Week 2 (due Jan. 21)

1. (20 pts) Consider a 4d gauge theory with gauge group $SU(N_c)$ and the following matter content: N_1 left-handed fermions and N_2 right-handed fermions, both in a representation R of the gauge group. Consider the following three possibilities: (1) R is the fundamental representation of $SU(N_c)$; (2) R is the adjoint representation; (3) R is a symmetric tensor of $SU(N_c)$ with two indices. In each of these cases determine the conditions on N_c, N_1, N_2 which ensure that the theory is consistent on the quantum level.

2. (10 pts) Consider a 4d gauge theory with gauge group $USp(2N_c)$ and N_f left-handed fermions in the fundamental ($2N_c$-dimensional) representation of $USp(2N_c)$. What is the global flavor symmetry of this theory? Can this theory have a mass gap?

3. Consider a 4d gauge theory with gauge group $SU(N_c)$, N_f left-handed fermions Q_i^a in the fundamental representation, N_f left-handed fermions $\tilde{Q}_{ai}$ in the complex-conjugate ("anti-fundamental") representation, and a single fermion λ_b^a in the adjoint representation. The index i runs from 1 to N_f, the indices a, b run from 1 to N_c, and λ_b^a is traceless. Global symmetry of this theory contains an obvious non-anomalous $SU(N_f) \times SU(N_f) \times U(1)_B$ subgroup, where $U(1)_B$ is the "baryon number symmetry" with respect to which Q and $\tilde{Q}$ have charges 1 and -1, respectively, and λ is uncharged.

(a) (10pts) Consider a $U(1)$ transformation group with respect to which λ has charge 1, and both Q and $\tilde{Q}$ have charge r. This is a symmetry on the classical level (for any r). For which values of r does it remain a symmetry on the quantum level?

(b) (10pts) It follows from the results of part (a) that the global symmmetry of the quantized theory is $SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R$. Compute all 't Hooft anomalies (including gravitational ones) for these currents.