Dynamical Selection of the Cosmological 'Constant' in the Cyclic Model

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Abstract

The cyclic model resolves the homogeneity and flatness problems of big bang cosmology via a period of ultra-slow contraction rather than the accelerated expansion found in inflationary schemes. Patches of space in the four-dimensional effective theory undergo repeated big bangs, thus allowing the introduction of timescales far greater than the 14 billion years since the last one. The conditions imposed by the requirement of continued cycling allow the possibility of dynamical selection, as opposed to the anthropic selection commonly used in inflation, in which the cycling conditions act as a sieve which repeatedly eliminates extreme parameter values without reference to the presence of observers. In this paper we examine the various constraints set on the cosmological constant within the context of a cyclic model, then combine this with the concept of a dynamical relaxation mechanism for the cosmological 'constant' to propose a natural, non-anthropic explanation for its present value.

1 Introduction

The Big Bang model of cosmology asserts that the universe expanded according to the laws of General Relativity from a globally homogeneous[1] hot, dense initial state. From these assumptions follow many predictions which have been verified experimentally, ranging from a nearly linear relationship between the distance and velocities of galaxies relative to a typical observer (Hubble's law) to the primordial abundances of light elements (big bang nucleosynthesis) to the presence of black body radiation nearly uniform in temperature and intensity in all directions (the cosmic microwave background). The Big Bang model is, however, widely considered to be *incomplete*–some of its assumptions (e.g. homogeneity) are not generically well-motivated, while it fails to explain some experimental observations (e.g. dark energy or flatness). A number of proposals

[1] For clarity, we will use the term "homogeneity" to denote both homogeneity and isotropy throughout this paper.

have been made to address these inadequacies, including inflation [6] and the cyclic model [1].

Under the Big Bang model combined with the presence of dark energy, galaxy formation happens for only a limited period of time before dark energy comes to dominate the universe. In the cyclic model, though, about which much more will be said in the remainder of this paper, patches of space can undergo multiple big bangs, and thus potentially multiple periods of galaxy formation. This allows the possibility of so-called *dynamical selection*–if a given parameter can vary from place to place[2], then choices of the parameter value which lead to more galaxy formation will be favored.

In this paper, we apply the methods of dynamical selection to argue that the observed value of the cosmological constant Λ is a probable one for a typical observer in a cyclic universe. To do so, we rely on a model [2] in which Λ is not in fact constant but rather relaxes discretely over extremely long timescales compared to the duration of a typical cycle, spending an exponentially longer time at each successive value. Under this scenario, providing certain extremely general conditions are met, an area of space which decays fully will have spent the vast majority of its galaxy-producing lifespan with a cosmological constant just above the lowest value at which cycling is still viable. This sets a definite expectation value on the value of Λ that should be observed by a typical galactic observer. As a result, we argue, this mechanism provides a natural–and entirely non-anthropic–explanation for the present value of the cosmological constant, eliminating the need for fine-tuning.

The remainder of this paper is laid out as follows. In Section 2, we discuss in detail the inadequacies of the Big Bang model which motivated inflation and the cyclic model. In Section 3, we provide a brief overview of the cyclic model, focusing on its resolution of these inadequacies and the features that will be relevant in this paper. In Section 4, we discuss both anthropic and dynamical selection generally, as well as in the context of the cyclic model. In Section 5, we present Abbott's original model of dynamical relaxation of Λ without reference to the cyclic model, and show how it fails in the inflationary scenario. In Section 6, we combine cyclic cosmology with Abbott's model to investigate the most probable value of Λ. Finally, we conclude in Section 7.

2 Motivation for the Cyclic Model

2.1 The Friedmann–Lemaître–Robertson–Walker Metric and the Time Evolution of the Universe

As mentioned above, Big Bang cosmology is predicated on the assumption (sometimes referred to as the Cosmological Principle) that the universe is spa-

[2]Or, of course, from time to time–general relativity makes the distinction meaningless on large scales.

tially homogeneous. The resulting four-dimensional metric[3] can then be written as

$$ds^2 = dt^2 - a^2(t)d\vec{x}^2 \tag{1}$$

where $\vec{x}$ parametrizes the three spatial dimensions. This is the *Friedmann-Lemaître-Robertson-Walker metric*, first considered by Friedmann [3] in 1922. Note that the scale factor $a(t)$ is independent of the spatial variables by the homogeneity assumption. Adopting spherical coordinates, the metric can be rewritten as

$$ds^2 = dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right] \tag{2}$$

where the possible spatial curvatures are parametrized by k, which is 0 in a Euclidean (or flat) geometry, 1 in a spherical (or closed) geometry, and -1 in a hyperbolic (or open) geometry. We can solve the Einstein field equations[4]

$$G_\alpha^\beta = T_\alpha^\beta \tag{3}$$

using the FLRW metric (1) to find the *Friedmann Equations*

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3} - \frac{k}{a^2} \tag{4}$$

$$\frac{\ddot{a}}{a} = -\frac{(\varepsilon - 1)\rho}{3} \tag{5}$$

where H is the Hubble constant and ρ the energy density. Here $\varepsilon = \frac{3}{2}(1+w)$, and $w = \frac{p}{\rho}$ (with p denoting pressure) is 0 for matter and $\frac{1}{3}$ for radiation. Hence the universe is decelerating when $\varepsilon > 1$, which is the case for both matter ($\varepsilon = \frac{3}{2}$) and radiation ($\varepsilon = 2$). Differentiating (4), substituting the result for the left side of (5), and solving for ρ shows that

$$\rho = \rho_0 a^{-2\varepsilon} \tag{6}$$

with ρ_0 the current energy density.

2.2 The Homogeneity Problem

The universe has consistently been observed to be very nearly homogeneous–for example, through measurements of the distribution of galaxies [25] and of the near-homogeneity and isotropy of the cosmic microwave background (CMB) [4]. It would seem, then, that the Friedmann equations should provide a very good description of the dynamics of the universe. But consider the situation more closely. By definition the volume observed today, which has radius $L_{obs}(t_0) =$

[3]Throughout the remainder of the paper, unless otherwise noted, we work in Planck units with $\hbar = c = k = 8\pi G = 1$.

[4]For simplicity, we temporarily set $\Lambda = 0$ in the discussion of the Friedmann equations that follows.

H_0^{-1}, is equal to the volume of space that is in causal contact with us, with radius $L_{causal}(t_0)$: $\frac{L_{obs}(t_0)}{L_{causal}(t_0)} = 1$. The observable radius at some earlier time t_e, though, is scaled by the ratio of scale factors: $L_{obs}(t_e) = H_0^{-1}\frac{a_e}{a_0}$. The causal horizon, meanwhile, is just the scale factor multiplied by the comoving distance: $L_{causal}(t_e) = a(t_e)\int_0^{t_e}\frac{dt}{a(t)}$. But in a nearly flat universe $k = 0$, so we can use (4) and (6) to solve for $a(t)$:

$$a(t) \propto t^{\frac{1}{\varepsilon}} \tag{7}$$

Then, when $\varepsilon > 1$ [5]

$$\frac{L_{obs}(t_e)}{L_{causal}(t_e)} = \frac{H_0^{-1}\frac{a_e}{a_0}}{a(t_e)\int_0^{t_e}\frac{dt}{a(t)}} = \frac{\varepsilon t_0\frac{a_e}{a_0}}{t_e^{\frac{1}{\varepsilon}}\frac{\varepsilon}{\varepsilon-1}t_e^{1-\frac{1}{\varepsilon}}} = (\varepsilon-1)\frac{\frac{a_e}{t_e}}{\frac{a_0}{t_0}} = (\varepsilon-1)\frac{\dot{a}_e}{\dot{a}_0} \gg 1 \tag{8}$$

So at earlier times portions of the universe which are now homogeneous were not in causal contact! But we have just stated that the CMB is very nearly homogeneous, which means that the universe as a whole was homogeneous *in the past*, at the time of recombination when the CMB was formed. The big bang model is unable to explain this fact, since we have just seen that homogenization by causal contact is ruled out for volumes as large as the observable universe at that time. This volume discrepancy only gets worse going even further back in time: taking $\varepsilon = 2$,

$$\frac{L_{obs}(t_e)^3}{L_{causal}(t_e)^3} = \left(\frac{\dot{a}_e}{\dot{a}_0}\right)^3 = \left(\frac{t_e}{t_0}\right)^{-\frac{3(\varepsilon-1)}{\varepsilon}} = \left(\frac{T_e}{T_0}\right)^{3(\varepsilon-1)} \tag{9}$$

Plugging in temperatures at the grand unified theory (GUT) scale, the volume ratio becomes

$$\frac{L_{obs}(t_e)^3}{L_{causal}(t_e)^3} = \left(\frac{T_e}{T_0}\right)^{3(\varepsilon-1)} \approx \left(\frac{10^{15}\text{ GeV}}{10^{-12}\text{ GeV}}\right)^3 = 10^{81} \approx \left(e^{60}\right)^3 \tag{10}$$

This is the *homogeneity problem*: $\frac{\dot{a}_e}{\dot{a}_0}$ is roughly e^{60} times too large for the universe to have achieved homogeneity by causal contact after temperatures fell below the GUT scale.

