Week 8 (due Nov. 23)

1. (a) Let us modify the Dirac Lagrangian by adding a term

$$\Delta L = \frac{ae}{2m}\bar{\psi}\sigma^{\mu\nu}\psi\, F_{\mu\nu},$$

where a is a constant, e electric charge, $F_{\mu\nu}$ is the electromagnetic field-strength tensor, and $\sigma^{\mu\nu}$ is defined by the usual expression

$$\sigma^{\mu\nu} = \frac{i}{4}[\gamma^{\mu}, \gamma^{\nu}].$$

(a) Consider the nonrelativistic limit of the modified Dirac equation and show that the resulting modification of the Pauli equation amounts to changing the magnetic moment of the electron. Express the modified gyromagnetic ration in terms of a.

(b) Now let modify the Dirac Lagrangian by adding a very-similar looking term:

$$\Delta L = \frac{ibe}{2m}\bar{\psi}\gamma_5\sigma^{\mu\nu}\psi\, F_{\mu\nu},$$

where b is a constant. Find the corresponding modification of the Pauli equation. What is its physical meaning?

2. Consider the Dirac equation in a constant magnetic field, say in the z-direction. Find a complete set of solutions of the Dirac equation and determine the energies of one-particle excitations (both for electrons and positrons). Express the energy of one-particle excitation in terms of the component of momentum in the z-direction and other quantum numbers.