All from the Srednicki solutions manual:

10.3) The vertex joins one dashed and two solid lines, with one arrow pointing towards the vertex and one away. The vertex factor is ig.

10.4) Using the method of problem 10.1, the vertex factor for three lines with arrows all pointing towards the vertex can be determined from the free-field theory matrix element

$$\begin{aligned}\langle 0|\varphi\partial^\mu\varphi\partial_\mu\varphi|k_1k_2k_3\rangle &= \partial_2\!\cdot\!\partial_3\langle 0|\varphi(x_1)\varphi(x_2)\varphi(x_3)|k_1k_2k_3\rangle\Big|_{x_1=x_2=x_3=x} \\ &= \partial_2\!\cdot\!\partial_3\Big[e^{i(k_1x_1+k_2x_2+k_3x_3)} + 5 \text{ perms of } k_i\text{'s}\Big]_{x_1=x_2=x_3=x} \\ &= i^2(2k_2\!\cdot\! k_3 + 2k_3\!\cdot\! k_1 + 2k_1\!\cdot\! k_2)e^{i(k_1+k_2+k_3)x}\ . \qquad (10.18)\end{aligned}$$

The vertex factor is then $\frac{1}{2}ig$ times the coefficient of the plane-wave factor on the right-hand side of eq. (10.18). Since $k_1+k_2+k_3=0$, we have $(k_1+k_2+k_3)^2=0$, and therefore the factor in parentheses on the the right-hand side of eq. (10.18) can be rewritten as $-(k_1^2+k_2^2+k_3^2)$. The overall vertex factor, for three lines with arrows all pointing towards the vertex, is then $\frac{1}{2}ig(k_1^2+k_2^2+k_3^2)$.

10.5) We take $\varphi \to \varphi + \lambda\varphi^2$. The lagrangian becomes

$$\begin{aligned}\mathcal{L} &= -\tfrac{1}{2}\partial^\mu(\varphi+\lambda\varphi^2)\partial_\mu(\varphi+\lambda\varphi^2) - \tfrac{1}{2}m^2(\varphi+\lambda\varphi^2)^2 \\ &= -\tfrac{1}{2}\partial^\mu\varphi\partial_\mu\varphi - \tfrac{1}{2}m^2\varphi^2 - 2\lambda\varphi\partial^\mu\varphi\partial_\mu\varphi - \lambda m^2\varphi^3 - 2\lambda^2\varphi^2\partial^\mu\varphi\partial_\mu\varphi - \tfrac{1}{2}\lambda^2m^2\varphi^4\ . \qquad (10.19)\end{aligned}$$

Using our results from problem 10.4, the three-point vertex factor is

$$\begin{aligned}\mathbf{V}_3 &= (-2i\lambda)(k_1^2+k_2^2+k_3^2) - 6i\lambda m^2 \\ &= (-2i\lambda)[(k_1^2+m^2)+(k_2^2+m^2)+(k_3^2+m^2)]\ , \qquad (10.20)\end{aligned}$$

and the four-point vertex factor is

$$\begin{aligned}\mathbf{V}_4 &= (-2i\lambda^2)(2!)(k_1^2+k_2^2+k_3^2+k_4^2) - 12i\lambda m^2 \\ &= (-4i\lambda^2)[(k_1^2+m^2)+\ldots+(k_4^2+m^2)] + 4i\lambda^2m^2\ , \qquad (10.21)\end{aligned}$$

where all momentum arrows point towards the vertex. The factor of 2! in the first term in $\mathbf{V}_4$ comes from matching external momenta with the two φ's without derivatives.

Now consider $\varphi\varphi \to \varphi\varphi$ scattering. We have the diagrams of fig. 10.2, plus a four-point vertex. In these diagrams, each three-point vertex connects two on-shell external lines with $k_i^2=-m^2$, and one internal line. In the s-channel diagram, the internal line has $k_i^2=-s$; thus each vertex in this diagram has the value $\mathbf{V}_3=(-2i\lambda)(-s+m^2)$. For the t- and u-channel diagrams, s is replaced by t or u. In the four-point diagram, all lines are external and on-shell, and so the value of the four-point vertex is $\mathbf{V}_4=4i\lambda^2m^2$. We therefore have

$$\begin{aligned}i\mathcal{T} &= [(-2i\lambda)(-s+m^2)]^2\,\frac{1}{i}\,\frac{1}{-s+m^2} + (s\to t) + (s\to u) + 4i\lambda^2m^2 \\ &= 4i\lambda^2[(-s+m^2)+(-t+m^2)+(-u+m^2)+m^2] \\ &= 4i\lambda^2(-s-t-u+4m^2) \\ &= 0\ . \qquad (10.22)\end{aligned}$$