2.3 The Flatness Problem

The first Friedmann equation (4) can be rewritten as

$$\frac{1}{2}a^2H^2 - \frac{\rho a^2}{6} = -\frac{k}{2} \tag{11}$$

[5] In fact, because of dark energy domination $\varepsilon < 1$ today–if dark energy is due to a cosmological constant (with $w = -1$), ε is approaching 0 as the proportion of energy density due to dark energy approaches unity. Accelerating expansion acts to exponentially decrease the comoving volume we observe. This does not spoil the overall argument, since the current rate of expansion is still much smaller than rates during the radiation-dominated period.

where the first term is identified with the kinetic energy of the universe and the second with the gravitational potential energy. Dividing both sides by $\frac{1}{2}a^2H^2$ and taking absolute values,

$$\left|-\frac{k}{a^2H^2}\right| = \left|1 - \frac{\rho}{3H^2}\right| = |1-\Omega| \tag{12}$$

so that the difference between 1 and $\Omega = \frac{\rho}{3H^2}$ measures the *deviation from flatness*. Results from WMAP [4], baryon acoustic oscillations [26], and supernovae [27] constrain the deviation from flatness today to be less than around one percent. But

$$|1-\Omega|_0 = |1-\Omega|_e \frac{a_e^2H_e^2}{a_0^2H_0^2} = |1-\Omega|_e \left(\frac{\dot{a}_e}{\dot{a}_0}\right)^2 \tag{13}$$

and at the GUT scale, $\left(\frac{\dot{a}_e}{\dot{a}_0}\right)^2 \approx \left(e^{60}\right)^2 \approx 10^{52}$. Since $|1-\Omega|_0 \leq 10^{-2}$,

$$|1-\Omega|_e \lesssim 10^{-54} \tag{14}$$

This is the *flatness problem*: flatness in the very early universe must have been incredibly finely tuned to produce the (lack of) curvature seen today.

3 An Introduction to the Cyclic Model

3.1 Resolving the Homogeneity and Flatness Problems

Both of the problems we have considered arise ultimately from the simple statement that

$$\frac{\dot{a}_e}{\dot{a}_0} \gg 1 \tag{15}$$

This means that the causal horizon and spatial curvature of the universe at earlier times, say at the GUT scale, must have been enormously smaller than they were today. There is, however, a simple way to resolve these problems: postulate that at some even earlier time t_i,

$$\frac{\dot{a}_i}{\dot{a}_0} \lesssim 1 \tag{16}$$

i.e. that although the universe was expanding much faster in the past than it is today[6], at some still earlier time it was expanding sufficiently slowly that the universe had time to come into causal contact, and that the universe at that time needed to be no flatter than it is today. This is the basic solution of both inflation and the cyclic model.

To see this, it is easiest to return to the first Friedmann equation (4), but allowing for the possibility of both matter and radiation, using the result of equation (6), and inserting an anisotropy term proportional to a^{-6}:

[6]Or, more precisely, than it was at the onset of dark energy domination.

$$H^2 = \frac{1}{3}\left(\frac{\rho_m^0}{a^3} + \frac{\rho_r^0}{a^4}\right) - \frac{k}{a^2} + \frac{\sigma^2}{a^6} \tag{17}$$

In an expanding universe ($H > 0$), the term with the smallest exponent in the denominator will dominate. Without any substances other than matter or radiation, the curvature will be the dominant term, contrary to observations. But now consider adding some other term $\rho_\varepsilon^0 a^{-2\varepsilon}$, perhaps a scalar field, with some ε. This term will dominate if $\varepsilon < 1$, or $w < -\frac{1}{3}$. This is, of course, the situation today–dark energy has just such properties, and this means that continued dark energy domination will bring the universe to a nearly flat and nearly homogeneous state[7].

On the other hand, consider allowing such a term to dominate for *a limited period of time*. This is the situation in inflation, which should ideally generate a period of acceleration lasting for at least 60 e-folds of expansion, bringing the universe to the required nearly flat and homogeneous state, and then cease, allowing recombination, nucleosynthesis, and the other predictions of Big Bang cosmology to proceed as normal. There are many ways of bringing this about, as well as a number of serious problems with inflationary schemes, but these will not be important in this paper.

Alternatively, instead consider a *contracting* universe. Then the dominant term will be the anisotropies, a situation that will lead to so-called *chaotic mixmaster* behavior [7] culminating in a *maximally* inhomogeneous and anisotropic universe! But, as above, we can add a new term, this time with $\varepsilon > 3$, or $w > 1$. Instead of the ultra-rapid expansion of inflation, the homogeneity and flatness problems are now solved by the ultra-slow contraction of the so-called *ekpyrotic mechanism*, the key ingredient of the cyclic model.

3.2 The Cyclic Potential and the Timeline of a Bounce

In the cyclic model, the universe does not begin at a Big Bang: rather, it undergoes multiple bangs in succession. The cyclic model was first formulated in the context of *heterotic M-theory*. In this picture, the universe is 11-dimensional, but six of the spatial dimensions are rolled up and inaccessible below the GUT length scale. In five dimensions, then, the universe consists of two 4-dimensional boundary branes separated by a fifth, finite dimension whose length can vary with time.

In the periods we are interested in, when φ, our scalar field, dominates the universe, the Friedmann equations (4, 5) can be written as

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\varphi}^2 + V(\varphi)\right) \tag{18}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{3}\left(\dot{\varphi}^2 - V(\varphi)\right) \tag{19}$$

[7] The original cyclic proposal [1] in fact relies on a period of dark energy domination to smooth the universe before a bounce, as does Roger Penrose's Conformal Cyclic Cosmology [5].

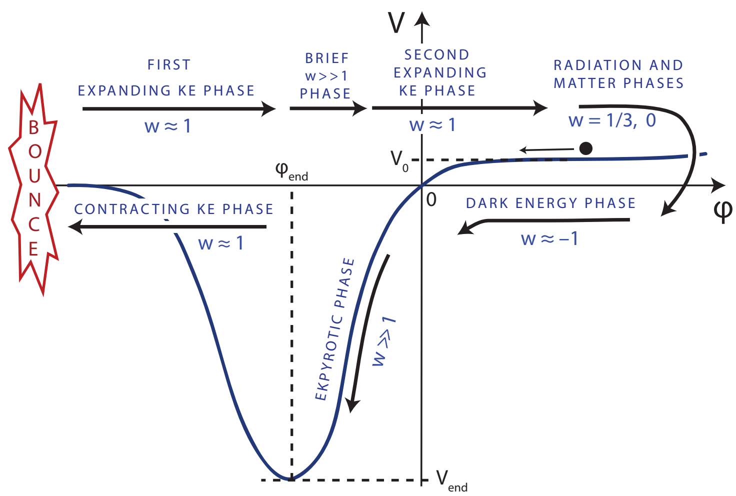


Figure 1: An example form of the cyclic potential. The only essential features are the shallow, increasing plateau at some positive V_0 and the minimum at some negative V_{end}; the potential need not approach 0 at $-\infty$. Reproduced from [8].

where $V(\varphi)$ is the *cyclic potential*, as shown in Figure 1.

Only two features are required of the cyclic potential. First, it must have a region where it rises with large negative curvature, i.e. $V''/V \gg 1$. This represents the ekpyrotic portion of the potential, where the universe is smoothed and the fluctuations visible in the CMB are generated. Second, it must have a shallow ($V''/V \ll 1$) plateau rising to some positive height V_0, which we identify with Λ, the cosmological constant. It is the field rolling down this plateau which gives rise to the era of dark matter domination.

For the purposes of our analysis, we will adopt the specific model

$$V(\varphi) = V_0 \left(1 - e^{-c\varphi}\right) F(\varphi) \tag{20}$$

for some positive c, with $c \approx 10$ from observations[8], and where $F(\varphi)$ goes to 0 super-exponentially as $\varphi \to -\infty$ but approaches 1 past the potential minimum φ_{end}; e.g. we might have

$$F(\varphi) = e^{-e^{-\gamma\varphi}} \tag{21}$$

for some $\gamma > 0$. This specific form for V was chosen in the original formulation of the cyclic model [10] because, in addition to exhibiting the two required features, it behaves purely exponentially in the regions of interest where $F(\varphi)$ can be neglected, making it very easy to analyze and allowing for near-exact solutions.

Now consider the timeline of a complete cycle. For reasons of convention, the location of the bounce is placed at $\varphi = -\infty$, so that following a bang φ first increases, than turns around and decreases back towards $-\infty$. Similarly, the bounce itself is at $t = 0$, and t increases from negative time values. At the onset of dark energy domination the field reaches its rightmost value φ_c and the turnaround occurs, when it begins to roll down the plateau. In this region the potential can be approximated $V(\varphi) \approx V_0(1 - e^{-c\varphi})$, so that the number of e-folds of inflation N_{DE} is given by the *slow-roll formula*

$$N_{DE} \approx \int_0^{\varphi_c} \frac{V(\varphi)}{V_\varphi} d\varphi = \int \frac{1 - e^{-c\varphi}}{ce^{-c\varphi}} \approx \frac{e^{c\varphi_c}}{c^2} \tag{22}$$

assuming that $e^{c\varphi_c} \gg 1$ [9].

After φ leaves the slow-roll regime, $V(\varphi)$ can be approximated by $-V_0 e^{-c\varphi}$. The Friedmann equations can be solved in this regime to find the *scaling solution* [9]

$$\varphi(t) = \frac{2}{c} \ln\left(-t\sqrt{\frac{c^2 V_0}{2}}\right) \tag{23}$$

$$a(t) = (-t)^{\frac{2}{c^2}} \propto e^{\frac{\varphi(t)}{c}} \tag{24}$$

[8]See equation 109 on page 27 for more precise constraints.

[9]This will be *required* for continued cycling to be viable under the entropic mechanism (see subsection 4.2.1 on page 14), so it is not a problematic assumption.

