

1. Let the initial electron and positron have four-momenta  $p_1$  and  $p_2$ , respectively, and the final electron and positron have four-momenta  $p'_1$  and  $p'_2$ . The relevant diagrams are shown in Srednicki fig. 45.8. The amplitude is then given by

$$i\mathcal{T}_{e^+e^- \rightarrow e^+e^-} = \frac{1}{i}(ig)^2 \left[ \frac{(\bar{u}'_1 i\gamma_5 u_1)(\bar{v}_2 i\gamma_5 v'_2)}{-t + M^2} - \frac{(\bar{v}_2 i\gamma_5 u_1)(\bar{u}'_1 i\gamma_5 v'_2)}{-s + M^2} \right], \quad (1)$$

where  $s = -(p_1 + p_2)^2$  and  $t = -(p_1 - p'_1)^2$ . Therefore

$$\mathcal{T}_{e^+e^- \rightarrow e^+e^-} = g^2 \left[ \frac{(\bar{u}'_1 i\gamma_5 u_1)(\bar{v}_2 i\gamma_5 v'_2)}{M^2 - t} - \frac{(\bar{v}_2 i\gamma_5 u_1)(\bar{u}'_1 i\gamma_5 v'_2)}{M^2 - s} \right]. \quad (2)$$

We then have

$$\bar{\mathcal{T}}_{e^+e^- \rightarrow e^+e^-} = g^2 \left[ \frac{(\bar{u}_1 i\gamma_5 u'_1)(\bar{v}'_2 i\gamma_5 v_2)}{M^2 - t} - \frac{(\bar{u}_1 i\gamma_5 v_2)(\bar{v}'_2 i\gamma_5 u'_1)}{M^2 - s} \right], \quad (3)$$

where we have used the relation  $\overline{i\gamma_5} = i\gamma_5$ . The squared amplitude  $|\mathcal{T}|^2 = \mathcal{T}\mathcal{T}^* = \bar{\mathcal{T}}\bar{\mathcal{T}}$  is given by

$$\begin{aligned} |\mathcal{T}|^2 = & + \frac{g^4}{(M^2 - t)^2} \Phi_{tt} + \frac{g^4}{(M^2 - s)^2} \Phi_{ss} \\ & - \frac{g^4}{(M^2 - t)(M^2 - s)} \Phi_{ts} - \frac{g^4}{(M^2 - s)(M^2 - t)} \Phi_{st}, \end{aligned} \quad (4)$$

where

$$\Phi_{tt} = \text{Tr} \left[ (u_1 \bar{u}_1) i\gamma_5 (u'_1 \bar{u}'_1) i\gamma_5 \right] \text{Tr} \left[ (v'_2 \bar{v}'_2) i\gamma_5 (v_2 \bar{v}_2) i\gamma_5 \right], \quad (5)$$

$$\Phi_{ss} = \text{Tr} \left[ (u_1 \bar{u}_1) i\gamma_5 (v_2 \bar{v}_2) i\gamma_5 \right] \text{Tr} \left[ (v'_2 \bar{v}'_2) i\gamma_5 (u'_1 \bar{u}'_1) i\gamma_5 \right], \quad (6)$$

$$\Phi_{st} = \text{Tr} \left[ (u_1 \bar{u}_1) i\gamma_5 (u'_1 \bar{u}'_1) i\gamma_5 (v'_2 \bar{v}'_2) i\gamma_5 (v_2 \bar{v}_2) i\gamma_5 \right], \quad (7)$$

$$\Phi_{ts} = \text{Tr} \left[ (u_1 \bar{u}_1) i\gamma_5 (v_2 \bar{v}_2) i\gamma_5 (v'_2 \bar{v}'_2) i\gamma_5 (u'_1 \bar{u}'_1) i\gamma_5 \right]. \quad (8)$$

Next we average over the two initial spins and sum over the two final spins to get

$$\langle |\mathcal{T}|^2 \rangle = \frac{1}{4} \sum_{s_1, s_2, s'_1, s'_2} |\mathcal{T}|^2. \quad (9)$$

