Week 3 (due Jan. 28)

Note: for basic definitions concerning differential forms you may consult the following books:

M. Nakahara, "Geometry, Topology, and Physics";

W. D. Curtis and F. R. Miller, "Differential Manifolds and Theoretical Physics";

B.A. Dubrovin, A.T. Fomenko, S.P. Novikov, "Modern Geometry - Methods and Applications," Part 1;

F. Warner, "Foundations of differentiable manifolds and Lie groups".

1. (a) The connection 1-form A corresponding to the gauge field A_μ is defined as $A = A_\mu dx^\mu$. The curvature 2-form F corresponding to the gauge-field strength $F_{\mu\nu}$ is defined as $F = \frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu$. Show that $F = dA + A \wedge A$.

(b) Show that the Bianchi identity for $F_{\mu\nu}$ is equivalent to $dF + [A, F] = 0$ in terms of differential forms.

2. (a) Compute the Chern-Simons forms and the gauge anomaly for $k = 0, 1, 2$ using the explicit formulas of Stora-Zumino. I remind that k is half the dimension of space-time, and one starts with a form of degree $2k + 2$.

(b) Verify by an explicit computation that the descent equations

$$d\Omega_{2k+1}(A) = \mathrm{Tr}\ F^{k+1}, \quad s\Omega_{2k+1}(A) = d\Omega^1_{2k}(\omega, A)$$

are satisfied in all three cases $(k = 0, 1, 2)$.