Then we can easily find $H(t)$ as well:

$$H(t) = \frac{\dot{a}(t)}{a(t)} = \frac{2}{c^2 t} \propto e^{\frac{-c\varphi(t)}{2}} \tag{25}$$

Since $c^2 \gg 1$, decreasing φ decreases a by only a small amount but increases the magnitude of H drastically. Hence the fractional energy densities $\Omega_k = a^{-2}H^{-2}$ and $\Omega_\sigma = a^{-6}H^{-2}$ in curvature and anisotropy, respectively, both drop sharply. As desired, the ekpyrotic mechanism has homogenized and flattened the universe; homogeneity (10) requires that $\dot{a} = aH$ must grow by at least 60 e-folds during this period.

Constraints on the variance of spatial curvature perturbations [9] require that the time between the end of the ekpyrotic phase and the bounce $|t_{end}| \approx 10^3$, and that the magnitude of the potential minimum $|V_{end}| \approx 10^{-16}$, the GUT scale. In addition, from (23) $V = \frac{2}{c^2 t^2}$. So

$$|t_{beg}| = \sqrt{\frac{|V_{end}|}{V_0}}\, |t_{end}| \approx 10^{57} \approx 5 \cdot 10^{13} s \approx 1 \text{ million years} \tag{26}$$

Again, homogeneity requires that

$$\frac{\dot{a}(t_{end})}{\dot{a}(t_{beg})} > e^{60} \tag{27}$$

But since $c^2 \gg 1$, $\dot{a} = aH \propto t \propto V^{\frac{1}{2}}$, so there is a bound on the number of e-folds of ekpyrosis N_{ek}:

$$N_{ek} \equiv \frac{1}{2} \ln \left(\frac{|V_{end}|}{V_0} \right) > 60 \tag{28}$$

and as a result an upper bound on V_0 and thus Λ:

$$V_0 \lesssim e^{-120} |V_{end}| \approx 10^{-68} \approx (100 \text{ GeV})^4 \tag{29}$$

taking $|V_{end}|$ of order the GUT scale. This is the largest possible value of Λ, the cosmological constant, consistent with the observed homogeneity of the universe.

As φ falls below φ_{end}, the effects of $F(\varphi)$ become important and take V to 0, so that kinetic energy becomes dominant. It is easy to solve the new Friedmann equations in the *kinetic phase*

$$H^2 = \frac{1}{6}\dot{\varphi}^2 \tag{30}$$

$$\frac{\ddot{a}}{a} = -\frac{1}{3}\dot{\varphi}^2 \tag{31}$$

to find

$$\varphi(t) = \sqrt{\frac{2}{3}} \ln(-t) + \hat{\varphi} \tag{32}$$

$$a(t) = \hat{a}\,(-t)^{\frac{1}{3}} \tag{33}$$

with $\hat{\varphi}$, $\hat{a}$ integration constants. Clearly as $t \to 0$ $\varphi \to -\infty$, $a \to 0$, as expected–i.e. the coordinates become singular. This does not, however, mean that the branes themselves experience singularities. We can transform to the *non-singular variables* [10]

$$a_0 = 2a \cosh\left(\frac{\varphi}{\sqrt{6}}\right) \tag{34}$$

$$a_1 = -2a \sinh\left(\frac{\varphi}{\sqrt{6}}\right) \tag{35}$$

where a_0 and a_1 are the scale factors on the two branes. As $\varphi \to -\infty$, $a \to 0$, both a_0 and a_1 go to the nonzero value of $\hat{a}e^{-\frac{\hat{\varphi}}{\sqrt{6}}}$, so that an observer on either brane would not experience a singularity.

Discussion of the details of the bounce is beyond the scope of this paper (but see [9, 10] for extensive discussions). For our purposes all that is required is that the scalar field receive a slight kick during the bounce in order to compensate for the Hubble damping of the radiation created at the bang. This can be achieved in several ways, for example [10] by producing slightly more matter on the negative-tension brane than the positive one.

After the bounce, as can be seen in Figure 1, the universe passes through two further kinetic energy dominated phases and a brief, unimportant period [8] with $w \gg 1$. It is convenient to treat all three kinetic phases as a single phase with net e-folds of growth $N_{kin} = \frac{2\gamma}{3}$, where [8]

$$\gamma = \ln\left(\frac{|V_{end}|^{\frac{1}{4}}}{T_r}\right) \tag{36}$$

and T_r is the temperature at radiation domination. Finally, there are the familiar periods of radiation and matter domination, during which there are roughly $N_{rad} \approx \left(\frac{T_r}{T_0}\right)$ e-folds of expansion. Then the total number of e-folds of expansion over a cycle is

$$N_{tot} = N_{DE} + N_{kin} + N_{rad} \tag{37}$$

Meanwhile, the exponential decrease in H during the radiation and kinetic dominated phases is roughly canceled by its rapid increase during ekpyrosis, so that the Hubble distance remains unchanged over a cycle [8].

4 Dynamical and Anthropic Selection

4.1 Anthropic Selection

4.1.1 An Example: The Anthropic Argument for Λ

In 1987, before it had been reliably measured, Weinberg used anthropic arguments to make a prediction [11] that the cosmological constant Λ should be small (no more than a few orders of magnitude larger than the matter density of the universe) rather than of order one. His argument has become perhaps the most famous claimed success of the anthropic principle, so we reproduce it here in some detail.

The basic intuition is that in order for observers to be present, Λ must be small enough for sufficiently large gravitationally bound systems to be formed. Here "sufficiently large" means large enough to allow both stars and planets around those stars, which in turn requires the existence of heavy elements. So the sufficiently large structures are galaxies, or perhaps globular clusters. Weinberg realized that this is the only way for the value of Λ to affect the presence of observers, since the exponential acceleration caused by a cosmological constant is incapable of breaking up gravitationally bound systems once they have formed. Hence there will be a point beyond which galaxies and thus observers cannot form, resulting in a definite upper bound on Λ. This bound will be relevant in any situation where the value of Λ can vary: examples might include a scalar field controlled by a potential like the one in Abbott's model, considered below, or the multiverse picture produced by eternal inflation, in which different bubble universes might have different values of fundamental constants. If these mechanisms *a priori* allow values of Λ both above and below this bound, the bound will have provided useful information by restricting which values are actually observable.

More quantitatively, consider a density perturbation of strength $\rho\delta$ (where we are adopting the notation of [12]), with $\delta \ll 1$, in a universe with cosmological constant $\Lambda > 0$. The first Friedmann equation (4) can now be written as

$$\dot{a}^2 + k = \frac{a^2}{3}\left(\rho(1+\delta) + \Lambda\right) \tag{38}$$

Here it will be convenient to normalize a, rather than k as above, so that instead of being set equal to -1, 0, or 1, k now denotes the radius of curvature today, and a is scaled such that $a_0 = 1$. We can solve the equation of motion to first order as $t \to 0$, and characterize the density perturbation by its strength at recombination $\rho_R\delta_R^3$, so that [11]

$$k = \frac{5}{9}a^2\left(\rho(1+\delta)^{\frac{2}{3}}\rho_R^{\frac{1}{3}}\delta_R\right) \tag{39}$$

In order for a perturbation to undergo gravitational condensation at its minimum, when $\rho(1+\delta) = 2\Lambda$, the right hand side of (38) must be less than k, or

$$\Lambda^{\frac{1}{3}}\left(\frac{\rho}{2}(1+\delta)\right)^{\frac{2}{3}}a^2 < k \tag{40}$$

Substitution for k gives the upper bound on Λ,

$$\Lambda < \frac{500\rho_R\delta_R^3}{729} \tag{41}$$

Now consider the time t_c required for the scale factor a of the density perturbation to reach its maximum and then recollapse back to $a = 0$. This will be smallest when $\Lambda \to 0$, so solving (38) in this case gives [11]

$$t_c > \frac{9\pi}{5\sqrt{5\rho_R\delta_R^3}} \tag{42}$$

In addition, the observed redshift z_c corresponding to the time t_c–that is, the largest redshift at which galaxies (or, equivalently quasars) are observed–will be greatest when $\Lambda \to 0$ [11]:

$$t_c < \frac{2}{3}\sqrt{\frac{3}{\rho_0\left(1+z_c\right)^3}} \tag{43}$$

where ρ_0 is the mass density (including dark matter) today. So the bound (41) can be rewritten:

$$\Lambda < \frac{\pi^2}{3}\rho_0\left(1+z_c\right)^3 \tag{44}$$

Taking $z_c \approx 5$ this predicts $\Lambda \lesssim 700\rho_0$, which only overestimates Λ by 2-3 orders of magnitude instead of 120.

