Week 8 (due March 3)

1. (a) Consider a scalar field theory in 1+1 dimensions with the following Lagrangian:

$$L = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - V(\phi),$$

where

$$V(\phi) = \lambda(\phi^2 - a^2)^2.$$

Find the one-kink solution and evaluate its energy.

(b) The same, for the potential $V(\phi) = \lambda(1 - cos(\beta\phi))$.

2. (ab) Consider the O(3) sigma-model in in 2+1 dimensions. It was shown in class that the absolute minima of the energy functional are solutions of a system of first-order differential equations:

$$\partial_i \phi^a \pm \epsilon^{abc}\epsilon_{ij}\phi^b \partial_j \phi^c = 0$$

Consider the following change of variables:

$$W = 2\frac{\phi^1 + i\phi^2}{1 - \phi^3}.$$

Show that the above equations then become equivalent to the Cauchy-Riemann equations for W. Use this fact to write down the most general n-soliton solution for this model. Determine the number of parameters in the most general n-soliton solution and interpret them in physical terms.