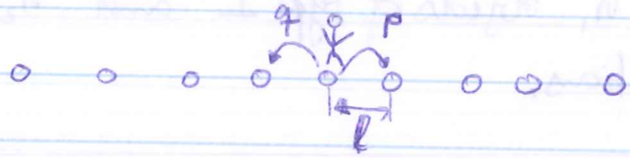
Recitation 2 – statistical physics

Probability and Ensembles

random walk in 1D



- N jumps
- probability to jump to the right is p
  " " " to the left is $q = 1-p$
- interested in the probability $P_N(m)$ to find the particle @ $x = m \cdot l$ after N jumps.
- Def: n_1 = # of jumps to the right, n_2 = # of jumps to the left $\Big\}$ $n_1 + n_2 = N$
- Probability for a particular sequence of n_1 jumps to the right and n_2 jumps to the left is given by:

$$\underbrace{p \cdots p}_{n_1} \cdot \underbrace{q \cdot q \cdots q}_{n_2}$$

- have to take into account equivalent configurations

$$\frac{N!}{n_1!\, n_2!}$$

understand this factor:

~~# of possibilities to order N objects of which are n_1 of typ I and n_2 of typ II~~

distribute n_1 objects of typ I and n_2 objects of typ II on N positions.

objects of the same type are indistinguishable.

→ probability, that the particle hops n_1 times to the right and n_2 times to the left is:

$$P_N(n_1) = \frac{N!}{n_1!\,n_2!}\, p^{n_1} q^{n_2}$$

$$= \frac{N!}{n_1!\,(N-n_1)!}\, p^{n_1} q^{N-n_1}$$

binomial distribution

$$(p+q)^N = \sum_{n=0}^{N} \frac{N!}{n!\,(N-n)!}\, p^n q^{N-n} = \sum \binom{N}{n} p^n q^{N-n}$$

together with $n_1 + n_2 = N$

and $n_1 - n_2 = m$

we obtain $P_N(m) = \dfrac{N!}{\left[\frac{1}{2}(N+m)\right]!\left[\frac{1}{2}(N-m)\right]!}\, p^{\frac{1}{2}(N+m)} q^{\frac{1}{2}(N-m)}$

Definition of important terms we encounter this year

System: part of physical reality under consideration

e.g. box of gas particles

later we distinguish between
<u>isolated</u> systems & <u>open</u> systems

- **Macrostate**: complete set of thermodynamic parameters of a system.

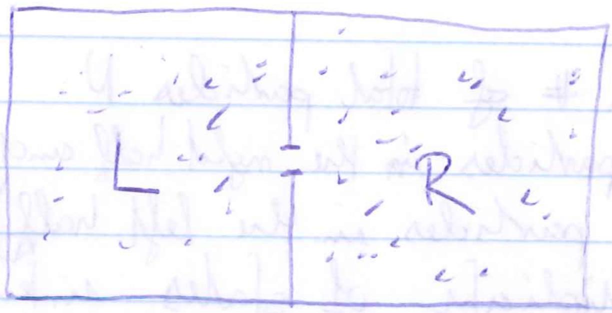
  e.g. pressure, temperature, etc.

- **microstate**: complete microscopic description of the system.

  e.g. spin system: know spin orientation of each site

$$\underset{1}{\uparrow}\;\underset{2}{\uparrow}\;\underset{3}{\uparrow}\;\underset{4}{\downarrow}\;\underset{5}{\uparrow} \quad \longleftrightarrow \quad \underset{1}{\uparrow}\;\underset{2}{\downarrow}\;\underset{3}{\uparrow}\;\underset{4}{\uparrow}\;\underset{1}{\uparrow}$$

different microstate



- box contains N noninteracting particles (distinguishable)
- divided into two equal parts, denoted by L & R
- particles can move back and forth between L & R in a statistically independent manner.

(a) if each ~~dist~~ distinguishable particle can have 2 states, L or R, depending on its position inside L or side R, what is the total number of different states of all particles considered together?

$$\# \text{states} = 2^{N}$$

N ← N-particles
2 ← 2 options

~~(b)~~ example : 3 particles ⇒ 2^3 different states

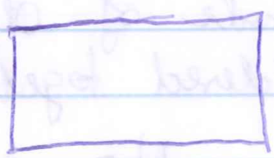
	particle 1	particle 2	particle 3
1	L	L	L
2	L	L	R
3	L	R	L
4	R	L	R
5	L	R	R
6	R ⊕R	L R	R L
8	R	R	R

✓

(b) For a given # of total particles N (with N_R particles in the right half and N_L particles in the left half), what is the <u>multiplicity</u> of states in terms of N, N_R & N_L?

$$\cancel{\frac{N!}{N_L!\,N_R!}}$$ ~~← divide out equivalent states~~

~~It does not matter how I enumerate the $N_{R/L}$ particles in the right/left box~~



$$\binom{N}{N_R} = \text{multiplicity}$$

$$\binom{N}{N_R} = \binom{N}{N-N_L} \;\hat{=}\; \binom{N}{N_L}$$

← symmetry of binomial coefficients

to find N_R - particles in the right and N_L particles in the left

$\hat{=}$ Lottery: have n particles with numbers attached to them from $1, \dots, n$ (distinguishable) draw N_R out of the N and place them in the right box!

(c) determine a quantity analog to spin excess!

$$\Delta N = N_R - N_L$$

(d) for $N = 10^{20}$, determine approximally the number of different states that have exactly the same number of particles on the two sides?

i.e. $N_R = N_L = 0.5 N$?

$$\binom{N}{\frac{N}{2}} = \frac{N!}{\frac{N}{2}!\left(N-\frac{N}{2}\right)!}$$

$$\# \equiv \underset{\text{with } N_R = N_L = \frac{N}{2}}{\#\text{ of states}} = \frac{N!}{\left[\left(\frac{N}{2}\right)!\right]^2}$$

$$\log \# = \log \frac{N!}{\left[\left(\frac{N}{2}\right)!\right]^2} = \log N! - 2\log\left(\frac{N}{2}\right)!$$

use Stirling approximation

$$\cong N\log N - \cancel{N} - \cancel{2}\frac{N}{\cancel{2}}\log\frac{N}{2} + \cancel{2}\frac{\cancel{N}}{\cancel{2}}$$

$$\Rightarrow \underline{\# \approx 2^N} \qquad \cong N\log N - N\log\frac{N}{2} = N\log 2$$

Multiplicity function of

N harmonic oscillators with total Energy

$E = n \cdot \hbar\omega$

$\uparrow$

n-quanta of Energy in the system

→ have to distribute n identical energy quanta on N identical particles

→ model N particles as the arrangement of N-1 walls

e.g. 3 particles ① | ② | ③

wall① wall②

→ Can arrange quanta and walls in a line

$(N-1+n)!$ possibilities

divide out $(N-1)!$ for identical particles

and $n!$ for identical energy quanta