Week 5 (due November 5)

1. The Callan-Gross relation $F_2 = 2xF_1$ holds because quarks are spinors. Suppose quarks were scalars instead, i.e. suppose the real world was described by scalar QCD. What would be the relation between F_1 and F_2 in such a world? (Use the naive parton model, i.e. neglect QCD corrections).

2. Leading-order DGLAP equations in QCD are written down on page 590 of Peskin and Schroeder.

(a) The Adler sum rule for the parton distribution in a proton states:

$$\int_0^1 dx(f_u(x) - f_{\bar{u}}(x) - f_d(x) + f_{\bar{d}}(x)) = 1.$$

What is the meaning of this relation? Show that the Adler sum rule is preserved by the DGLAP evolution, i.e. if it is valid at some scale Q_0^2, then it is valid at all scales.

(b) The Gottfried sum rule for the parton distribution in a proton states:

$$\int_0^1 dx(f_{\bar{u}}(x) - f_{\bar{d}}(x)) = 0.$$

This relation is badly violated by Nature. Why is this suprprising? Is this sum rule preserved by the DGLAP evolution? What does this suggest about the origin of the violation of the Gottfried sum rule?

3. (a) Show that the Mellin transform

$$\tilde{f}(n) = \int_0^1 dx\ x^{n-1} f(x)$$

takes DGLAP equations into a system of ordinary differential equations for the moments of the parton distribution functions.

(b) Compute the anomalous dimensions for the zeroth moments of the parton distribution functions $\int f(x)dx$ (this corresponds to $n = 1$). What does this computation tell you about the total number of gluons in a proton?