Week 6 (due November 12)

1. Problem 18.1 in Peskin and Schroeder.

2. (a) The scalar part of the Operator Product Expansion for the product of two quark currents starts as follows:

$$J_\mu(x)J_\nu(0) \sim C^1_{\mu\nu}(x)\cdot 1 + C^{m\bar{q}q}_{\mu\nu}(x)(m\bar{q}q(0)) + \ldots$$

where m is the quark mass. Determine the coefficient functions at leading order in α_s. (Hint: evaluate the expectation value of both sides in the vacuum and the 1-quark state.)

(b) The OPE of quark currents also contains non-scalar operators. The leading (dimension 3) operators are all of the form $\bar{q}\Gamma q$, where Γ is some product of the Dirac gamma-matrices. Show that if $m = 0$, then only two Γ's are allowed, $\Gamma = \gamma_\rho$ and $\Gamma = \gamma_\rho\gamma_5$.

(c) Determine the coefficient functions multiplying $\bar{q}\gamma_\rho q$ and $\bar{q}\gamma_\rho\gamma_5 q$ at leading order in α_s. (Hint: evaluate both sides in the 1-quark state and use the following identity:

$$\gamma_\alpha\gamma_\beta\gamma_\nu = g_{\alpha\beta}\gamma_\nu + g_{\beta\nu}\gamma_\alpha - g_{\alpha\nu}\gamma_\beta - i\epsilon_{\alpha\beta\nu\eta}\gamma^\eta\gamma_5.$$