Week 4 (due October 29)

1. Suppose we added to the QCD Lagrangian scalar fields which transform in a representation R of $SU(3)$. Let us assume that these scalar fields are minimally-coupled, i.e. they only interact with the gauge-field, and all the interaction comes from replacing ordinary derivatives with covariant derivatives.

(a) Write down the Feynman rules describing the interaction of the scalars with the gluons.

(b) Compute the contribution to the one-loop beta-function (for the Yang-Mills coupling) from these scalars.

2. Consider a gauge theory with gauge group $U(N)$ and quarks in the defining (N-dimensional) representation. The classical Lagrangian is taken to be

$$L = -\frac{1}{2g^2}\mathrm{Tr}F_{\mu\nu}F^{\mu\nu} + \bar{q}\left(i\gamma^\mu D_\mu - m\right)q.$$

Since the Lie algebra of $U(N)$ is a direct sum of the Lie algebras of $U(1)$ and $SU(N)$, we can decompose the $U(N)$ gauge field into a $U(1)$ gauge field and an $SU(N)$ gauge field. Note though that the same coupling constant g controls the strength of both $U(1)$ and $SU(N)$ interactions. Does this remain true on the quantum level? That is, does the relative strength of $U(1)$ and $SU(N)$ interactions remain the same in the quantum theory? Justify your answer.

3. The decay of the τ-lepton is described by the following term in the effective Lagrangian:

$$\delta L = \frac{g^2}{2m_W^2}\left(\bar{\nu}\gamma^\mu\frac{1-\gamma_5}{2}\tau\right)\left(\bar{d}\gamma_\mu\frac{1-\gamma_5}{2}u\right),$$

where τ is the τ-lepton field, ν is the neutrino field, u, d are the quark fields, and m_W is the W-boson mass. The coupling g is defined by

$$\frac{g^2}{8m_W^2} = \frac{G_F}{\sqrt{2}},$$

where $G_F = 1.166 \times 10^{-5}$ GeV^{-2} is the Fermi constant. Compute the total decay rate of the τ-lepton. The mass of the τ-lepton is 1.777 GeV. You may neglect the masses of u, d and ν.