Instead of predicting only that Λ should saturate its upper bound, we may further adopt the so-called *principle of mediocrity*, which states that we should expect to find ourselves in a universe that is typical of those that support life. This is just the statement that the probability $dP_{obs}(\Lambda)$ of an observer measuring a value of the cosmological constant between Λ and $\Lambda + d\Lambda$ is

$$dP_{obs}(\Lambda) = N(\Lambda)P(\Lambda)d\Lambda \tag{45}$$

where $P(\Lambda)$ is some *a priori* probability, independent of the existence of observers, of a universe with cosmological constant Λ being formed, and $N(\Lambda)$ is the number of observers that form in universes with such a Λ. Weinberg made the simplifying observations that there is only a narrow range of values of Λ that support observers, such that $P(\Lambda)$ is constant in that range, and that $N(\Lambda)$ is proportional to the total number of baryons within galaxies in that universe. The resulting problem then becomes tractable: one way to calculate $N(\Lambda)$ is to use the *spherical infall method*, which assumes a slight positive density fluctuation within a spherical region compensated by a slight underdensity in a shell outside it to represent the infall of matter into the region. Using this method and the assumption of Gaussian initial fluctuations, $P_{obs}(\leq \Lambda) =$ [28, 12]

$$1+(1+\beta)\,e^{-\beta}+\frac{1}{2\ln 2-1}\int_{\beta}^{\infty} e^{-x}dx\left\{-2\sqrt{\beta x}+\beta+2x\ln\left(\sqrt{\frac{\beta}{x}}+1\right)\right\} \quad (46)$$

with $\beta = \frac{1}{2\sigma^2}\left(\frac{729\Lambda}{500\rho_R}\right)^{\frac{2}{3}}$, ρ_R the average mass density at recombination, and σ the root-mean-square fractional density perturbation at recombination. Depending on parameter values, Weinberg [12] reports the probability of a cosmological constant at least as small as the actually-measured Λ at 5-12%. This number, however, depends on the assumption that the behavior of $P(\Lambda)$ is constant in the small range being considered and can thus be ignored. In cases where this is not true, such as a scalar field with a power-law potential, $P_{obs}(\leq \Lambda)$ can be highly sensitive to the shape of $P(\Lambda)$ within the range [13].

4.1.2 Conceptual and Theoretical Problems with Anthropic Selection

The argument given above captures the essential features of arguments based on the Anthropic Principle (though more complicated extensions [14] have also been considered). Certainly, the result of the argument seems impressive–it's much better than the *a priori* prediction that $\Lambda \approx 1$ in Planck units. But the argument as formulated raises a number of problems, which we will consider here in turn.

Smolin [15] notes that the anthropic portion of the argument is strictly unnecessary to come to a conclusion about the magnitude of Λ. To see this, we can separate the argument above into three steps. First, we claim that anthropic considerations require the existence of galaxies–i.e. galaxies are necessary for observers to exist. Second, we note that, given our theory of structure formation, Λ must not be larger than a specific value in order for galaxies to form. Then, since galaxies have in fact formed, we assert that Λ is below the upper bound that we have found.

Assuming each of our assumptions are correct, this argument is correct as well–but it would be correct even if the first step was not. *Because we observe the existence of galaxies, we know, tautologically, that there must have been a time when the formation of galaxies was allowed.* This would be true *even if galaxy formation is not necessary for the existence of observers* (which, in fact, it may not be). Note, also, that if we had found a value of Λ larger than the anthropic upper bound, this would falsify the second step, and therefore our theory of structure formation, rather than the first step. If we only use it to make arguments of this type, the anthropic principle is unfalsifiable.

By contrast, the second part of the argument, which employs the principle of mediocrity to argue that we should be near the center of some probability distribution, is, at least in principle, falsifiable. But it contains two further assumptions which can be easily challenged. First, the final result depends critically on the choice of probability distribution. But only one point on this distribution can be observed–namely, our own universe. In the absence of a fundamental theory, which would itself presumably render anthropic arguments moot, it is easy to construct other plausible distributions, or just change parameter values, to arrive at drastically different answers.

More important, though, is the second assumption–that *only* Λ *should be allowed to vary.* We know, for example, that increasing Q, the amplitude of primordial density fluctuations, would produce universes with exponentially greater structure and volume, and that Q can be increased by $2-3$ orders of magnitude without affecting anthropic viability [16]. If Q and Λ are allowed to vary simultaneously, then the probability of living in a universe with the observed parameter values is reduced significantly, perhaps to $10^{-3} - 10^{-4}$ if anthropic constraints on Q are also considered [29, 17]. Even if this were not the case, it would remain true that the most natural way to arrive at a probability distribution would be to allow *all possible parameters*, including coupling constants, particle masses, etc. to vary simultaneously. If an anthropic argument only considers varying one parameter at a time, then it is essentially making the assumption that all other parameters are held constant. This leads to a serious problem of parsimony, since making a successful prediction requires many inputs for only one output.

4.2 Dynamical Selection

4.2.1 An Example: N_{DE} from the "Phoenix Universe"

The original cyclic model [1], which contained only one scalar field φ, had what seemed like the attractive property that cycling was a *dynamical attractor*. This ensured that the behavior of brane collisions was both regular and predictable, with little sensitivity on initial conditions or the detailed physics of the bounce. However, such a model proved unable [18] to generate the necessary scale-invariant spectrum of curvature perturbations required by observations of the CMB. It was realized, though, that a so-called *entropic mechanism* [19] with *two* scalar fields φ_1, φ_2 and a potential $V(\varphi_1, \varphi_2)$ would generate scale-invariant *entropic* perturbations before the bounce that would then be converted to the necessary curvature perturbations after it.

The details will not be considered here, but it is important to note that with the addition of a second scalar field the cyclic solution is no longer an attractor; rather, the solution exhibits a so-called *tachyonic instability* [20], with the result that without exponential fine-tuning of initial conditions most of the universe fails to complete the ekpyrotic period and instead falls into the singularity-laden and highly inhomogeneous chaotic mixmaster behavior.

This would seem to be a fatal problem–only an exponentially tiny portion of the universe manages to bounce and return to a homogeneous and isotropic state! However, let us consider the situation more carefully. In the effective low-dimension theory, the full potential is [21] $V(\varphi_1, \varphi_2) = V_{ek}(\varphi_1, \varphi_2) + V_{rep}(\varphi_2)$, where V_{rep}, a repulsive potential which corresponds to the negative-tension brane being repelled by a singularity, is not relevant to us here and where V_{ek} is just the two-dimensional generalization of the ekpyrotic potential considered earlier:

$$V_{ek} = V_0 - V_1 e^{-c_1\varphi_1} - V_2 e^{-c_2\varphi_2} \tag{47}$$

During the ekpyrotic phase the trajectory is given by the two-dimensional form of the scaling solution (23),(24) discussed earlier:

$$\varphi_i(t) = \frac{2}{c_i}\ln\left(-t\sqrt{\frac{c_i^2 V_i}{2}}\right) \tag{48}$$

$$a(t) = (-t)^p \tag{49}$$

with $p = p_1 + p_2 \ll 1$, $p_i = \frac{2}{c_i}$. Changing variables [21] to point parallel and transverse to the trajectory, respectively:

$$\sigma = \frac{c_2\varphi_1 + c_1\varphi_2}{\sqrt{c_1^2 + c_2^2}} = \sqrt{2p}\ln\left(-t\sqrt{\frac{V_0}{p(1-3p)}}\right) \tag{50}$$

$$s = \frac{c_2\varphi_1 - c_1\varphi_2}{\sqrt{c_1^2 + c_2^2}} = 0 \tag{51}$$

where φ_1, φ_2 have been shifted such that s is zero on the ridge followed by the exact cyclic solution (rather than merely constant) and that $V_0 = \left(1 + \frac{p_2}{p_1}\right) V_1$. Then the potential around the cyclic trajectory can be expanded to find [21]

$$V_{ek} = V_0\left(1 - e^{-\sigma\sqrt{\frac{2}{p}}}\left(1 + \frac{s^2}{p} + ...\right)\right) \tag{52}$$

It is clear that V_{ek} is unstable in the s direction: in fact

$$|s\left(t_{end}\right)| \lesssim p \tag{53}$$

is required, where t_{end} is the end of ekpyrosis, as in Section 3.2, in order for the fields to stay on the cyclic trajectory for the entire ekpyrotic period. If the fields are near the trajectory, this means that almost all of the contribution to the total energy density comes from σ, so that

$$0 = \frac{\dot{s}^2}{2} + \left.\frac{\partial^2 V_{ek}}{\partial s^2}\right|_{s=0}\frac{s^2}{2} = \frac{\dot{s}^2}{2} - \frac{s^2}{t^2} \tag{54}$$

using (52) and (50). Then the equation of motion is

$$\ddot{s} - \frac{2}{t^2}s = 0 \tag{55}$$

which has a growing mode solution that behaves as t^{-1}, i.e. as $H(t)$ in the scaling solution (25). So during ekpyrosis s will grow by the same number of

e-folds as H–that is, staying on the cyclic trajectory until the end of ekpyrosis requires

$$|s(t_{beg})| \lesssim pe^{-N_{ek}} \tag{56}$$

where (28)

$$N_{ek} = \frac{1}{2}\ln\left(\frac{|V_{end}|}{V_0}\right) \approx \frac{1}{2}\ln\left(10^{108}\right) \approx 124 \tag{57}$$

if $|V_{end}|$ is at the GUT scale $\left(10^{15}\text{ GeV}\right)^4 \approx 10^{-16}$.

Now, consider some volume of space that has survived the entire ekpyrotic period. This volume expands (37) by $e^{N_{tot}} = e^{\frac{2}{3}\gamma + N_{rad} + N_{DE}}$ over a cycle, but only $e^{-N_{ek}}$ of it will survive through the next ekpyrotic period. This means that the condition for net growth over a cycle is [21]

$$N_{de} > N_{ek} - \frac{2}{3}\gamma - N_{rad} \tag{58}$$

If the value of N_{de} can vary spatially or temporally, net growth will occur only when this inequality is met–that is, regions with N_{de} higher than this critical value will be *dynamically selected*. Now the origin of the term *phoenix universe* is clear: although only an exponentially small portion of the volume survives, its subsequent growth is sufficient to more than recreate the original volume.

