we raise the pressure, the boiling temperature rises too. Push hard enough, and you reach a point where you can no longer tell liquid and gas apart so clearly. This is known as water's critical point – a feature of a different sort of phase transition that remains poorly understood.

Over the past century or so, we have gradually realised that similar critical points crop up all over the place. One example has garnered particular interest: how magnetic materials lose their magnetism above a certain temperature.

In 1920, German physicist Wilhelm Lenz challenged his doctoral student Ernst Ising to have a stab at modelling this transition. Ising's approach was to imagine the material as being made up of millions of tiny atomic magnets that could be aligned either north-south or south-north. Each would initially tend to point in the same direction as its neighbours, giving the material an overall magnetism. But each could also flip randomly – and the higher the temperature, the more likely they were to flip, breaking the magnetism.

The model didn't work. As Ising eventually showed in his PhD thesis in 1924, it lacked the hoped-for phase transition. It also only applied to a simple one-dimensional row of atoms. With anything more complex, the couplings between neighbouring atoms were just too complex for Ising's approach.

That didn't stop others trying, but it wasn't until 1944 that the Norwegian physicist Lars Onsager solved Ising's model in two dimensions. That revealed the model to be a better fit to reality than anyone had realised. Not only could it acquire magnetism, but above a critical temperature it could also lose it.

For such a simple approximation, the 2D Ising model has proven far more powerful than it has any right to be. It has since been used to simulate a bewildering array of other phenomena that flip between states: from the way infectious diseases suddenly spread through a community to signalling between neurons in the brain.

Small wonder that many people have yearned to solve the Ising model in three dimensions. "It's something that could open up entirely new fields in mathematical physics," says Zohar Komargodski, a physicist at the Weizmann Institute of Science in Rehovot, Israel.

A big indicator of what that might bring came in the late 1960s, when Russian theorist Alexander Polyakov, now at Princeton University, was studying interactions between fundamental particles. Polyakov realised that they too represented strongly coupled systems that could undergo sudden phase transitions. For example, the fundamental particles known as quarks are usually bound by the strong nuclear force into particles such as the protons and neutrons of the atomic nucleus. But raised to higher and higher energies, quarks may reach a critical point where they overcome the strong force, allowing them to exist independently. So solve the 3D Ising model, and you might solve fundamental problems such as why protons and neutrons exist with the masses they do – and therefore why atomic matter as we know it exists.

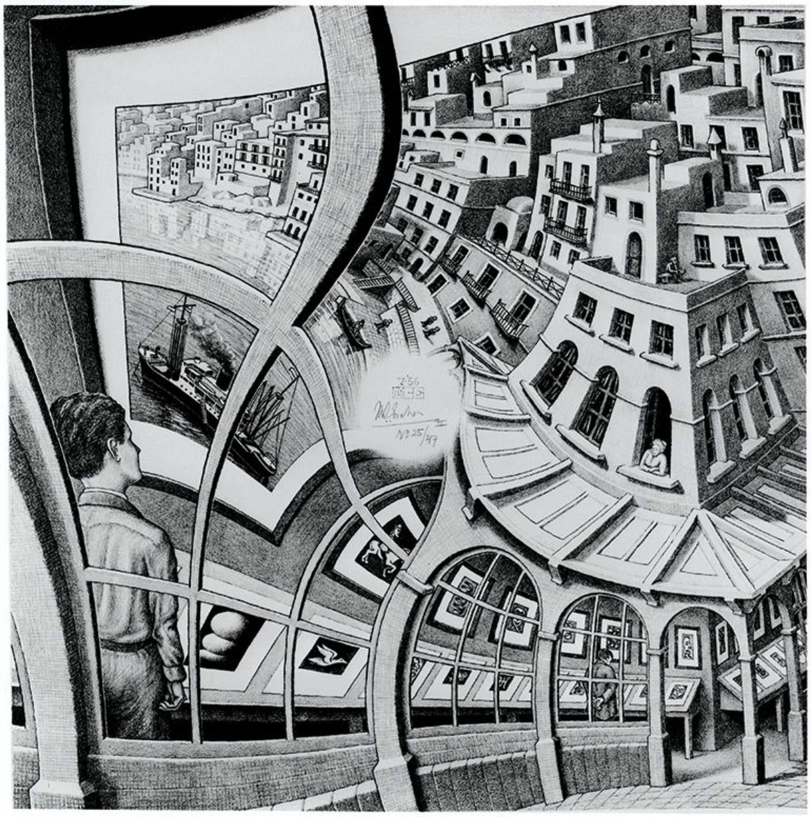
Bootstrap it

Not everyone was convinced. "I remember in the 60s, some senior physicist, a very good one actually, asked me what I'm working on now," says Polyakov. "I said, I'm trying to understand elementary particles by looking at a boiling kettle. I got a very strange look; obviously he thought that I'm a crackpot. Nobody believed that it was serious."

What he was suggesting was a shot in the dark. Mathematically, the 2D Ising model and the equations that govern the behaviour of elementary particles are linked by certain symmetries. In particular, at their critical points they share a property known as conformal symmetry, meaning that they look the same under conformal transformations – complicated mathematical functions that distort space but, within a small region, leave angles unchanged, as in M. C. Escher's famous *Print Gallery* drawing. If the same were true of the 3D Ising model, a complete mathematical description of it at its critical point might describe any other strongly coupled system with the same symmetries.

Polyakov's approach was certainly a radical one. Rather than start out with a sense of what the equations describing the particle system should look like, Polyakov first described its overall symmetries and other properties required for his model to make mathematical sense. Then, he worked backwards to the equations. The more symmetries he could describe, the more he could constrain how the underlying equations should look.

