Week 6 (due Nov. 14)

Reading: Srednicki, sections 8,9,10.

1. (a) Consider the free real scalar field (with mass m). Use the path-integral representation to compute the expectation values

$$A = \langle 0|e^{ia\phi(x)}|0\rangle, \quad B(x-y) = \langle 0|T\left(e^{ia\phi(x)}e^{ib\phi(y)}\right)|0\rangle.$$

Here a and b are numbers. Show that the result is infinite. Show nevertheless that the ratio $B(x-y)/A^2$ is finite and evaluate it explicitly in the special case $m = 0$.

(b) Consider the free real scalar field ϕ in two dimensions (i.e. one spatial dimension, one time dimension). Show that the propagator for small $(x-y)^2$ behaves as $\log(x-y)^2$ and use this fact to evaluate the short-distance behavior of $B(x-y)/A^2$. If a two-point function $\langle 0|O(x)O^\dagger(0)|0\rangle$ of an operator $O(x)$ behaves for small x^2 as $(x^2)^{-d}$, then one says that $O(x)$ has short-distance scaling dimension d. Find the short-distance scaling dimension of the operator $e^{ia\phi}$ in the free scalar theory.

(c) Show that in the limit $m \to 0$ the propagator for the free scalar field in two dimensions diverges. Show that nevertheless the two-point function

$$\langle 0|T\left(\partial_\mu\phi(x)\partial_\nu\phi(y)\right)|0\rangle$$

is well-defined even for $m = 0$. Compute this two-point function.

2. Consider the two-point function for the free real scalar field of mass m:

$$G(x-y) = i\langle 0|T\left(\phi(x)\phi(y)\right)|0\rangle.$$

We have shown by an explicit computation that $G(x-y)$ is the Green's function for the Klein-Gordon equation:

$$(-\partial_x^2 + m^2)G(x-y) = \delta^4(x-y).$$

Rederive this fact without using the Fourier representation of $G(x-y)$; instead, write the time-ordered product in terms of ordinary products multiplied by appropriate step-functions, act with the Klein-Gordon operator, and make use of equation of motion for ϕ and the equal-time commutation relations for ϕ and $\dot{\phi}$.

3. Problem 9.2 (abc) in Srednicki.

4. Problem 9.3 (ab) in Srednicki.