4.2.2 Dynamical Selection, the Phoenix Universe, and the Concept of a Sieve

It is clear that the preceding argument is quite different in character than the anthropic argument above. Most obviously, there is no role in it for observers–the cutoff for N_{DE} is based only on the condition for continued cycling, or more precisely for continued volume expansion. It is simple to add an anthropic element, by noting that it is hard to imagine the existence of an observer in a region of the universe with chaotic mixmaster properties, so that observers are overwhelmingly likely to measure N_{de} greater than the critical value[10], but this is not an essential feature of the argument. Furthermore, the conclusion is in principle falsifiable: although we cannot measure N_{de} directly at this time, the cyclic model predicts there should be some non-zero (but probably very small) value of $\dot{\Lambda}$ that a sufficiently sensitive experiment could measure and relate to N_{de}.

The defining characteristic of dynamical selection, as in the phoenix universe, is that it happens not once, but many times in succession. This is in sharp contrast to the usual application of anthropic selection, which is typically invoked in the landscape of *eternal inflation*, in which different bubble universes with different parameter values can nucleate in the expanding backdrop of the

[10]They are not *certain* to be found there, because a region of space with these properties will not become hostile to life until it has passed through the ekpyrotic phase for the first time.

inflaton field. It is difficult to define a probability distribution on this landscape because differentiating among the infinite number of universes that are produced is often highly dependent on the specific choice of one of many different viable *volume measures*. In the context of the cyclic model, though, we have a natural way to eliminate parameter values or construct a probability distribution: wait for an arbitrarily long time and then see what choices produce a universe which is still viable.

In particular, if we now restrict ourselves to considering universes that match our observations–that, for example, contain galaxies–viable cyclic universes should be generically preferred over inflating ones, since all inflating universes can undergo *only one* period of galaxy formation before they either undergo heat death or recollapse. By contrast, viable cyclic universes can undergo very many phases of galaxy formation–perhaps even an infinite number–and there is the potential for each subsequent phase to produce a greater number of galaxies than the last.

Ideally, then, we can think of the phoenix universe as acting like a sieve. Just as in inflation, we can consider a complex parameter landscape, which is a generic prediction of the string-theoretic principles that the cyclic model is rooted in. In inflation, this landscape remains static, once set. But in the phoenix universe, areas of space with unfavorable parameter values fail to have the necessary characteristics for cycling, so they collapse into a chaotic mixmaster solution. This happens multiple times, over timescales far longer than a typical cycle length, so that even slightly unfavorable parameter values will eventually become highly disfavored. If we follow this model to its logical conclusion, it is not unreasonable to imagine finding natural explanations for all the parameter values measured today, without the need for anthropic considerations.

5 Abbott's Model for Dynamical Relaxation of Λ

The cosmological constant is measured to be [22]

$$\Lambda = \rho_c \Omega_\Lambda \approx 7.21 \cdot 10^{-30}\ \frac{\mathrm{g}}{\mathrm{cm}^3} \approx (2.36\ \mathrm{meV})^4 \approx 1.40 \cdot 10^{-123} \tag{59}$$

where ρ_c is the critical energy density at which the universe has zero curvature. This is problematic because Λ should receive contributions proportional to the fourth powers of various mass scales, including the electroweak and QCD mass scales, but these terms are much larger (on the order of MeV or GeV) than the observed value of Λ. Arriving at such a small value only by the summation of much larger terms would require massive fine-tuning.

To remedy this problem, Abbott [2] introduced a new scalar field φ weakly coupled to some gauge field F that only couples to ordinary matter through gravity and is hence essentially undetectable. Before symmetry breaking the scalar field has the classical shift symmetry

$$\varphi \to \varphi + \text{constant} \tag{60}$$

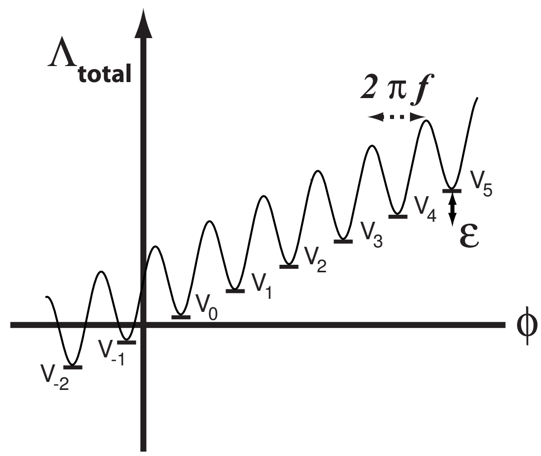


Figure 2: The cosmological constant for a washboard potential. Λ_{tot} moves down the potential in discrete steps via tunneling by bubble nucleation, spending an exponentially longer time at each successively lower step. Reproduced from [23].

and the coupling

$$\frac{\alpha_\varphi}{4\pi}\frac{\varphi}{f_\varphi}\varepsilon^{\mu\nu\alpha\beta}\mathrm{Tr}\left\{F_{\mu\nu}F_{\alpha\beta}\right\} \tag{61}$$

where α_φ is the coupling constant at the Planck scale and f_φ is some high energy mass scale. The addition of non-perturbative quantum corrections to the classical potential gives an overall potential [2]

$$V_1(\varphi) = -\Lambda_\varphi^4 \cos\left(\frac{\varphi}{f_\varphi}\right) \tag{62}$$

where the symmetry has been broken to the discrete subgroup

$$\varphi \to 2\pi N f_\varphi \tag{63}$$

Λ_φ^4, which characterizes the coupling strength of F, must be smaller than the observed value of Λ, but this is technically natural. In QCD with six flavors, for example, [23]

$$\Lambda_{QCD} = e^{-\frac{2\pi}{7\alpha_{QCD}}} \tag{64}$$

A coupling of this same form, but with $\alpha_\varphi = \frac{1}{80}$, results in $\Lambda_\varphi^4 \approx 2\cdot 10^{-125}$, which is adequately small. A second term is now added to tilt the cosine potential and break the discrete symmetry, producing a "washboard:"

$$V_2(\varphi) = \varepsilon\frac{\varphi}{2\pi f_\varphi} \tag{65}$$

where $\varepsilon < \Lambda_\varphi^4$ is required. So ε is naturally small, with no fine-tuning necessary because, since it breaks a symmetry, all radiative corrections are proportional to ε [2]. $V_2(\varphi)$ need not be linear–in fact, all that is required is that it have no minima over the relevant range of φ–but we choose a linear form for simplicity. The total vacuum energy is then given by

$$\Lambda_{tot} = V(\varphi) = -\Lambda_\varphi^4 \cos\left(\frac{\varphi}{f_\varphi}\right) + \varepsilon\frac{\varphi}{2\pi f_\varphi} + V_{other} \tag{66}$$

where V_{other} aggregates all of the other contributions to Λ, presumably of order 1. A plot of the potential is shown in Figure 2. Note that, if $\varepsilon \ll \Lambda_\varphi^4$, local minima are at $\varphi_N \approx 2\pi N f_\varphi$, so that

$$V_N \approx N\varepsilon - \Lambda_\varphi^4 + V_{other} \tag{67}$$

and the values of Λ_{tot} are spaced by ε. Then there must be some minimum V_0 such that $0 \leq V_0 < \varepsilon$. When Λ_{tot} drops below zero in a particular area of space, that region becomes anti-De Sitter, shrinking rather than growing, and collapses into a black hole.

Assume that the universe is at some large initial value of Λ_{tot}, perhaps of order 1. Then because of the washboard potential φ will gradually roll downwards, but with different behavior in different energy regimes. When the Hawking temperature [2]

$$T_H = \sqrt{\frac{2V_N}{3\pi}} \gtrsim \Lambda_\varphi \tag{68}$$

or $V_N \gtrsim \Lambda_\varphi^2$, then the barriers have no effect and the field proceeds downhill without pause. Once $V_N \lesssim \Lambda_\varphi^2$, though, the field must proceed by tunneling. Then the tunneling rate per unit volume is [2]

$$\frac{\Gamma}{V} \approx \Lambda_\varphi^4 e^{-\frac{3}{8V_N}} \tag{69}$$

This leads to extremely long decay times for the lower rungs: taking $\Lambda_\varphi^4 = 2 \cdot 10^{-125}$ as above and $V_0 = \varepsilon = 10^{-2}\Lambda_\varphi^4$ (where here V_0 denotes the lowest positive value of V_N, as above, *not* V_0 in the cyclic potential!), then

$$\left(\frac{\Gamma}{V}\right)_0^{-1} \approx 10^{10^{126}} \tag{70}$$

It is clear that a comoving volume of space that eventually undergoes gravitational collapse will have spent the vast majority of its time on the bottom rung of the potential.

In the standard setting of big bang cosmology (or in inflationary scenarios), as Abbott realized, his proposal is not practicable. This is because the universe will be expanding exponentially during the entire time φ is moving down the potential, so that it will have diluted matter and radiation to nothingness by the time it reaches the observed value of Λ_{tot}. This will not be a problem in cyclic cosmologies, however, as we will see below.

