Week 9 (due March 12)

1. Consider a theory of a gauge field A_μ and a real scalar σ with the Lagrangian

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - M^2 D_\mu \sigma D^\mu \sigma,$$

where $D_\mu \sigma = \partial_\mu \sigma + A_\mu$.

(a) Compute the propagator for the fields A and σ in the Lorenz gauge and more generally in the α-gauge. I remind that the α-gauge is equivalent to adding to the action a term

$$\frac{-1}{2\alpha} (\partial^\mu A_\mu)^2.$$

The Lorenz gauge is recovered in the limit $\alpha \to 0$. Note that there will be "mixed" propagators between A_μ and σ; don't forget about them. Determine the locations of the poles of the propagators.

(b) Now consider what is called an 't Hooft gauge.This is actually a family of gauge conditions given by

$$\partial^\mu A_\mu - \beta \sigma = 0,$$

where β is a parameter. This gauge interpolates between the Lorenz gauge $\beta = 0$ and the unitary gauge which is obtained in the limit $\beta = \infty$. Compute the propagators for A_μ and σ in this gauge by adding a gauge-fixing term of the form

$$B(\partial^\mu A_\mu - \beta \sigma),$$

where B is a Lagrange multiplier field. Determine the locations of the poles in the propagators as a function of β. Make conclusions about which poles are physical and which are unphysical.

2. Consider a theory of a gauge field in three-dimensional space-time with a Lagrangian

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + k \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho.$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. The second term in the Lagrangian is called the Chern-Simons term, and the theory is called Chern-Simons-Maxwell theory. Derive the propagator for the field A in the Lorenz gauge $\partial_\mu A^\mu = 0$ by adding a term

$$B \partial_\mu A^\mu$$

to the Lagrangian, where B is a new independent field (the Lagrange multiplier field).