Then we use Srednicki Eq. (48.5) to get

$$\langle \Phi_{ss} \rangle = \frac{1}{4} \text{Tr} \left[ (-\not{p}_1 + m) i\gamma_5 (-\not{p}_2 - m) i\gamma_5 \right] \text{Tr} \left[ (-\not{p}'_2 - m) i\gamma_5 (-\not{p}'_1 + m) i\gamma_5 \right], \quad (10)$$

$$\langle \Phi_{tt} \rangle = \frac{1}{4} \text{Tr} \left[ (-\not{p}'_1 + m) i\gamma_5 (-\not{p}'_1 + m) i\gamma_5 \right] \text{Tr} \left[ (-\not{p}'_2 - m) i\gamma_5 (-\not{p}_2 - m) i\gamma_5 \right],$$

$$\langle \Phi_{st} \rangle = \frac{1}{4} \text{Tr} \left[ (-\not{p}'_1 + m) i\gamma_5 (-\not{p}'_1 + m) i\gamma_5 (-\not{p}'_2 - m) i\gamma_5 (-\not{p}_2 - m) i\gamma_5 \right],$$

$$\langle \Phi_{ts} \rangle = \frac{1}{4} \text{Tr} \left[ (-\not{p}'_1 + m) i\gamma_5 (-\not{p}_2 - m) i\gamma_5 (-\not{p}'_2 - m) i\gamma_5 (-\not{p}'_1 + m) i\gamma_5 \right].$$

With the help of *FeynCalc.m*, we obtain that

$$\begin{aligned}\langle \Phi_{ss} \rangle &= s^2, & \langle \Phi_{tt} \rangle &= t^2, \\ \langle \Phi_{st} \rangle &= -\frac{st}{2}, & \langle \Phi_{ts} \rangle &= -\frac{st}{2}.\end{aligned}\tag{11}$$

Putting all of these together, we get

$$\langle |\mathcal{T}|^2 \rangle = g^4 \left[ \frac{s^2}{(M^2 - s)^2} + \frac{st}{(M^2 - s)(M^2 - t)} + \frac{t^2}{(M^2 - t)^2} \right].\tag{12}$$

## Ph205b HW4 Problem I

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```
In[47]:= Import["http://www.feyncalc.org/install.m"]
```

Considering the process  $p_1+p_2 \rightarrow q_1+q_2$ , write four-vector products in terms of the Mandelstam variables

```
In[49]:= ScalarProduct[p1] = m^2;
```

```
ScalarProduct[p2] = m^2;
```

```
ScalarProduct[q1] = m^2;
```

```
ScalarProduct[q2] = m^2;
```

```
ScalarProduct[p1, p2] = ScalarProduct[q1, q2] = - 1/2 (s - 2 m^2);
```

```
ScalarProduct[p1, q1] = ScalarProduct[p2, q2] = 1/2 (t - 2 m^2);
```

```
ScalarProduct[p1, q2] = ScalarProduct[q1, p2] = 1/2 (u - 2 m^2);
```

Note that FeynCalc uses the metric signature  $(+,-,-,-)$  while Srednicki uses  $(-,+,+,+)$ , therefore we can set  $m \rightarrow im$  and consider the relevant sign change while tracing to get the results consistent with Srednicki convention.

```
In[56]:= TrickMandelstam[1/4 Tr[(-GS[p1] + i m) . (i GA[5]) . (-GS[p2] - i m) . (i GA[5])]
Tr[(-GS[q2] - i m) . (i GA[5]) . (-GS[q1] + i m) . (i GA[5])], {u, t, s, 4 m^2}]
```

```
Out[56]= s^2
```

```
In[57]:= TrickMandelstam[1/4 Tr[(-GS[p1] + i m) . (i GA[5]) . (-GS[q1] + i m) . (i GA[5])]
Tr[(-GS[q2] - i m) . (i GA[5]) . (-GS[p2] - i m) . (i GA[5])], {u, t, s, 4 m^2}]
```

```
Out[57]= t^2
```

```
In[60]:= TrickMandelstam[
1/4 Tr[(-GS[p1] + i m) . (i GA[5]) . (-GS[q1] + i m) . (i GA[5]) . (-GS[q2] - i m) .
(i GA[5]) . (-GS[p2] - i m) . (i GA[5])], {u, t, s, 4 m^2}]
```

```
Out[60]= -s t/2
```

```
In[61]:= TrickMandelstam[
1/4 Tr[(-GS[p1] + i m) . (i GA[5]) . (-GS[p2] - i m) . (i GA[5]) . (-GS[q2] - i m) .
(i GA[5]) . (-GS[q1] + i m) . (i GA[5])], {u, t, s, 4 m^2}]
```

