Week 2 (due Oct. 17)

Reading: Srednicki, sections 1 and 3.

1. Consider free nonrelativistic bosons (in Fock space formalism).

(a) Compute the commutator of the creation and annihilation operators

$$[\Psi(t,\vec{x}),\Psi^\dagger(t',\vec{x}')]$$

for arbitrary t,t' and $\vec{x},\vec{x}'$. (Hint: use the solution for Ψ and $\Psi^\dagger$ in terms of time-independent creation-annihilation operators in momentum space a_p and $a_p^\dagger$).

(b) Consider the operator

$$P_k(t) = -i\int d^3x \Psi(t,\vec{x})^\dagger \partial_k \Psi(t,\vec{x}), \quad k=1,2,3.$$

Show that $[H,P_k]=0$, i.e. P_k is an integral of motion. Show that

$$[P_k,\Psi(t,\vec{x})] = i\partial_k\Psi(t,\vec{x}), \quad [P_k,\Psi(t,\vec{x})^\dagger] = i\partial_k\Psi(t,\vec{x})^\dagger.$$

(This means that P_k is the generator of spatial translations, i.e. the momentum operator).

2. The Hamiltonian for the free complex scalar is given by

$$H = \int d^3x \left(pp^\dagger + \partial_i\phi^\dagger\partial_i\phi + m^2\phi^\dagger\phi\right).$$

The equal-time commutation relations are

$$\begin{aligned}
[\phi(t,\vec{x}),p(t,\vec{y})] &= i\delta^3(\vec{x}-\vec{y}),\\
[\phi(t,\vec{x})^\dagger,p(t,\vec{y})^\dagger] &= i\delta^3(\vec{x}-\vec{y}),\\
[\phi(t,\vec{x})^\dagger,p(t,\vec{y})] &= 0,\\
[\phi(t,\vec{x}),p(t,\vec{y})^\dagger] &= 0,\\
[\phi(t,\vec{x}),\phi(t,\vec{y})] &= 0,\\
[\phi(t,\vec{x})^\dagger,\phi(t,\vec{y})^\dagger] &= 0,\\
[\phi(t,\vec{x}),\phi(t,\vec{y})^\dagger] &= 0,\\
[p(t,\vec{x}),p(t,\vec{y})] &= 0,\\
[p(t,\vec{x})^\dagger,p(t,\vec{y})^\dagger] &= 0,\\
[p(t,\vec{x}),p(t,\vec{y})^\dagger] &= 0.
\end{aligned}$$

Show that the Heisenberg equations of motion

$$i\partial_0\phi(x) = [\phi(x), H], \quad i\partial_0 p(x) = [p(x), H]$$

are equivalent to the Klein-Gordon equation for $\phi(x)$.