Physics 12c: Problem Set 8

Due: Thursday, June 6, 2019

1. Problem 2 in chapter 9 of Kittel and Kroemer.

2. **Latent heat of melting**

   Consider a substance which can exist in either one of two phases, labeled A and B. If the number of molecules is N, the heat capacity of the A phase at temperature τ is

   $$C_A = N\alpha\tau^3, \tag{1}$$

   while the heat capacity of the B phase is

   $$C_B = N\beta\tau. \tag{2}$$

   (a) Assuming the entropy is zero at zero temperature, find the entropies σ_A, σ_B of the two phases at temperature τ.

   (b) Suppose that, for both the A phase and the B phase, the internal energy is $U_0 = N\epsilon_0$ at zero temperature. Find the internal energies U_A, U_B of the two phases at temperature τ.

   (c) Using the thermodynamic identity $dU = \tau d\sigma + \mu dN$, find the chemical potentials μ_A, μ_B of the two phases, expressed as functions of τ.

   (d) Which phase is favored at low temperature? At what "melting" temperature τ_m does a transition occur to the other phase?

   (e) At the melting temperature, how much heat must be added to transform the low-temperature phase to the high-temperature phase?

3. **Latent heat of BEC**

   Bose-Einstein condensation of an ideal gas may be regarded as a sort of first-order phase transition, where the two coexisting phases are the "liquified" condensate (the particles in the ground orbital) and the "normal" gas (the particles in excited orbitals). The purpose of this problem is to calculate the latent heat of this transition. Assume that the particles are spinless bosons with mass m. For this problem, you may wish to use

   $$\int_0^\infty dx \frac{x^\alpha}{e^x - 1} = \Gamma(1 + \alpha)\zeta(1 + \alpha). \tag{3}$$

   (a) As for a nonrelativistic ideal gas in the classical regime, the pressure P of a bosonic ideal gas is related to its internal energy and volume V by $P = \frac{2}{3}\frac{U}{V}$ even when the temperature τ is below the Einstein condensation temperature τ_E. Why?

(b) For a fixed particle number $N \gg 1$ and a nonzero temperature τ, the gas is uncondensed when its volume is large enough. But as the gas is compressed at constant temperature, Bose-Einstein condensation occurs at some critical concentration n^*. Find n^*.

(c) As we continue to compress the gas at constant temperature beyond the condensation concentration, the pressure remains constant as more and more of the gas liquifies. Find this coexistence pressure $P_{\text{coex}}(\tau)$.

(d) Condensation continues until all the particles are in the ground orbital, which occurs at essentially zero volume if the temperature is nonzero. Use the Clausius-Clapeyron relation to express the latent heat L of the transition in terms of the temperature τ. (The latent heat is the amount of heat released during the liquefaction of the gas at temperature τ.)

(e) Now check your expression for L by computing it another way. Calculate the work done as the gas is compressed from concentration n^* to zero volume, and calculate how the internal energy U changes as the gas is compressed. Use the first law to find the latent heat L.

4. **Boiling water on Mount Everest**

(a) The boiling temperature τ_0 of water at sea level is such that its vapor pressure matches the atmospheric pressure P_0. At what temperature τ_1 would water boil if the atmospheric pressure were P_1 instead? Treat the water vapor as a classical ideal gas, and assume that the latent heat of vaporization per particle L is independent of temperature. Also assume that the volume per particle in liquid water is negligible compared to the volume per particle in water vapor.

(b) Suppose that the atmosphere is an isothermal ideal gas of molecular nitrogen at 300°K. Find the boiling temperature of water at the top of Mount Everest. The latent heat of vaporization of water is $41\,\text{kJ}$ per mole, and Mount Everest is $8.8\,\text{km}$ high.