```
Out[61]= -s t/2
```

2. Srednicki 48.2

48.2) From eq. (45.23), we have

$$\mathcal{T}_{e^+e^- \rightarrow \varphi\varphi} = g^2 \bar{v}_2 \left[ \frac{-\not{p}_1 + \not{k}'_1 + m}{-t + m^2} + \frac{-\not{p}_1 + \not{k}'_2 + m}{-u + m^2} \right] u_1, \quad (48.31)$$

where  $t = -(p_1 - k'_1)^2 = -(p_2 - k'_2)^2$  and  $u = -(p_1 - k'_2)^2 = -(p_2 - k'_1)^2$ . We can use  $-\not{p}_1 u_1 = m u_1$  to simplify this to

$$\mathcal{T} = g^2 \bar{v}_2 \left[ \frac{\not{k}'_1 + 2m}{-t + m^2} + \frac{\not{k}'_2 + 2m}{-u + m^2} \right] u_1. \quad (48.32)$$

We then have

$$\bar{\mathcal{T}} = g^2 \bar{u}_1 \left[ \frac{\not{k}'_1 + 2m}{-t + m^2} + \frac{\not{k}'_2 + 2m}{-u + m^2} \right] v_2. \quad (48.33)$$

Therefore

$$\begin{aligned} |\mathcal{T}|^2 = & + \frac{g^4}{(m^2 - t)^2} \text{Tr} \left[ (v_2 \bar{v}_2) (\not{k}'_1 + 2m) (u_1 \bar{u}_1) (\not{k}'_1 + 2m) \right] \\ & + \frac{g^4}{(m^2 - u)^2} \text{Tr} \left[ (v_2 \bar{v}_2) (\not{k}'_2 + 2m) (u_1 \bar{u}_1) (\not{k}'_2 + 2m) \right] \\ & + \frac{g^4}{(m^2 - t)(m^2 - u)} \text{Tr} \left[ (v_2 \bar{v}_2) (\not{k}'_1 + 2m) (u_1 \bar{u}_1) (\not{k}'_2 + 2m) \right] \\ & + \frac{g^4}{(m^2 - t)(m^2 - u)} \text{Tr} \left[ (v_2 \bar{v}_2) (\not{k}'_2 + 2m) (u_1 \bar{u}_1) (\not{k}'_1 + 2m) \right]. \end{aligned} \quad (48.34)$$

Averaging over the initial spins, we get

$$\langle |\mathcal{T}|^2 \rangle = g^4 \left[ \frac{\langle \Phi_{tt} \rangle}{(m^2 - t)^2} + \frac{\langle \Phi_{uu} \rangle}{(m^2 - u)^2} + \frac{\langle \Phi_{tu} \rangle + \langle \Phi_{ut} \rangle}{(m^2 - t)(m^2 - u)} \right], \quad (48.35)$$

where

$$\begin{aligned} \langle \Phi_{tt} \rangle &= \frac{1}{4} \text{Tr} \left[ (-\not{p}_2 - m) (\not{k}'_1 + 2m) (-\not{p}_1 + m) (\not{k}'_1 + 2m) \right], \\ \langle \Phi_{uu} \rangle &= \frac{1}{4} \text{Tr} \left[ (-\not{p}_2 - m) (\not{k}'_2 + 2m) (-\not{p}_1 + m) (\not{k}'_2 + 2m) \right], \\ \langle \Phi_{tu} \rangle &= \frac{1}{4} \text{Tr} \left[ (-\not{p}_2 - m) (\not{k}'_1 + 2m) (-\not{p}_1 + m) (\not{k}'_2 + 2m) \right], \\ \langle \Phi_{ut} \rangle &= \frac{1}{4} \text{Tr} \left[ (-\not{p}_2 - m) (\not{k}'_2 + 2m) (-\not{p}_1 + m) (\not{k}'_1 + 2m) \right]. \end{aligned} \quad (48.36)$$

We have

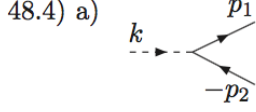
$$\begin{aligned} \langle \Phi_{tt} \rangle &= \frac{1}{4} \text{Tr} [\not{p}_2 \not{k}'_1 \not{p}_1 \not{k}'_1] + \frac{1}{4} m^2 \text{Tr} [4 \not{p}_1 \not{p}_2 + 2 \not{p}_1 \not{k}'_1 + 2 \not{p}_1 \not{k}'_1 - 2 \not{p}_2 \not{k}'_1 - 2 \not{p}_2 \not{k}'_1 - \not{k}'_1 \not{k}'_1] - m^2 \text{Tr} 1 \\ &= 2(p_1 k'_1)(p_2 k'_1) - (p_1 p_2) k_1'^2 - m^2 (4p_1 p_2 + 4p_1 k'_1 - 4p_2 k'_1 - k_1'^2) - 4m^4 \\ &= \frac{1}{2} (t - m^2 - M^2)(u - m^2 - M^2) - \frac{1}{2} (s - 2m^2) M^2 \\ &\quad - m^2 [4(m^2 - \frac{1}{2}s) + 2(t - m^2 - M^2) - 2(u - m^2 - M^2) + M^2] - 4m^4 \\ &= -\frac{1}{2} [-tu + m^2(9t + u) + 7m^4 - 8m^2 M^2 + M^4] \end{aligned} \quad (48.37)$$