Polyakov's technique is now known as the bootstrap method, for its ability to pull itself up by its own bootstraps and generate knowledge from only a few general properties. "You get something out of nothing," says Komargodski. Polyakov and his colleagues soon managed to bootstrap their way to replicating Onsager's achievement with the 2D Ising



The distorted symmetry of Escher's *Print Gallery* is a feature of many physics problems

model – but try as they might, they still couldn't crack the 3D version. "People just thought there was no hope," says David Poland, a physicist at Yale University. Frustrated, Polyakov moved on to other things, and bootstrap research went dormant.

It remained that way until 2008, when Slava Rychkov, a physicist at CERN near Geneva, Switzerland, and the école Normale Supérieure in Paris, and his colleagues were beginning to wonder whether the Higgs boson – the particle thought to give mass to all other fundamental particles – might not actually exist. They were trying to build an alternative theory without the Higgs when they stumbled across the bootstrap method. "It was one of those lucky moments," says Rychkov. "We basically said, either we can try to tackle it using the bootstrap, or we will never be able to solve this problem." And solve it they did.

They proved to be on the wrong side of history: the Higgs boson was discovered at CERN in 2012. But the success reignited interest in bootstrap research. Simmons-Duffin got wind of Rychkov's work and, together with Poland, used the same technique to analyse mathematical functions describing how quantities such as the orientations of neighbouring atomic magnets are related. They hadn't actually set out to solve the 3D Ising model – but bizarrely, their work started to reproduce its characteristic features. "It seemed to know about the 3D Ising model," Simmons-Duffin says. "This was a big surprise."

He and Poland teamed up with Rychkov and others to find out how much more they might learn. Using bootstrap methods, they constrained key properties of the model over a thousand times more tightly than had ever been achieved before, while providing the first rigorous mathematical foundation for describing systems at their critical point. "For 30 years it was all based on voodoo," says Komargodski.

The achievement is impressive but raises its own questions, says Polyakov. "It's not obvious why it should be so precise. There's something hidden which we don't understand."

It's not just particle physics that could benefit from the bootstrap approach. The problem of modelling turbulent fluid flows, one of the thorniest in all of mathematics, might also be susceptible (see "Turbulence ahead"). Theorists are already applying the method to materials research, for example to probe the critical points that may be involved in high-temperature superconductivity. At present, the highest temperature at which any known material can conduct electricity without resistance is around -140 °C. The cost of cooling to such temperatures limits us to a few speciality applications, such as levitating trains that float above magnets made of superconducting coils. A superconductor that works at or near room temperature could enable lossless electricity transmission and cheap, powerful magnets, potentially revolutionising the power industry.

As yet we have no general theory of how these materials work, and no way to predict new combinations of elements that could be superconductors at even higher temperatures. But simulations suggest that high-temperature superconductors may show conformal symmetry at their critical points – making them amenable to the new approach.

A new realm

One practical problem remains: the bootstrap tends to generate predictions for appropriate combinations that are so precise that they outstrip our ability to create samples with sufficient purity or uniformity to test them. "It's like going from a Mercedes to a Rolls-Royce," says Subir Sachdev, a theoretical physicist at Harvard. But he is optimistic. "I don't think they quite have a home run yet," he says. "But I think they will."

Certainly some are betting big on the bootstrap's potential. This August, the Simons Foundation, a private organisation based in New York that funds maths and physics research collaborations, awarded $10 million to a group of physicists, including Rychkov, Komargodski, Poland and Simmons-Duffin, to build on bootstrap techniques. Top of their priorities is a complete catalogue of all theories that have conformal symmetry, which would serve as a road map of unsolved problems the bootstrap could tackle.

Simmons-Duffin also hopes the bootstrap can help to unify gravity with quantum mechanics. So far, no single theory has been able to couch Einstein's century-old general relativity in quantum language, but physicists haven't stopped trying. One of the major breakthroughs came in 1997, when it was shown that some theories of quantum gravity would acquire conformal symmetry if you recast them in one fewer spatial dimension. This result means that such theories could also be studied using bootstrap techniques.

Small wonder, then, that bootstrap researchers feel they could be on the cusp of a new realm of physics. But they are clear-eyed about the scope of their techniques. For a start, in many of the complex problems physicists grapple with now, the 3D Ising model is not enough. Questions such as how supercold fluids begin to flow without viscosity, or how to construct alternative theories of gravity, will require tackling more complicated variants of the Ising model, ones that span more dimensions and that have so far proved resistant to bootstrapping. The connection between boiling water and the forces that hold matter together might not be the only hidden pattern bubbling away beneath the surface of reality.

Turbulence ahead

In May 2000, the Clay Mathematics Institute in Peterborough, New Hampshire, published a list of seven particularly fiendish problems, and offered a million-dollar reward for the first correct solution to each. Only one of the Millennium prizes has so far been claimed, leaving six up for grabs.

One with particular practical significance concerns the Navier-Stokes equations, which describe the complex behaviour of fluid under turbulent conditions. They become difficult to solve at the transition from smooth to turbulent flow, says Zohar Komargodski of the Weizmann Institute of Science in Israel, because the fluid particles start interacting too vigorously to model.

This is another example of strong coupling of the sort that crops up in situations as diverse as particle physics, magnetism and boiling water (see main story). Some physicists believe that if we could understand strong coupling in one of these domains, cracking the Navier-Stokes equations could be next.

Komargodski notes that previous attempts to make connections between turbulence and this branch of physics have been unsuccessful, but there is still room for hope. “It might not be so different in the end,” he says.

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