6 Dynamical Selection of Λ in the Cyclic Model

6.1 Perturbing the Cyclic Potential

Let us now consider combining the cyclic potential with Abbott's washboard potential. Since density perturbations are beyond the scope of this paper, we will revert to a one-field model and add in the effects of the phoenix universe during ekpyrosis by hand. The total potential is then

$$V(\varphi) = V_{cyclic}(\varphi) + V_{washboard}(\varphi) \tag{71}$$

Since $V_{washboard}$ changes on much longer timescales than V_{cyclic} does, it will be convenient to begin by considering the former's effects as a constant perturbation $\triangle V$ to V_{cyclic}:

$$V(\varphi) = V_0 \left(1 - e^{-c\varphi}\right) F(\varphi) + \triangle V \tag{72}$$

Hence the "height" of the potential, $V_0 - V_{end}$, is left fixed, so that the perturbation is equivalent to taking $V_0 \to V_0 + \triangle V$, $|V_{end}| \to |V_{end} - \triangle V|$. Note that V_0 now refers to two distinct quantities–an overall scale applied to the entire potential and the height approached by the plateau for large φ–and that the perturbation affects only the second of these quantities, not the first.

Throughout this section, we will consider the change in various quantities that measure growth over a cycle from adding our perturbation $\triangle V$.

6.1.1 $\triangle N_{tot}$

The number of e-folds of growth in the scale factor over a cycle is (37)

$$N_{tot} = N_{DE} + N_{kin} + N_{rad} \tag{73}$$

neglecting the (very) small change in scale factor during the ekpyrotic phase and letting N_{rad} denote the expansion from the beginning of radiation domination to the beginning of dark energy domination. We wish to calculate the change in expansion over a cycle $\triangle N_{tot}$ in terms of $\triangle V$; let us compute each term in turn.

First, approximating $V(\varphi) = V_0(1 - e^{-c\varphi}) \Rightarrow V_\varphi = V_0 c e^{-c\varphi}$ and using (22) $N_{DE} = \frac{e^{c\varphi_c}}{c^2}$,

$$\triangle N_{DE} = \int \frac{V(\varphi) + \triangle V}{V_\varphi} d\varphi - \int \frac{V(\varphi)}{V_\varphi} \approx \int \frac{\triangle V}{V_\varphi} d\varphi \tag{74}$$

$$= \frac{\triangle V}{V_0} \int \frac{e^{c\varphi}}{c} d\varphi \approx \frac{\triangle V}{V_0} \frac{e^{c\varphi_c}}{c^2} = \frac{N_{DE}}{V_0} \triangle V \tag{75}$$

Note that φ_c depends only on radiation damping at the bounce and the Hubble parameter at the onset of radiation domination, so $\triangle\varphi_c \approx 0$ to a good approximation [1].

By definition $N_{kin} = \frac{2}{3}\gamma$, with γ defined (36) by

$$\gamma \equiv \ln\left(\frac{|V_{end}|^{\frac{1}{4}}}{T_r}\right) \tag{76}$$

where T_r denotes the temperature at the beginning of the radiation-dominated period. The radiation energy density, however, should be proportional to the energy of the collision, which itself is proportional to the depth of the potential well, so that $T_r^4 \propto |V_{end}|$, and

$$\frac{\triangle N_{kin}}{\triangle V} = 0 \tag{77}$$

Now we need only compute N_{rad}, given by

$$N_{rad} \equiv \ln\left(\frac{T_r}{T_{DE}}\right) = \ln\left(|V_{end}|^{\frac{1}{4}}\right) - \ln\left(T_{DE}\right) \tag{78}$$

with T_{DE} the temperature at the beginning of dark energy domination. Let us consider perturbing each of these terms in turn. Clearly

$$\triangle\left(\ln\left(|V_{end}|^{\frac{1}{4}}\right)\right) \approx -\frac{1}{4\,|V_{end}|}\triangle V \tag{79}$$

which is negligible compared to $\triangle N_{DE}$ since $V_0 \ll |V_{end}|$.

Evaluation of the second term, however, requires the specification of when the next cycle has ended. If this point is taken to be when the universe has returned to its current temperature, then $T_{DE} \approx T_0$, the temperature today, which is of course a constant. Hence

$$\triangle\left(-\ln\left(T_{DE}\right)\right) \approx \triangle\left(-\ln\left(T_0\right)\right) \approx 0 \tag{80}$$

so that this term makes no contribution to $\triangle N_{tot}$. If, however, the cycle is instead defined as the interval between two successive times when dark energy domination begins, $T_{DE}^4 \propto V_0$, so that

$$\triangle\left(-\ln\left(T_{DE}\right)\right) = \triangle\left(-\ln\left(V_0^{\frac{1}{4}}\right)\right) \approx -\frac{1}{4V_0}\triangle V \tag{81}$$

In either case, all contributions not proportional to V_0^{-1} are negligible, so

$$\triangle N_{tot} \approx \begin{cases} \frac{N_{DE}}{V_0}\triangle V & \text{(cycling between equal temperatures)} \\ \frac{N_{DE}-\frac{1}{4}}{V_0}\triangle V & \text{(cycling between beginnings of dark energy domination)} \end{cases} \tag{82}$$

So a positive perturbation $\triangle V$ leads to more total growth over a cycle. Since N_{DE} is not known to sufficient accuracy to make the addition of $\frac{1}{4}$ in the numerator meaningful, we will henceforth write

$$\triangle N_{tot} \approx \frac{N_{DE}}{V_0}\triangle V \tag{83}$$

6.1.2 $\triangle N_{net}$

Recall from Section 4.2.1 that the effect of the entropic mechanism is to throw off all but a portion $e^{-3N_{ek}}$ of the volume of the universe from cyclic trajectories over the N_{ek} e-folds of contraction in the ekpyrotic phase, rendering the rest of the volume uninhabitable (i.e. unable to form new galaxies). We thus want to calculate the change in the net increase in habitable volume over a cycle from perturbing the height of the potential,

$$\triangle N_{net} = \triangle N_{tot} - \triangle N_{ek} \tag{84}$$

N_{ek} is given (28) as

$$N_{ek} \equiv \frac{1}{2} \ln \left(\frac{|V_{end}|}{V_0} \right) \tag{85}$$

so that

$$\triangle N_{ek} \approx -\frac{1}{2} \left(\frac{1}{|V_{end}|} + \frac{1}{|V_0|} \right) \triangle V \tag{86}$$

Then, again neglecting contributions not proportional to V_0^{-1},

$$\triangle N_{net} \approx \frac{N_{DE} + \frac{1}{2}}{V_0} \triangle V \tag{87}$$

A positive perturbation $\triangle V$ also means a smaller proportion of the universe is thrown off the cyclic trajectories during ekpyrosis, so that *net* growth over a cycle is positive as well. We will again neglect the $\frac{1}{2}$ in the numerator, which is small compared to the uncertainty in N_{DE}, after this subsection, and write

$$\triangle N_{net} \approx \frac{N_{DE}}{V_0} \triangle V \tag{88}$$

6.1.3 $\triangle G_{tot}$

We have seen that the growth of habitable volume over a cycle N_{net} increases when we increase the potential; what we would really like to calculate, however, is the increase in growth of the number of galaxies, which requires a discussion of density perturbations.

The variance of the spatial curvature perturbations in the entropic mechanism is approximately [16]

$$Q^2 \approx \frac{\varepsilon \, |V_{end}|}{10^3} \tag{89}$$

with ε here the fast-roll parameter $\varepsilon \approx \frac{c^2}{2}$. The probability of finding a region with a certain variance given such perturbations may be found by employing the *Press-Schechter formalism* [24]. The basic idea is to start with a gas consisting of point masses at small scales; as density perturbations on these scales grow sufficiently large, the particles are then treated as having collapsed to form new point particles with larger masses. Starting from an initial distribution of particles, then, the Press-Schechter formalism produces a *self-similar* spectrum on all scales. In the case that the positions of particles within the gas are independent at large distances, the probability of finding a variance δ^2 within a given volume is [24]

$$p(\delta) = \frac{1}{\sqrt{2\pi}} \delta_*^{-1} e^{-\frac{\delta^2}{2\delta_*^2}} \tag{90}$$

Identifying δ_* with Q and substituting,

$$p(\delta) = \frac{1}{\sqrt{2\pi}} \frac{10^{3/2}}{\sqrt{\varepsilon \left|V_{end}\right|}} e^{-\frac{10^3 \delta^2}{2\varepsilon \left|V_{end}\right|}} \tag{91}$$

Now allow a given volume to expand over an entire cycle. Clearly the total number of bodies with density variance δ that will have been produced from this “seed” is proportional (with positive proportionality constant) to the total volume increase multiplied by the probability of such structures within it

$$e^{3N_{net}} p(\delta) = \frac{1}{\sqrt{2\pi}} \frac{10^{3/2}}{\sqrt{\varepsilon \left|V_{end}\right|}} e^{3N_{net} - \frac{10^3 \delta^2}{2\varepsilon \left|V_{end}\right|}} \tag{92}$$

This exponential increase corresponds to a number of e-folds of growth in the number of bodies

$$G_{tot} \approx \frac{1}{2} \ln \left(\frac{1}{\left|V_{end}\right|} \right) + 3N_{net} - \frac{10^3 \delta^2}{2\varepsilon \left|V_{end}\right|} \tag{93}$$

dropping the negligible logarithms of constant terms (and writing G because we are ultimately interested in galaxies). Applying the usual perturbation $\triangle V$ to the potential and substituting for $\triangle N_{net}$ (88) gives the result

$$\triangle G_{tot} = \left(3\frac{N_{DE}}{V_0} + \frac{1}{2\left|V_{end}\right|} - \frac{10^3 \delta^2}{2\varepsilon \left|V_{end}\right|^2} \right) \triangle V \tag{94}$$

But $\left|V_{end}\right| \approx 10^{-16}$, the GUT scale, so that $\left|V_{end}\right|^2 \gg V_0$ [9]. So $\triangle G_{tot}$ is just $3\triangle N_{net}$:

$$\triangle G_{tot} = 3\frac{N_{DE}}{V_0} \triangle V \tag{95}$$

So a larger potential value leads to exponentially more galaxies produced. In fact $\triangle G_{tot}$ is ultimately just $3\triangle N_{DE}$ (75): the increase in the number of e-folds of dark energy domination is the dominant effect.

