Week 2 (due Oct. 16)

Reading: Srednicki, sections 1 and 3.

1. Consider free nonrelativistic bosons of mass m (in the Fock space formalism).

(a) Compute the commutator of the field operators

$$[\Psi(t, \mathbf{x}), \Psi^\dagger(t', \mathbf{x}')]$$

for arbitrary $t, t', \mathbf{x}, \mathbf{x}'$. Hint: use solutions for Ψ and $\Psi^\dagger$ in terms of momentum space operators a_p and $a_p^\dagger$.

(b) Consider the vector-valued operator $\mathbf{P}$ with components

$$P_k(t) = -i \int \Psi^\dagger(t, \mathbf{x}) \partial_k \Psi(t, \mathbf{x}) d^3x, \quad k = 1, 2, 3.$$

Let H be the usual Hamiltonian in Fock space, i.e.

$$H = \frac{1}{2m} \int \partial_k \Psi^\dagger \partial_k \Psi d^3x.$$

Show that $[H, \mathbf{P}] = 0$, i.e. $\mathbf{P}$ is an integral of motion. Show that

$$[\mathbf{P}(t), \Psi(t, \mathbf{x})] = i\nabla\Psi(t, \mathbf{x}), \quad [\mathbf{P}(t), \Psi^\dagger(t, \mathbf{x})] = i\nabla\Psi^\dagger(t, \mathbf{x}),$$

i.e. $\mathbf{P}$ is the generator of translations.