and

$$\begin{aligned}
\langle \Phi_{tu} \rangle &= \frac{1}{4} \text{Tr}[\not{p}_2 \not{k}'_1 \not{p}'_1 \not{k}'_2] + \frac{1}{4} m^2 \text{Tr}[4\not{p}_1 \not{p}_2 + 2\not{p}_1 \not{k}'_1 + 2\not{p}_1 \not{k}'_2 - 2\not{p}_2 \not{k}'_1 - 2\not{p}_2 \not{k}'_2 - \not{k}'_1 \not{k}'_2] - m^2 \text{Tr} 1 \\
&= (p_1 k'_1)(p_2 k'_2) + (p_1 k'_2)(p_2 k'_1) - (p_1 p_2)(k'_1 k'_2) \\
&\quad - m^2(4p_1 p_2 + 2p_1 k'_1 + 2p_1 k'_2 - 2p_2 k'_1 - 2p_2 k'_2 - k'_1 k'_2) - 4m^4 \\
&= \frac{1}{4}(t - m^2 - M^2)^2 + \frac{1}{4}(u - m^2 - M^2)^2 - \frac{1}{4}(s - 2m^2)(s - 2M^2) \\
&\quad - m^2[4(m^2 - \frac{1}{2}s) - (M^2 - \frac{1}{2}s)] - 4m^4 \\
&= -\frac{1}{2}[tu + 3m^2(t + u) + 9m^4 - 8m^2 M^2 - M^4]. \tag{48.38}
\end{aligned}$$

The extra factor of one-half compared to the crossing-related process  $e^- \varphi \rightarrow e^- \varphi$  arises because we are summing (rather than averaging) over the final electron spin in the latter case.

3. Srednicki 48.4



For notational convenience we omit primes on the final momenta. The amplitude is then  $\mathcal{T} = g\bar{u}_1 v_2$ , and so  $\bar{\mathcal{T}} = g\bar{v}_2 u_1$ ,  $|\mathcal{T}|^2 = g^2 \text{Tr}[u_1 \bar{u}_1 v_2 \bar{v}_2]$ ,  $\langle |\mathcal{T}|^2 \rangle = g^2 \text{Tr}[(-\not{p}_1 + m)(-\not{p}_2 - m)] = g^2(-4p_1 p_2 - 4m^2) = 2g^2(M^2 - 4m^2)$ .

Using our results from problem 11.1b, we have  $\Gamma = (\langle |\mathcal{T}|^2 \rangle / 16\pi M)(1 - 4m^2/M^2)^{1/2} = (g^2 M / 8\pi)(1 - 4m^2/M^2)^{3/2}$ .

b) From eq. (38.28), with spin quantized along the  $x$ -axis we have

$$\begin{aligned} u_1 \bar{u}_1 &= \frac{1}{2}(1 - s_1 \gamma_5 \not{x})(-\not{p}_1 + m), \\ v_2 \bar{v}_2 &= \frac{1}{2}(1 - s_2 \gamma_5 \not{x})(-\not{p}_2 - m), \end{aligned} \quad (48.49)$$

and we take  $\mathbf{p}_1 = -\mathbf{p}_2 = p\hat{\mathbf{z}}$  with  $p = \frac{1}{2}M(1 - 4m^2/M^2)^{1/2}$ . Thus we have

$$\begin{aligned} |\mathcal{T}|^2 &= g^2 \text{Tr}[u_1 \bar{u}_1 v_2 \bar{v}_2] \\ &= \frac{1}{4}g^2 \text{Tr}[(1 - s_1 \gamma_5 \not{x})(-\not{p}_1 + m)(1 - s_2 \gamma_5 \not{x})(-\not{p}_2 - m)]. \end{aligned} \quad (48.50)$$

Since a trace with a single  $\gamma_5$  and three or fewer gamma matrices vanishes, we have

$$|\mathcal{T}|^2 = \frac{1}{4}g^2 \text{Tr}[(-\not{p}_1 + m)(-\not{p}_2 - m) + s_1 s_2 (\gamma_5 \not{x})(-\not{p}_1 + m)(\gamma_5 \not{x})(-\not{p}_2 - m)]. \quad (48.51)$$