6.2 Applying Abbott’s Model

Combining the results of this section and the last one, we see that there are two different fundamental mechanisms at work. First, it is clear that the decay rate on the washboard potential (69) is exponentially lower for lower rungs of the potential. Using this, we can compute how much longer on average a region of space will stay in one rung than the one above it:

$$\frac{t_N}{t_{N+1}} = e^{\frac{3}{8}\left(\frac{1}{V_N} - \frac{1}{V_{N+1}}\right)} \approx e^{\frac{3}{8\varepsilon}\left(\frac{1}{N} - \frac{1}{N+1}\right)} \tag{96}$$

where the subscript N now denotes *height above* $\Lambda_{tot} = 0$, so that $N\varepsilon \leq V_N < (N+1)\varepsilon$. Because ε, the spacing between rungs, is so small, this ratio is huge: using the previous choice of $\varepsilon = 2 \cdot 10^{-127}$ and approximating $V_N \approx N\varepsilon$, then $\frac{t_1}{t_2} \approx 10^{10^{125}}$.

Second, we have seen (95) that more volume, and more galaxies, are formed when $V_0 = \Lambda_{tot}$ is large, or equivalently when N is large. As a result, the two mechanisms represent two competing tendencies. There are three possible outcomes. It is possible that one of the tendencies will overwhelm the other, so that an observer finding himself in a galaxy in a cyclic universe will be overwhelmingly likely to be at either the highest value Λ_{max} or the lowest value Λ_{min} where cycling is viable. (We have already seen (29) that $\Lambda_{max} \approx 10^{-68} \approx (100 \text{ GeV})^4$ if the current level of homogeneity is observed; Λ_{min} will be calculated in subsection 6.2.2 below.) Alternatively, it could be that the two tendencies are finely balanced, so that observers will tend to be at some median value. In this case the outcome might depend sensitively on the choice of volume measure.

6.2.1 Λ_{max} or Λ_{min}?

We have seen that there are two competing effects which combine to determine the most likely observed value of the cosmological constant, given the existence of galaxies. First, under the cyclic model increasing Λ, or equivalently V_0, by a small amount increases the growth of galaxies exponentially: there should be a factor of $e^{\triangle G_{tot}}$ more galaxies produced in a given comoving volume, where (95)

$$e^{\triangle G_{tot}} = e^{3\frac{N_{DE}}{V_0}\triangle V} \tag{97}$$

At the same time, though, in Abbott's model for dynamical relaxation of Λ, each part of a given comoving volume of space should spend an exponentially longer time at each successive lower rung of the washboard potential: on average a region should spend a time at the $(N+1)st$ rung that is a factor of $\frac{t_{N+1}}{t_N}$ less than the time it will spend at the next lower one, where (96)

$$\frac{t_{N+1}}{t_N} \approx e^{-\frac{3}{8\varepsilon}\left(\frac{1}{N} - \frac{1}{N+1}\right)} \tag{98}$$

These tendencies can be compared by multiplying the two exponential factors together: if the result is near 1, then specific choices of volume measure might alter the result, but if the result approaches 0 or ∞, then it will be clear that the lowest viable value of Λ, Λ_{min}, or the highest viable value of Λ, Λ_{max}, respectively, should be observed with near-certain probability. Here "viable" means that a universe with this value of the cosmological constant allows cycling solutions, with an increasing number of galaxies at each step; inflationary solutions, which only produce galaxies once, and cycling solutions with a shrinking number of galaxies, such that the total number of galaxies in any given region should go to zero, are strongly disfavored because the timescale for a given cycle is exponentially shorter than the time for the scalar field to decay one rung on the washboard potential.

When the two factors are multiplied, and $\triangle V$, the perturbation size, set equal to ε, the step size between rungs, the result is

$$e^{-\frac{3}{8\varepsilon}\left(\frac{1}{N}-\frac{1}{N+1}\right)+\frac{3N_{DE}}{V_0}\varepsilon} \tag{99}$$

All that need be done now is to compare the relative magnitudes of $\varepsilon^{-1}\left(\frac{1}{N}-\frac{1}{N+1}\right)$ and εV_0^{-1}, ignoring small constants like $\frac{3}{8}$ and N_{DE}. It is clear that Λ should be driven to its lowest allowed rung Λ_{min} if $\varepsilon \ll V_0^{\frac{1}{2}}\left(\frac{1}{N}-\frac{1}{N+1}\right)^{-\frac{1}{2}}$ and to its highest allowed rung Λ_{max} if $\varepsilon \gg V_0^{\frac{1}{2}}\left(\frac{1}{N}-\frac{1}{N+1}\right)^{-\frac{1}{2}}$.

If $V_0 \approx \Lambda$, the observed value of the cosmological constant, as indeed it was originally chosen to be in [1], then from Section 5 $\varepsilon \ll V_0$ by assumption, so[11] $\varepsilon \ll V_0^{\frac{1}{2}}$. Since $\left(\frac{1}{N}-\frac{1}{N+1}\right)^{-\frac{1}{2}} > 1$ for positive values of N, it is clear that the lowest value of Λ that allows cycling, Λ_{min}, will be overwhelmingly favored regardless of N's value[12]. All that must be done is to compute the lowest viable value of the cosmological constant, Λ_{min}, and determine when it is equal to the observed value, Λ_{obs}.

6.2.2 Finding Λ_{min}

We have determined that the value of Λ that is overwhelmingly likely to be observed, given the existence of galaxies, is the smallest viable value of the cosmological constant that allows cycling, Λ_{min}. Finding Λ_{min} is therefore equivalent to placing a lower bound on V_0, the value of the cosmological constant in the cyclic model.

The most general constraint on V_0 in the cyclic model is that cycling should actually occur: that is, that the scalar field φ should have returned from $-\infty$

[11] Recall that in Planck units $0 < V_0 < 1$, so $V_0 < V_0^{\frac{1}{2}}$.

[12] This need not be true if it is not the case that $\varepsilon \ll V_0$. A full analysis of this condition is beyond the scope of this paper, but some brief remarks can be made. If $V_0 \lesssim \varepsilon \ll V_0^{\frac{1}{2}}$, it is still the case that Λ_{min} will be favored. However, as the value of ε increases in this range, it becomes increasingly unlikely that V_{other} in (66) has the precise value needed so that the cosmological constant is permitted to take on the observed value of Λ rather than simply jumping from a greater value to a lesser one–in effect, we have re-introduced the need for fine-tuning to explain the smallness of Λ. If $\varepsilon \approx V_0^{\frac{1}{2}}$, then Λ should take on some value somewhere in the middle of the range of values where cycling is allowed, and a more detailed analysis is required. However, we will still have a fine-tuning problem, as above. Finally, if $\varepsilon \gg V_0^{\frac{1}{2}}$, we should find $\Lambda_{obs} \approx \Lambda_{max} \approx 10^{-68}$ (29) and the required fine-tuning is even more extreme.

past $|V_{end}|$ and rolled up the potential well onto the plateau at V_0 before the beginning of radiation domination, at time t_r after the bounce. The trajectory of φ during this period is given by the solution to the Friedmann equations in the kinetic phase (32), but after the bounce, so that the sign of t is now positive:

$$\varphi(t) = \sqrt{\frac{2}{3}} \ln(t) + \hat{\varphi} = \varphi(t) = \sqrt{\frac{2}{3}} \ln\left(\frac{t}{t_{end}}\right) \tag{100}$$

where t_{end}, the time from the bounce to when φ again crosses its minimum, has the same magnitude as t_{end} in subsection 3.2, assuming the bounce is nearly symmetric. Then the cycling condition states that

$$\frac{t_r}{t_{end}} > \left(\frac{|V_{end}|}{V_0}\right)^{\sqrt{\frac{3}{2c^2}}} \tag{101}$$

since $V(\varphi) \propto e^{-c\varphi}$. We have $t_r \approx H_r^{-1} \approx T_r^{-2}$, $t_{end} \approx |V_{end}|^{-\frac{1}{2}}$, so

$$T_r \lesssim |V_{end}|^{\frac{1}{4}} \left(\frac{V_0}{|V_{end}|}\right)^{\sqrt{\frac{3}{8c^2}}} \tag{102}$$

This is equivalent to a lower bound on V_0:

$$V_0 \gtrsim T_r^{\frac{4c}{\sqrt{6}}} |V_{end}|^{1-\frac{c}{\sqrt{6}}} \tag{103}$$

So Λ_{min} is just this lowest allowed value:

$$\Lambda_{min} \approx T_r^{\frac{4c}{\sqrt{6}}} |V_{end}|^{1-\frac{c}{\sqrt{6}}} \tag{104}$$

So Λ_{min} depends on three parameters: T_r, the temperature at radiation, c, which denotes the slope of the potential well, and $|V_{end}|$, the magnitude of the potential minimum. We can also write Λ_{min} in terms of T_r, c, and the number of e-folds of ekpyrosis $N_{ek} \equiv \frac{1}{2} \ln\left(\frac{|V_{end}|}{V_0}\right)$ (28):

$$\Lambda_{min} \approx T_r^4 e^{2N_{ek}\left(\frac{\sqrt{6}}{c}-1\right)} \tag{105}$$

This bound, together with the analysis that suggests Λ should be found at it, is the main result of this paper.

