

## Homework 3

**Charged pion decay:** In order to incorporate charged pions  $\pi^\pm$  in the Fermi theory of weak interactions, it is natural to add a term  $f_\pi p_\mu \pi^\pm$  to the charged current  $J_\mu^-$  (remember that pions couple derivatively). Here,  $p_\mu$  is the pion momentum and  $f_\pi$  is the pion decay constant. This leads to the following interaction Lagrangian:

$$\mathcal{L}_\pi = \frac{G_F}{\sqrt{2}} f_\pi p_\rho \pi^\pm \bar{\mu} \gamma^\rho (1 - \gamma_5) \nu_\mu + h.c.$$

Show that the total rate for  $\pi^- \rightarrow \mu^- \bar{\nu}_\mu$  decay is

$$\Gamma(\pi^- \rightarrow \mu^- \bar{\nu}_\mu) = \frac{G_F^2}{8\pi} f_\pi^2 m_{\pi^-} m_\mu^2 \left(1 - \frac{m_\mu^2}{m_{\pi^-}^2}\right)^2$$

where  $m_{\pi^-}$  (resp.  $m_\mu$ ) is the pion (resp. muon) mass.

**Kaon decay:** Predict the ratio of the  $K^- \rightarrow e^- \bar{\nu}_e$  and  $K^- \rightarrow \mu^- \bar{\nu}_\mu$  decay rates. Given that the lifetime of the  $K^-$  is  $\tau = 1.2 \times 10^{-8}$  sec and the  $K^- \rightarrow \mu^- \bar{\nu}_\mu$  branching ratio is 64%, estimate the decay constant  $f_K$ .