Using  $\gamma_5 \not{x} = -\not{x} \gamma_5$  and  $\gamma_5^2 = 1$ , we have

$$|\mathcal{T}|^2 = \frac{1}{4}g^2 \text{Tr}[(-\not{p}_1 + m)(-\not{p}_2 - m) + s_1 s_2 \not{x}(-\not{p}_1 - m)\not{x}(-\not{p}_2 - m)]. \quad (48.52)$$

Then using  $\not{x}\not{x} = -\not{x}\not{x} - 2ab$  along with  $xp_1 = xp_2 = 0$  and  $\not{x}\not{x} = -x^2 = -1$ , we have

$$\begin{aligned} |\mathcal{T}|^2 &= \frac{1}{4}g^2 \text{Tr}[(-\not{p}_1 + m)(-\not{p}_2 - m) - s_1 s_2 (-\not{p}_1 + m)(-\not{p}_2 - m)] \\ &= \frac{1}{4}g^2(1 + s_1 s_2) \text{Tr}[(-\not{p}_1 + m)(-\not{p}_2 - m)] \\ &= \frac{1}{4}g^2(1 + s_1 s_2)(-4p_1 p_2 - 4m^2) \\ &= \frac{1}{2}g^2(1 + s_1 s_2)(M^2 - 4m^2). \end{aligned} \quad (48.53)$$

This vanishes if  $s_1 = -s_2$  or if  $M = 2m$ . Reason: since  $\bar{\Psi}\Psi$  has even parity, so must  $\varphi$ . An electron-positron pair with orbital angular momentum  $\ell$  has parity  $-(-1)^\ell$ . Thus  $\ell$  must be odd. A particle with zero three-momentum cannot have nonzero orbital angular momentum, so  $|\mathcal{T}|^2$  vanishes if  $M = 2m$ . Also, since the initial particle has spin zero, the total angular momentum must be zero. Thus there must be spin angular momentum to cancel the orbital

angular momentum, and so the spins must be aligned; thus  $|\mathcal{T}|^2$  vanishes if the spins are opposite.

c) For helicities  $s_1$  and  $s_2$ , we have

$$\begin{aligned} p_1 &= (E, 0, 0, +p) , \\ p_2 &= (E, 0, 0, -p) , \\ z_1 &= (p, 0, 0, +E)/m , \\ z_2 &= (p, 0, 0, -E)/m , \end{aligned} \tag{48.54}$$

with  $E = \frac{1}{2}M$  and  $p = \frac{1}{2}M(1 - 4m^2/M^2)^{1/2}$ . We have  $z_1 p_1 = z_2 p_2 = 0$ , and so

$$\begin{aligned} |\mathcal{T}|^2 &= \frac{1}{4}g^2 \text{Tr}[(1 - s_1 \gamma_5 \not{z}_1)(-\not{p}_1 + m)(1 - s_2 \gamma_5 \not{z}_2)(-\not{p}_2 - m)] \\ &= \frac{1}{4}g^2 \text{Tr}[(-\not{p}_1 + m)(-\not{p}_2 - m) + s_1 s_2 \not{z}_1(-\not{p}_1 - m)\not{z}_2(-\not{p}_2 - m)] \\ &= \frac{1}{4}g^2 \text{Tr}[(-\not{p}_1 + m)(-\not{p}_2 - m) + s_1 s_2 \not{z}_1(-\not{p}_1 - m)\not{z}_2(-\not{p}_2 - m)] \\ &= -g^2(p_1 p_2 + m^2) + g^2 s_1 s_2 [(z_1 p_2)(z_2 p_1) - (z_1 z_2)(p_1 p_2) + m^2 z_1 z_2] . \end{aligned} \tag{48.55}$$

From eq. (48.54), we see that  $z_1 p_2 = z_2 p_1 = -2Ep/m$  and  $z_1 z_2 = p_1 p_2 / m^2 = -(E^2 + p^2)/m^2$ . Plugging these in, we find

$$|\mathcal{T}|^2 = \frac{1}{2}g^2(1 + s_1 s_2)(M^2 - 4m^2) . \tag{48.56}$$