6.2.3 Constraints on Parameters

We want to find a consistent set of parameter values that gives the result $\Lambda_{min} \approx \Lambda_{obs} \approx 10^{-123}$ given that (104, 105)

$$\Lambda_{min} \approx T_r^{\frac{4c}{\sqrt{6}}} |V_{end}|^{1-\frac{c}{\sqrt{6}}} \approx T_r^4 e^{2N_{ek}\left(\frac{\sqrt{6}}{c}-1\right)} \tag{106}$$

Let us consider the constraints on each parameter in turn. Recovering the predictions of big bang nucleosynthesis requires that $T_r \gtrsim 1$ MeV $\approx 10^{-22}$.

However, the energy density of the radiation produced at the bounce must be small compared to the kinetic energy of the colliding branes themselves, which implies that [30]

$$T_r \lesssim \sqrt{\frac{v_{coll}}{L}} \tag{107}$$

where v_{coll}, the brane velocity at collision, should be non-relativistic and L, the curvature scale of the extra spatial dimension, is typically in the range $L = 10^{4-6}$, so that $10^{-22} \lesssim T_r \lesssim 10^{-2}$. Note, though, that avoiding the production of monopoles requires that T_r be less than the GUT scale 10^{15} GeV $\approx 10^{-4}$, leading to the more stringent constraint that

$$10^{-22} \lesssim T_r \lesssim 10^{-4} \tag{108}$$

The scalar spectral index n_S is related to c for our model potential by [10] $n_S \approx 1 - \frac{4}{c^2}$ and constrained by observations to be [4] 0.968 ± 0.012 (68% CL), so that

$$9 \lesssim c \lesssim 15 \tag{109}$$

The requirement of at least 60 folds of ekpyrosis means that (29)$|V_{end}| \gtrsim e^{120}V_0 \approx 10^{52}V_0$; in order not to spoil big bang nucleosynthesis, the energy density of gravitational waves must be less than 10% of the radiation density at nucleosynthesis, implying that [30] $|V_{end}| \lesssim 10c^2T_r$. Hence if $\Lambda_{min} \approx 10^{-123}$, so that the observed value of the cosmological constant is indeed the natural one in Abbott's model, we need

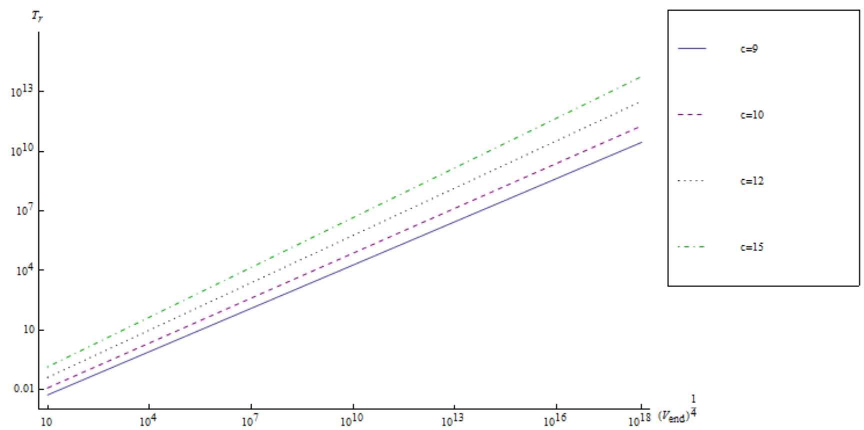
$$10^{-71} \lesssim |V_{end}| \lesssim 10^{-1} \tag{110}$$

Finally, it is clear that (28) $N_{ek} > 60$. There is no maximum number of e-folds of ekpyrosis, but $|V_{end}|$ must be below the Planck scale, meaning that if $\Lambda_{min} \approx 10^{-123} \approx e^{-284}$ then

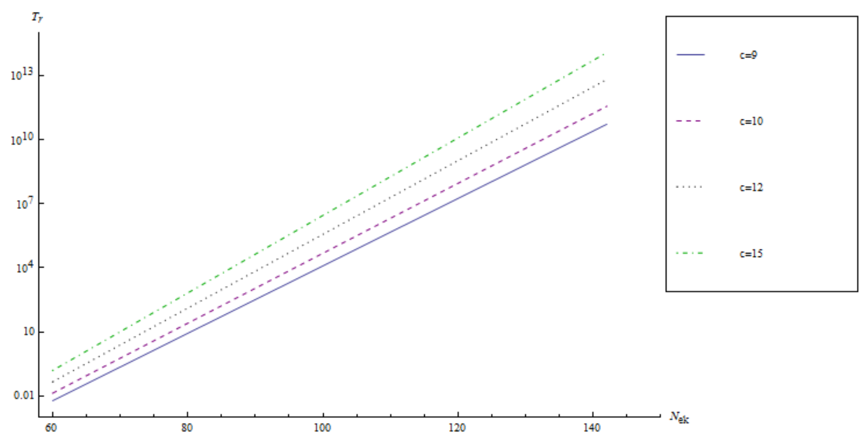
$$60 \lesssim N_{ek} \lesssim 142 \tag{111}$$

We seek parameter values compatible with (108-111) that give the desired result $\Lambda_{min} \approx \Lambda_{obs} \approx 1.4 \cdot 10^{-123}$. Figure 3 on the following page shows the full range of the parameter space which allows this equality. In general, as can be seen from the form of the expressions (104, 105), increasing V_{end} or N_{ek} requires increasing T_r to compensate, while increasing c requires a slight increase in T_r or a large decrease in V_{end}.

The most natural choice of parameters is to set $V_{end}^{\frac{1}{4}} \approx 10^{15}$ GeV, i.e. at the GUT scale. Table 1 on page 29 shows the predicted values of T_r, the temperature at radiation-kinetic equality, if this is true for values of c across the entire allowed range. It is clear that each of these temperatures is well below the GUT scale, preventing the disastrous overproduction of monopoles. More accurate measurements of the scalar spectral index n_s should enable the precise determination of c, via $n_S \approx 1 - \frac{4}{c^2}$ as above, which will have the effect of fixing a one-to-one correspondence between V_{end} and T_r in this model, a clear prediction to be verified or falsified.



(a)



(b)

Figure 3: (a) A plot of the values of the values of $V_{end}^{\frac{1}{4}}$ (in GeV), the fourth root of the potential minimum, and T_r, the temperature at kinetic-radiation equality (also in GeV), required to produce $\Lambda_{min} \approx \Lambda_{obs}$ for four different values of c. T_r is constrained slightly beyond its existing bounds in the cyclic model; even at extreme values of c and T_r we require 6 MeV $\lesssim T_r \lesssim 7.3 \cdot 10^{13}$ GeV, compared to the previous bound (108) 1 MeV $\lesssim T_r \lesssim 10^{15}$ GeV.
(b) A plot of the values of the values of N_{ek} (dimensionless), the number of e-folds of ekpyrosis, and T_r (in GeV) required to produce $\Lambda_{min} \approx \Lambda_{obs}$ for four different values of c.

c	T_r (GeV)
9	$6.55 \cdot 10^7$
10	$3.49 \cdot 10^8$
11	$1.37 \cdot 10^9$
12	$4.30 \cdot 10^9$
13	$1.13 \cdot 10^{10}$
14	$2.59 \cdot 10^{10}$
15	$5.30 \cdot 10^{10}$

Table 1: Table of the temperature at kinetic-radiation equality T_r (in GeV) required such that $\Lambda_{min} \approx \Lambda_{obs}$, with the potential minimum $V_{end} \approx \left(10^{15}\ \text{GeV}\right)^4$, the GUT scale, (or, equivalently, the number of e-folds of ekpyrosis $N_{ek} \approx 123$) for various values of the parameter c, which controls the slope of the cyclic potential.

7 Conclusion

In this paper, we have presented a worked example of the concept of dynamical selection, which offers the possibility of making concrete and rigorously non-anthropic deductions of the parameters of the universe we find ourselves in. We have combined Abbott's model of dynamical relaxation of the cosmological constant, which provides a mechanism for reducing the value of Λ to its observed value without the need for extreme fine tuning, with the framework of the cyclic model, whose cycling conditions provide a natural sieve for selection of parameter values which lead to the most galaxy formation.

Abbott's model, which requires extremely large timescales for Λ to decay from the Planck scale to its current value, is incompatible with an inflationary framework, where only one generation of galaxies is ever produced in a given volume of space. It fits naturally, however, into the cyclic picture, where repeated cycling means that galaxies can be created again and again; now it is perfectly acceptable for the period of time after any individual bang where galaxy formation is allowed to be much shorter than the time required for Λ to relax.

Calculation suggests that the most likely value of Λ at which galaxies can be observed is the smallest value at which cycling is still viable; in other words, the lowest viable value of the cosmological constant is *dynamically selected* to be the most likely one. This value can be easily computed within the cyclic model in terms of other parameters; encouragingly, the observed value of Λ can be produced by very natural choices of these parameters, with no need for fine-tuning. At the same time, though, a precise enough determination of these parameters, or evidence requiring the rejection of the cyclic model, could easily rule out this means of generating the cosmological constant; unlike the superficially similar anthropic argument for Λ, which also relies on computing bounds, this method is *falsifiable*. Regardless of whether it is ultimately successful, the non-anthropic determination of Λ amply demonstrates the utility of dynamical selection methods[13].

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