Week 9 (due Dec. 3)

(Note: chiral perturbation theory is explained in Section 19.7 of Weinberg.)

4. Consider the following alternative to the orthodox scenario of chiral symmetry breaking in QCD. Suppose the $SU(3)\times SU(3)$ symmetry is broken down not to the diagonal subgroup $SU(3)_V$, but to a larger group $SU(3)_V\times G$, where $G\simeq \mathbb{Z}_3$ acts as follows:

$$q_L^i \to e^{2\pi i/3} q_L^i,\;\; q_R^i \to q_R^i, \quad i=u,d,s.$$

Assume that all quark masses are exactly zero, unless stated otherwise.

(a) What is the expected spectrum of the Goldstone bosons in this case? What is the vacuum expectation value of the quark bilinear $\bar{q}_L q_R$?

(b) Does the axial $SU(3)$ current have a non-zero matrix element between the vacuum and the state with one Goldstone boson?

(c) Find the lowest-dimension operator which could serve as an order parameter in this scenario and determine its transformation properties under $SU(3)\times SU(3)$.

(d) Now imagine turning on small quark masses. They explicitly break the discrete symmetry G. Show that in this scenario the masses of Goldstone bosons scale linearly with quark masses (while in the orthodox scenario they scale like square root of quark masses). Estimate the size of quark masses and the size of the the vacuum expectation value of the quark bilinear $\bar{q}_L q_R$. How do they compare with the orthodox scenario? (It is sufficient to give order-of-magnitude estimates).

(e) Show that the Goldstone bosons in this scenario are invariant under the discrete G-symmetry. Further, show that the lightest pseudoscalar meson which is NOT invariant must have a scalar partner with the same mass, provided that the quark masses are exactly zero. This is an example of "parity-doubling." Does this observation rule out the unorthodox scenario of chiral symmetry breaking? (Hint: use tables from the Review of Particle Properties to find the observed masses of various mesons.)

2. (a) Consider a gauge theory with gauge group $SO(4)$ and a real scalar which transforms as a 2-index antisymmetric tensor under gauge transformations. What patterns of gauge symmetry breaking are possible in this theory?

(b) The same, but with a complex scalar in the spinor representation of $SO(4)$.