There can be no orbital angular momentum parallel to the linear momentum, and so the  $z$  component of the spin angular momentum must vanish. The total spin along the  $\hat{z}$  axis is  $s_1 - s_2$ , and so the helicities must be the same to get a nonzero  $|\mathcal{T}|^2$ . Parity again explains why  $|\mathcal{T}|^2 = 0$  if  $M = 2m$ .

d) Now the amplitude is  $\mathcal{T} = ig\bar{u}_1 \gamma_5 v_2$ , and so  $\bar{\mathcal{T}} = ig\bar{v}_2 \gamma_5 u_1$ ,  $|\mathcal{T}|^2 = -g^2 \text{Tr}[v_2 \bar{v}_2 \gamma_5 u_1 \bar{u}_1 \gamma_5]$ ,  $\langle |\mathcal{T}|^2 \rangle = -g^2 \text{Tr}[(-\not{p}_2 - m)\gamma_5(-\not{p}_1 + m)\gamma_5] = -g^2 \text{Tr}[(-\not{p}_2 - m)(\not{p}_1 + m)] = -g^2(4p_1 p_2 - 4m^2) = 2g^2 M^2$ . This is larger by a factor of  $M^2/(M^2 - 4m^2)$ . It is larger because  $i\bar{\Psi}\gamma_5\Psi$  has odd parity, and therefore so must  $\varphi$ . Thus the orbital angular momentum of the electron-positron pair must be even, and in particular must be zero, since  $\ell = 2$  or larger could not be cancelled by spin. With zero orbital angular momentum,  $|\mathcal{T}|^2$  need not vanish for zero electron three-momentum, leading to a larger decay rate.

e) Redoing part (b) yields

$$\begin{aligned} |\mathcal{T}|^2 &= -\frac{1}{4}g^2 \text{Tr}[(1 - s_1 \gamma_5 \not{z}_1)(-\not{p}_1 + m)\gamma_5(1 - s_2 \gamma_5 \not{z}_2)(-\not{p}_2 - m) - \gamma_5] \\ &= -\frac{1}{4}g^2 \text{Tr}[(1 - s_1 \gamma_5 \not{z}_1)(-\not{p}_1 + m)(1 + s_2 \gamma_5 \not{z}_2)(\not{p}_2 - m)] \end{aligned} \tag{48.57}$$

Comparing with the second line of eq. (48.50), we see that eq. (48.57) has an extra overall minus sign,  $s_2 \rightarrow -s_2$ , and  $p_2 \rightarrow -p_2$ . Therefore, comparing with the third line of eq. (48.53), we get

$$\begin{aligned} |\mathcal{T}|^2 &= -\frac{1}{4}g^2(1 - s_1 s_2)(4p_1 p_2 - 4m^2) \\ &= \frac{1}{2}g^2(1 - s_1 s_2)M^2 . \end{aligned} \tag{48.58}$$

This vanishes if  $s_1 = s_2$ . We know the electron-positron pair has even orbital angular momentum, and the total angular momentum must be zero. The only possibility is  $\ell = 0$  and  $s = s_1 + s_2 = 0$ , so  $|\mathcal{T}|^2$  vanishes if  $s_1 = s_2$ .

Redoing part (c) yields the same changes. Therefore, comparing with the last line of eq. (48.55) yields

$$\begin{aligned}
 |\mathcal{T}|^2 &= g^2(-p_1 p_2 + m^2) + g^2 s_1 s_2 [-(z_1 p_2)(z_2 p_1) + (z_1 z_2)(p_1 p_2) + m^2 z_1 z_2] \\
 &= \frac{1}{2} g^2 (1 + s_1 s_2) M^2 .
 \end{aligned}
 \tag{48.59}$$

As in part (c), there can be no orbital angular momentum parallel to the linear momentum, and so the  $z$  component of the spin angular momentum must vanish. The total spin along the  $\hat{z}$  axis is  $s_1 - s_2$ , and so the helicities must be the same to get a nonzero  $|\mathcal{T}|^2$ .