Week 1 (due April 9)

Reading: Srednicki, sections 54, 55.

1. In any spacetime dimension $n > 1$ one can define a theory of a massless vector field A_μ by letting

$$L = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.$$

This Lagrangian is invariant under gauge transformations

$$A_\mu \mapsto A_\mu + \partial_\mu f,$$

where f is an arbitrary real function.

(a) Quantize this theory in the Coulomb gauge and show that it describes massless particles with $n-2$ polarization states.

(b) In particular, show that for $n = 2$ the only solution of the classical equations of motion is $F_{12} = const$, and that for fixed F_{12} A_μ is defined uniquely up to a freedom to make gauge transformations.

(c) For $n = 3$ this theory is actually equivalent to a theory of a free massless scalar. Prove this by showing that there exists a scalar σ such that $F_{\mu\nu} = \epsilon_{\mu\nu\rho}\partial^\rho \sigma$, that σ satisfies the equation

$$\partial_\mu \partial^\mu \sigma = 0,$$

and that the equal-time commutation relations for σ are the standard ones.

2. Compute the equal-time commutation relations between electric and magnetic fields in 4d electrodynamics. Use this to deduce the uncertainty relations for smeared electric and magnetic fields. That is, pick two smooth vector fields $a_i(\mathbf{x})$ and $b_i(\mathbf{x})$ on $\mathbb{R}^3$, define smeared electric and magnetic fields by

$$E_a = \int d^3x E_i(\mathbf{x}) a_i(\mathbf{x}), \quad B_b = \int d^3x B_i(\mathbf{x}) b_i(\mathbf{x}),$$

compute their commutators, and deduce the uncertainty relations in the usual way.